12.2 Chemical Calculations

Connecting to Your World Air bags inflate almost instantaneously upon a car's impact. The effectiveness of air bags is based on the rapid conversion of a small mass of sodium azide into a large volume of gas. The gas fills an air bag, preventing the driver from hitting the steering wheel or dashboard. The entire reaction occurs in less than a second. In this section you will learn how to use a balanced chemical equation to calculate the amount of product formed in a chemical reaction.

Guide for Reading

Key Concepts

- How are mole ratios used in chemical calculations?
- What is the general procedure for solving a stoichiometric problem?

Vocabulary

mole ratio

Reading Strategy

Relating Text and Visuals As you read, look closely at Figure 12.8. Explain how this illustration helps you understand the relationship between known and unknown quantities in a stoichiometric problem.

Writing and Using Mole Ratios

As you just learned, a balanced chemical equation provides a great deal of quantitative information. It relates particles (atoms, molecules, formula units), moles of substances, and masses. A balanced chemical equation also is essential for all calculations involving amounts of reactants and products. For example, suppose you know the number of moles of one substance. The balanced chemical equation allows you to determine the number of moles of all other substances in the reaction.

Look again at the balanced equation for the production of ammonia from nitrogen and hydrogen:

$$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$$

The most important interpretation of this equation is that 1 mol of nitrogen reacts with 3 mol of hydrogen to form 2 mol of ammonia. Based on this interpretation, you can write ratios that relate moles of reactants to moles of product. A **mole ratio** is a conversion factor derived from the coefficients of a balanced chemical equation interpreted in terms of moles. **In chemical calculations, mole ratios are used to convert between moles of reactant and moles of product, between moles of reactants, or between moles of products.** Three mole ratios derived from the balanced equation above are:

$$\frac{1 \text{ mol } N_2}{3 \text{ mol } H_2} \qquad \frac{2 \text{ mol } NH_3}{1 \text{ mol } N_2} \qquad \frac{3 \text{ mol } H_2}{2 \text{ mol } NH_3}$$

Mole-Mole Calculations In the mole ratio below, W is the unknown quantity and G is the given quantity. The values of a and b are the coefficients from the balanced equation. Thus a general solution for a mole-mole problem, such as Sample Problem 12.2, is given by

$$\underbrace{x\,\cancel{\text{mol } G}}_{\text{Given}} \times \underbrace{\frac{b \text{ mol } W}{a\,\cancel{\text{mol } G}}}_{\text{Mole ratio}} = \underbrace{\frac{xb}{a} \text{ mol } W}_{\text{Calculated}}$$

Figure 12.4 Manufacturing plants produce ammonia by combining nitrogen with hydrogen. Ammonia is used in cleaning products, fertilizers, and in the manufacture of other chemicals.

Figure 12.5 To determine the number of moles in a sample of a compound, first measure the mass of the sample. Then use the molar mass to calculate the number of moles in that mass.

Math Handbook

For help with dimensional analysis, go to page R66.

Interactive Textbook

Problem-Solving 12.12 Solve Problem 12 with the help of an interactive guided tutorial.

with ChemASAP

SAMPLE PROBLEM 12.2

Calculating Moles of a Product

How many moles of ammonia are produced when 0.60 mol of nitrogen reacts with hydrogen?

1 Analyze ***List the known and the unknown.***

Known

- moles of nitrogen = 0.60 mol N_2

Unknown

- moles of ammonia = ? mol NH_3

The conversion is mol $N_2 \longrightarrow$ mol NH_3. According to the balanced equation, 1 mol N_2 combines with 3 mol H_2 to produce 2 mol NH_3. To determine the number of moles of NH_3, the given quantity of N_2 is multiplied by the form of the mole ratio from the balanced equation that allows the given unit to cancel. This mole ratio is 2 mol NH_3/1 mol N_2.

2 Calculate ***Solve for the unknown.***

$$0.60\ \cancel{\text{mol } N_2} \times \frac{2 \text{ mol } NH_3}{1\ \cancel{\text{mol } N_2}} = 1.2 \text{ mol } NH_3$$

3 Evaluate ***Does the result make sense?***

The ratio of 1.2 mol NH_3 to 0.60 mol N_2 is 2:1, as predicted by the balanced equation.

Practice Problems

11. This equation shows the formation of aluminum oxide, which is found on the surface of aluminum objects exposed to the air.

$$4Al(s) + 3O_2(g) \longrightarrow 2Al_2O_3(s)$$

a. Write the six mole ratios that can be derived from this equation.

b. How many moles of aluminum are needed to form 3.7 mol Al_2O_3?

12. According to the equation in Problem 11:

a. How many moles of oxygen are required to react completely with 14.8 mol Al?

b. How many moles of Al_2O_3 are formed when 0.78 mol O_2 reacts with aluminum?

Mass-Mass Calculations No laboratory balance can measure substances directly in moles. Instead, the amount of a substance is usually determined by measuring its mass in grams, as shown in Figure 12.5. From the mass of a reactant or product, the mass of any other reactant or product in a given chemical equation can be calculated. The mole interpretation of a balanced equation is the basis for this conversion. If the given sample is measured in grams, the mass can be converted to moles by using the molar mass. Then the mole ratio from the balanced equation can be used to calculate the number of moles of the unknown. If it is the mass of the unknown that needs to be determined, the number of moles of the unknown can be multiplied by the molar mass. As in mole-mole calculations, the unknown can be either a reactant or a product.

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SAMPLE PROBLEM 12.3

Calculating the Mass of a Product

Calculate the number of grams of NH_3 produced by the reaction of 5.40 g of hydrogen with an excess of nitrogen. The balanced equation is

$$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$$

1 Analyze ***List the knowns and the unknown.***

Knowns

- mass of hydrogen = 5.40 g H_2
- 3 mol H_2 = 2 mol NH_3 (from balanced equation)
- 1 mol H_2 = 2.0 g H_2 (molar mass)
- 1 mol NH_3 = 17.0 g NH_3 (molar mass)

Unknown

- mass of ammonia = ? g NH_3

The mass in grams of hydrogen will be used to find the mass in grams of ammonia:

$$\text{g } H_2 \longrightarrow \text{g } NH_3$$

The following steps are necessary to determine the mass of ammonia:

$$\text{g } H_2 \longrightarrow \text{mol } H_2 \longrightarrow \text{mol } NH_3 \longrightarrow \text{g } NH_3$$

The coefficients of the balanced equation show that 3 mol H_2 reacts with 1 mol N_2 to produce 2 mol NH_3. The mole ratio relating mol NH_3 to mol H_2 is 2 mol NH_3/3 mol H_2.

2 Calculate ***Solve for the unknown.***

This following series of calculations can be combined:

$$\text{g } H_2 \longrightarrow \text{mol } H_2 \longrightarrow \text{mol } NH_3 \longrightarrow \text{g } NH_3$$

$$\underbrace{5.40\ \cancel{\text{g } H_2}}_{\text{Given quantity}} \times \underbrace{\frac{1\ \cancel{\text{mol } H_2}}{2.0\ \cancel{\text{g } H_2}}}_{\text{Change given unit to moles}} \times \underbrace{\frac{2\ \cancel{\text{mol } NH_3}}{3\ \cancel{\text{mol } H_2}}}_{\text{Mole ratio}} \times \underbrace{\frac{17.0\text{ g } NH_3}{1\ \cancel{\text{mol } NH_3}}}_{\text{Change moles to grams}} = 31\text{ g } NH_3$$

3 Evaluate ***Does the result make sense?***

Because there are three conversion factors involved in this solution, it is more difficult to estimate an answer. However, because the molar mass of NH_3 is substantially greater than the molar mass of H_2, the answer should have a larger mass than the given mass. The answer should have two significant figures.

Practice Problems

13. Acetylene gas (C_2H_2) is produced by adding water to calcium carbide (CaC_2).

$$CaC_2(s) + 2H_2O(l) \longrightarrow C_2H_2(g) + Ca(OH)_2(aq)$$

How many grams of acetylene are produced by adding water to 5.00 g CaC_2?

14. Using the same equation, determine how many moles of CaC_2 are needed to react completely with 49.0 g H_2O.

CHEMath

Significant Figures

The significant figures in a measurement are all the digits known with certainty plus one estimated digit. The number of significant figures in the measurements used in a calculation determines how you round the answer.

When multiplying and dividing measurements, the rounded answer can have no more significant figures than the least number of significant figures in any measurement in the calculation.

The product of 3.6 m × 2.48 m = 8.928 m^2 is rounded to 8.9 m^2 (2 significant figures).

When adding and subtracting measurements, the answer can have no more decimal places than the least number of decimal places in any measurement in the problem. The difference of 8.78 cm − 2.2 cm = 6.58 cm is rounded to 6.6 cm (one decimal place).

Math Handbook

For help with significant figures, go to page R59.

Interactive Textbook

Problem-Solving 12.13 Solve Problem 13 with the help of an interactive guided tutorial.

with ChemASAP

Figure 12.6 In this Hubble Space Telescope image, clouds of condensed ammonia are visible covering the surface of Saturn.

If the law of conservation of mass is true, how is it possible to make 31 g NH_3 from only 5.40 g H_2? Looking back at the equation for the reaction, you will see that hydrogen is not the only reactant. Another reactant, nitrogen, is also involved. If you were to calculate the number of grams of nitrogen needed to produce 31 g NH_3 and then compare the total masses of reactants and products, you would have an answer to this question. Go ahead and try it!

Mass-mass problems are solved in basically the same way as mole-mole problems. Figure 12.7 reviews the steps for the mass-mass conversion of any given mass (G) and any wanted mass (W).

Steps in Solving a Mass-Mass Problem

1. Change the mass of G to moles of G (mass $G \longrightarrow$ mol G) by using the molar mass of G.

$$\text{mass } G \times \frac{1 \text{ mol } G}{\text{molar mass } G} = \text{mol } G$$

2. Change the moles of G to moles of W (mol G $\longrightarrow$ mol W) by using the mole ratio from the balanced equation.

$$\text{mol } G \times \frac{b \text{ mol } W}{a \text{ mol } G} = \text{mol } W$$

3. Change the moles of W to grams of W (mol W $\longrightarrow$ mass W) by using the molar mass of W.

$$\text{mol } W \times \frac{\text{molar mass } W}{1 \text{ mol } W} = \text{mass } W$$

Figure 12.7 also shows the steps for doing mole-mass and mass-mole stoichiometric calculations. For a mole-mass problem, the first conversion (from mass to moles) is skipped. For a mass-mole problem, the last conversion (from moles to mass) is skipped. You can use parts of the three-step process shown in Figure 12.7 as they are appropriate to the problem you are solving.

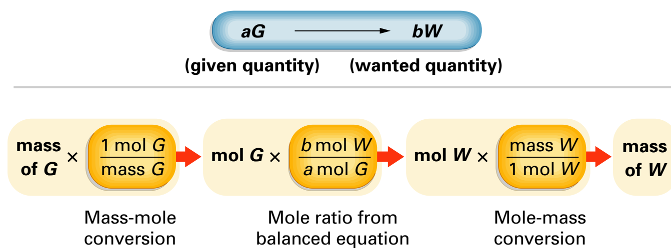

Figure 12.7 This general solution diagram indicates the steps necessary to solve a mass-mass stoichiometry problem: convert mass to moles, use the mole ratio, and then convert moles to mass. **Inferring** ***Is the given always a reactant?***

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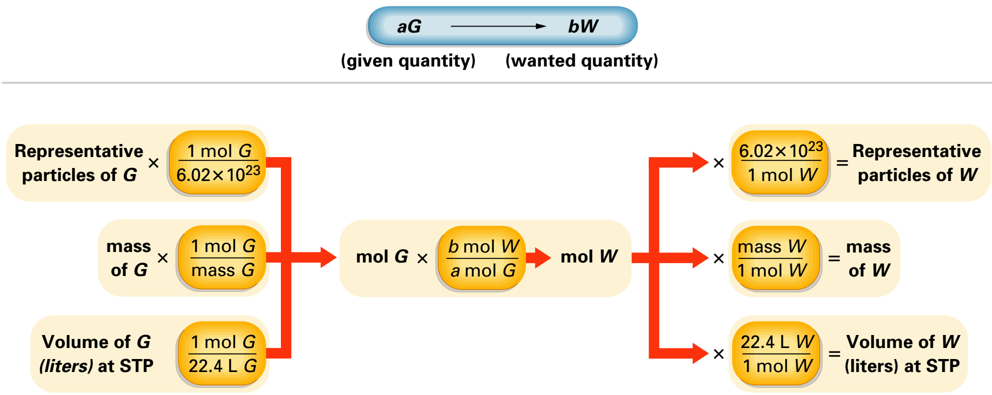


Figure 12.8 With your knowledge of conversion factors and this problem-solving approach, you can solve a variety of stoichiometric problems.
Identifying ***What conversion factor is used to convert moles to representative particles?***

Other Stoichiometric Calculations

As you already know, you can obtain mole ratios from a balanced chemical equation. From the mole ratios, you can calculate any measurement unit that is related to the mole. The given quantity can be expressed in numbers of representative particles, units of mass, or volumes of gases at STP. The problems can include mass-volume, particle-mass and volume-volume calculations. For example, you can use stoichiometry to relate volumes of reactants and products in the reaction shown in Figure 12.8. **In a typical stoichiometric problem, the given quantity is first converted to moles. Then the mole ratio from the balanced equation is used to calculate the number of moles of the wanted substance. Finally, the moles are converted to any other unit of measurement related to the unit mole, as the problem requires.**

Thus far, you have learned how to use the relationship between moles and mass (1 mol = molar mass) in solving mass-mass, mass-mole, and mole-mass stoichiometric problems. The mole-mass relationship gives you two conversion factors.

$$\frac{1\ \text{mol}}{\text{molar mass}} \quad \text{and} \quad \frac{\text{molar mass}}{1\ \text{mol}}$$

Recall from Chapter 10 that the mole can be related to other quantities as well. For example, 1 mol = 6.02×10^{23} representative particles, and 1 mol of a gas = 22.4 L at STP. These two relationships provide four more conversion factors that you can use in stoichiometric calculations.

$$\frac{1\ \text{mol}}{6.02 \times 10^{23}\ \text{particles}} \quad \text{and} \quad \frac{6.02 \times 10^{23}\ \text{particles}}{1\ \text{mol}}$$

$$\frac{1\ \text{mol}}{22.4\ \text{L}} \quad \text{and} \quad \frac{22.4\ \text{L}}{1\ \text{mol}}$$

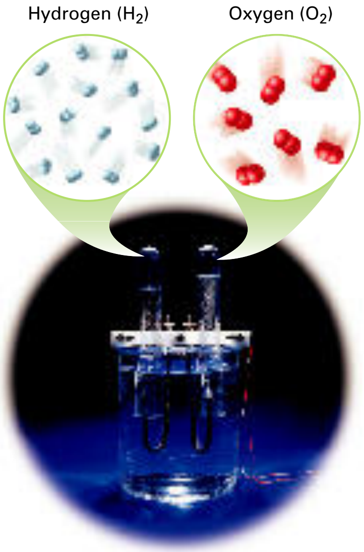
Figure 12.8 summarizes the steps for a typical stoichiometric problem. Notice that the units of the given quantity will not necessarily be the same as the units of the wanted quantity. For example, given the mass of *G*, you might be asked to calculate the volume of *W* at STP.

Checkpoint ***What conversion factors can you write based on the mole-mass and mole-volume relationships?***

Interactive Textbook

Simulation 13 Strengthen your analytical skills by solving stoichiometric problems.

with ChemASAP



The electrolysis of water causes it to decompose into hydrogen and oxygen.

Math Handbook

For help with dimensional analysis, go to page R66.

SAMPLE PROBLEM 12.4

Calculating Molecules of a Product

How many molecules of oxygen are produced when 29.2 g of water is decomposed by electrolysis according to this balanced equation?

$$2H_2O(l) \xrightarrow{\text{electricity}} 2H_2(g) + O_2(g)$$

1 Analyze ***List the knowns and the unknown.***

Knowns

- mass of water = 29.2 g H_2O
- 2 mol H_2O = 1 mol O_2 (from balanced equation)
- 1 mol H_2O = 18.0 g H_2O (molar mass)
- 1 mol O_2 = 6.02×10^{23} molecules O_2

Unknown

- molecules of oxygen = ? molecules O_2

The following calculations need to be done:

$$\text{g } H_2O \longrightarrow \text{mol } H_2O \longrightarrow \text{mol } O_2 \longrightarrow \text{molecules } O_2$$

The appropriate mole ratio relating mol O_2 to mol H_2O from the balanced equation is 1 mole O_2/2 mol H_2O.

2 Calculate ***Solve for the unknown.***

$$\underbrace{29.2\ \cancel{\text{g } H_2O}}_{\text{Given quantity}} \times \underbrace{\frac{1\ \cancel{\text{mol } H_2O}}{18.0\ \cancel{\text{g } H_2O}}}_{\text{Change to moles}} \times \underbrace{\frac{1\ \cancel{\text{mol } O_2}}{2\ \cancel{\text{mol } H_2O}}}_{\text{Mole ratio}} \times \underbrace{\frac{6.02 \times 10^{23}\text{ molecules } O_2}{1\ \cancel{\text{mol } O_2}}}_{\text{Change to molecules}}$$

$$= 4.88 \times 10^{23}\text{ molecules } O_2$$

3 Evaluate ***Does the result make sense?***

The given mass of water should produce a little less than 1 mol of oxygen, or a little less than Avogadro's number of molecules. The answer should have three significant figures.

Practice Problems

15. How many molecules of oxygen are produced by the decomposition of 6.54 g of potassium chlorate ($KClO_3$)?

$$2KClO_3(s) \longrightarrow 2KCl(s) + 3O_2(g)$$

16. The last step in the production of nitric acid is the reaction of nitrogen dioxide with water.

$$3NO_2(g) + H_2O(l) \longrightarrow 2HNO_3(aq) + NO(g)$$

How many grams of nitrogen dioxide must react with water to produce 5.00×10^{22} molecules of nitrogen monoxide?

Interactive Textbook

Problem-Solving 12.15 Solve Problem 15 with the help of an interactive guided tutorial.

with ChemASAP

The coefficients in a chemical equation indicate the relative number of particles and the relative number of moles of reactants and products. For a reaction involving gaseous reactants or products, the coefficients also indicate relative amounts of each gas. As a result, you can use volume ratios in the same way you have used mole ratios.

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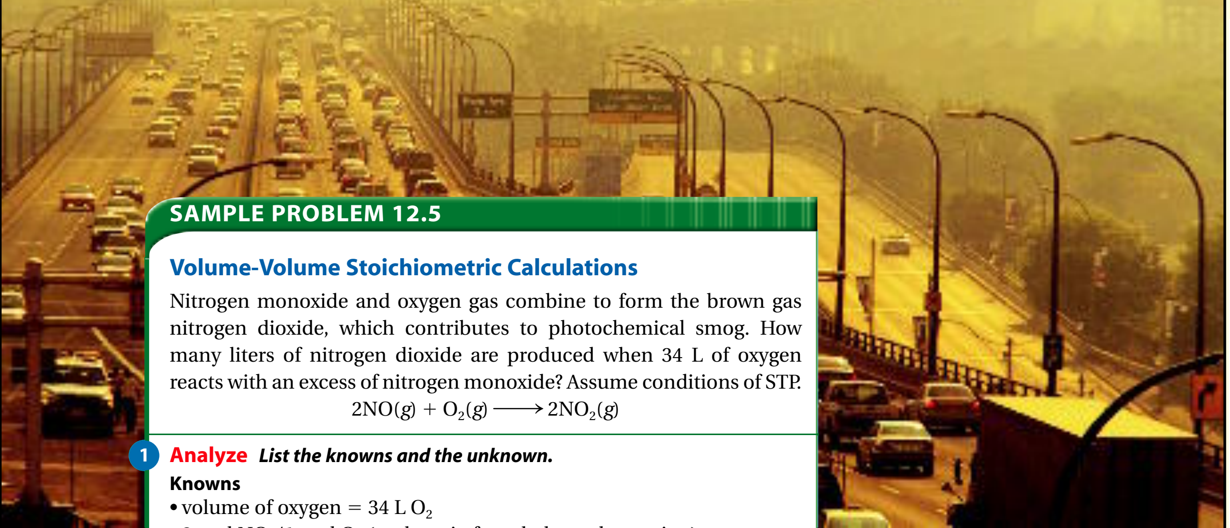

SAMPLE PROBLEM 12.5

Volume-Volume Stoichiometric Calculations

Nitrogen monoxide and oxygen gas combine to form the brown gas nitrogen dioxide, which contributes to photochemical smog. How many liters of nitrogen dioxide are produced when 34 L of oxygen reacts with an excess of nitrogen monoxide? Assume conditions of STP.

$$2NO(g) + O_2(g) \longrightarrow 2NO_2(g)$$

1 Analyze ***List the knowns and the unknown.***

Knowns

- volume of oxygen = 34 L O_2
- 2 mol NO_2/1 mol O_2 (mole ratio from balanced equation)
- 1 mol O_2 = 22.4 L O_2 (at STP)
- 1 mol NO_2 = 22.4 L NO_2 (at STP)

Unknown

- volume of nitrogen dioxide = ? L NO_2

2 Calculate ***Solve for the unknown.***

$$\underbrace{34\ \cancel{\text{L}\ O_2}}_{\text{Given quantity}} \times \underbrace{\frac{1\ \cancel{\text{mol}\ O_2}}{22.4\ \cancel{\text{L}\ O_2}}}_{\text{Change to moles}} \times \underbrace{\frac{2\ \cancel{\text{mol}\ NO_2}}{1\ \cancel{\text{mol}\ O_2}}}_{\text{Mole ratio}} \times \underbrace{\frac{22.4\ \text{L}\ NO_2}{1\ \cancel{\text{mol}\ NO_2}}}_{\text{Change to liters}} = 68\ \text{L}\ NO_2$$

3 Evaluate ***Does the result make sense?***

Because 2 mol NO_2 is produced for each 1 mol O_2 that reacts, the volume of NO_2 should be twice the given volume of O_2. The answer should have two significant figures.

Practice Problems

17. The equation for the combustion of carbon monoxide is

$$2CO(g) + O_2(g) \longrightarrow 2CO_2(g)$$

How many liters of oxygen are required to burn 3.86 L of carbon monoxide?

18. Phosphorus and hydrogen can be combined to form phosphine (PH_3).

$$P_4(s) + 6H_2(g) \longrightarrow 4PH_3(g)$$

How many liters of phosphine are formed when 0.42 L of hydrogen reacts with phosphorus?

Interactive Textbook

Problem-Solving 12.18 Solve Problem 18 with the help of an interactive guided tutorial.

with **ChemASAP**

Did you notice that in Sample Problem 12.5 the 22.4 L/mol factors canceled out? This will always be true in a volume-volume problem. Remember that coefficients in a balanced chemical equation indicate the relative numbers of moles. The coefficients also indicate the relative volumes of interacting gases.

SAMPLE PROBLEM 12.6

Finding the Volume of a Gas Needed for a Reaction

Assuming STP, how many milliliters of oxygen are needed to produce 20.4 mL SO_3 according to this balanced equation?

$$2SO_2(g) + O_2(g) \longrightarrow 2SO_3(g)$$

1 Analyze ***List the knowns and the unknown.***

Knowns

- volume of sulfur trioxide = 20.4 mL
- 2 mL SO_3/1 mL O_2 (volume ratio from balanced equation)

Unknown

- volume of oxygen = ? mL O_2

2 Calculate ***Solve for the unknown.***

$$20.4\ \cancel{\text{mL } SO_3} \times \frac{1 \text{ mL } O_2}{2\ \cancel{\text{mL } SO_3}} = 10.2 \text{ mL } O_2$$

3 Evaluate ***Does the result make sense?***

Because the volume ratio is 2 volumes SO_3 to 1 volume O_2, the volume of O_2 should be half the volume of SO_3. The answer should have three significant figures.

Math Handbook

For help with dimensional analysis, go to page R66.

Practice Problems

Consider this equation:

$$CS_2(l) + 3O_2(g) \longrightarrow CO_2(g) + 2SO_2(g)$$

19. Calculate the volume of sulfur dioxide produced when 27.9 mL O_2 reacts with carbon disulfide.

20. How many deciliters of carbon dioxide are produced when 0.38 L SO_2 is formed?

Interactive Textbook

Problem-Solving 12.19 Solve Problem 19 with the help of an interactive guided tutorial.

with ChemASAP

12.2 Section Assessment

21. **Key Concept** How are mole ratios used in chemical calculations?

22. **Key Concept** Outline the sequence of steps needed to solve a typical stoichiometric problem.

23. Write the 12 mole ratios that can be derived from the equation for the combustion of isopropyl alcohol.

$$2C_3H_7OH(l) + 9O_2(g) \longrightarrow 6CO_2(g) + 8H_2O(g)$$

24. The combustion of acetylene gas is represented by this equation:

$$2C_2H_2(g) + 5O_2(g) \longrightarrow 4CO_2(g) + 2H_2O(g)$$

How many grams of CO_2 and grams of H_2O are produced when 52.0 g C_2H_2 burns in oxygen?

Connecting Concepts

Chemical Quantities Review the "mole road map" at the end of Section 10.2. Explain how this road map ties into the summary of steps for stoichiometric problems shown in Figure 12.8.

Interactive Textbook

Assessment 12.2 Test yourself on the concepts in Section 12.2.

with ChemASAP

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Small-Scale LAB

Analysis of Baking Soda

Purpose
To determine the mass of sodium hydrogen carbonate in a sample of baking soda using stoichiometry.

Materials
- baking soda
- 3 plastic cups
- soda straw
- balance
- pipets of HCl, NaOH, and thymol blue
- pH sensor (optional)

Procedure

Probeware version available in the Probeware Lab Manual.

A. Measure the mass of a clean, dry plastic cup.

B. Using the straw as a scoop, fill one end with baking soda to a depth of about 1 cm. Add the sample to the cup and measure its mass again.

C. Place two HCl pipets that are about 3/4 full into a clean cup and measure the mass of the system.

D. Transfer the contents of both HCl pipets to the cup containing baking soda. Swirl until the fizzing stops. Wait 5–10 minutes to be sure the reaction is complete. Measure the mass of the two empty HCl pipets in their cup again.

E. Add 5 drops of thymol blue to the plastic cup.

F. Place two full NaOH pipets in a clean cup and measure the mass of the system.

G. Add NaOH slowly to the baking soda/HCl mixture until the pink color just disappears. Measure the mass of the NaOH pipets in their cup again.

Analyze
Using your experimental data, record the answers to the following questions below your data table.

1. Write a balanced equation for the reaction between baking soda ($NaHCO_3$) and HCl.
2. Calculate the mass in grams of the baking soda.
   (Step B − Step A)
3. Calculate the total mmol of 1*M* HCl.
   Note: Every gram of HCl contains 1 mmol.
   (Step C − Step D) × 1.00 mmol/g
4. Calculate the total mmol of 0.5*M* NaOH.
   Note: Every gram of NaOH contains 0.5 mmol.
   (Step F − Step G) × 0.500 mmol/g
5. Calculate the mmol of HCl that reacted with the baking soda. *Note:* The NaOH measures the amount of HCl that did not react.
   (Step 3 − Step 4)
6. Calculate the mass of the baking soda from the reaction data.
   (0.084 g/mmol × Step 5)
7. Calculate the percent error of the experiment.
   $$\frac{(\text{Step 2} - \text{Step 6})}{\text{Step 2}} \times 100\%$$

You're the Chemist
The following small-scale activities allow you to develop your own procedures and analyze the results.

1. **Analyze It!** For each calculation you did, substitute each quantity (number and unit) into the equation and cancel the units to explain why each step gives the quantity desired.
2. **Design It!** Baking powder consists of a mixture of baking soda, sodium hydrogen carbonate, and a solid acid, usually calcium dihydrogen phosphate ($Ca(H_2PO_4)_2$). Design and carry out an experiment to determine the percentage of baking soda in baking powder.