10-4 Common Logarithms

What YOU'LL LEARN

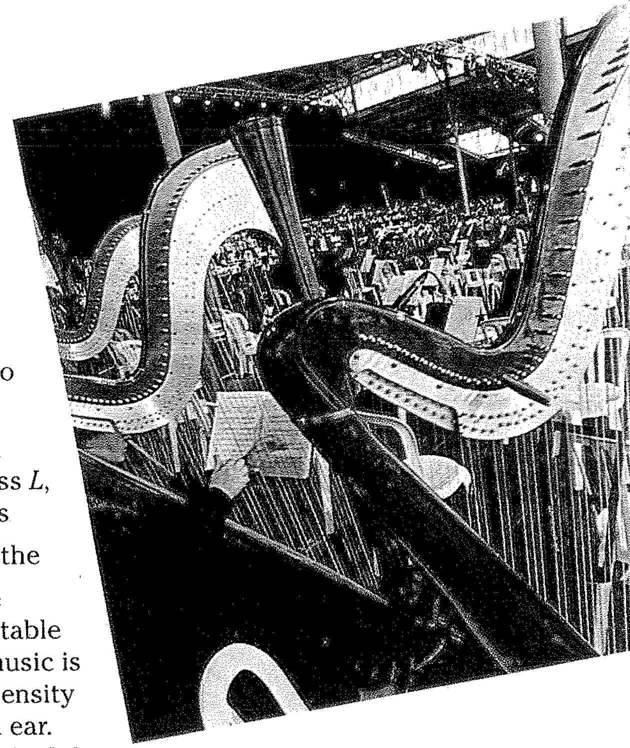
- To identify the characteristic and the mantissa of a logarithm, and
- to find common logarithms and antilogarithms.

Why IT'S IMPORTANT

You can use common logarithms to solve problems involving astronomy and acoustics.

Real World APPLICATION

Acoustics



One of the more useful logarithms is base 10, because our number system is base 10. Base 10 logarithms are called **common logarithms.** These are usually written without the subscript 10, so $\log_{10} x$ is written as $\log x$.

Common logarithms are used in the measure of sound. The loudness L, in decibels, of a particular sound is defined as $L = 10 \log \frac{I}{I_0}$, where I is the intensity of the sound and I_0 is the minimum intensity of sound detectable by the human ear. Soft recorded music is about 4000 times the minimum intensity of sound detectable by the human ear. Use the definition of logarithms to find the loudness in decibels.

$$L = 10 \log \frac{I}{I_0}$$

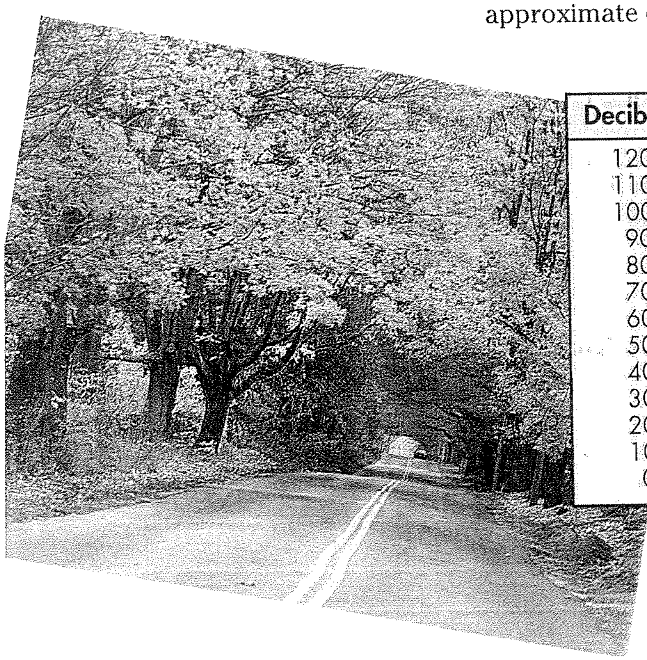
$$= 10 \log \frac{4000 I_0}{I_0} \quad \text{Substitution}$$

$$= 10 \log 4000$$

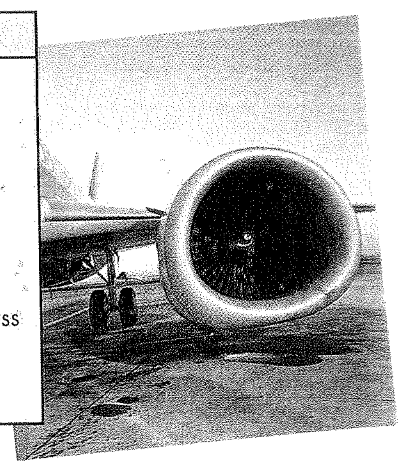
$$\approx 10(3.602)$$

$$\approx 36$$

Soft recorded music is about 36 decibels. Other common sounds and their approximate decibels levels are listed in the chart below.



Decibels	Sounds
120	Jet engine/Threshold of pain
110	Pneumatic drill
100	Food blender
90	Moderate discotheque
80	Noisy city street
70	Accounting office
60	Normal conversation (4 feet)
50	Average residence area
40	City night noises
30	Broadcast studio – no program in progress
20	Average whisper (4 feet)
10	Rustle of leaves
0	Threshold of hearing



Expressing a number in scientific notation is helpful when working with common logarithms.

Example 1 **If $\log 1.2 \approx 0.0792$, find each of the following.**

a. $\log 120$ **b. $\log 0.12$**

$$\log 120 = \log (1.2 \times 10^2) = \log 1.2 + \log 10^2 \approx 0.0792 + 2 \text{ or } 2.0792$$

$$\log 0.12 = \log (1.2 \times 10^{-1}) = \log 1.2 + \log 10^{-1} \approx 0.0972 + (-1)$$

Look closely at the results of Example 1. Notice that log 120 and log 0.12 have the same decimal part, or **mantissa,** 0.0792, but different integer parts, 2 and -1. The integer part of the common logarithm of a number is called its **characteristic** and indicates the magnitude of the number. The characteristic is the exponent of 10 when the original number is expressed in scientific notation.

$$\log (1.2 \times 10^2) = \log 1.2 + \log 10^2 = 0.0792 + 2$$

mantissa (0.0792) characteristic (2)

The mantissa is usually expressed as a positive number. To avoid negative mantissas, we rewrite the negative mantissa as the difference of a positive number and an integer, usually 10.

Example 2 **Use a scientific calculator to find log 0.0038. Write the result with a positive mantissa.**

The [LOG] key is used to find common logarithms.

Enter: .0038 [LOG] −2.420216403

The value of log 0.0038 is approximately -2.4202.

To write the logarithm with a positive mantissa, add and subtract 10.

$(-2.4202 + 10) - 10 = 7.5798 - 10$ *$10 - 10 = 0$ and $a + 0 = a$.*

The characteristic of the logarithm is $7 - 10$ or -3. Thus, the mantissa is 0.5798.

Sometimes an application of logarithms requires that you use the inverse of logarithms, exponentiation. When you are given the logarithm of a number and asked to find the number, you are finding the **antilogarithm.** That is, if $\log x = a$, then $x = \text{antilog } a$.

Example 3 **Use a scientific calculator to find the antilogarithm of 3.073.**

To find the antilogarithm on your calculator, use the [10^x] key.

Enter: 3.073 [2nd] [10^x] 1183.041556

The antilogarithm of 3.073 is approximately 1183.
This means that $10^{3.073} \approx 1183$.

Example 4 **Real World APPLICATION** **Acoustics**

The intensity of the music at a rock concert registered 66.6 decibels several miles away. How many times the minimum intensity of sound detectable by the human ear was this sound, if I_0 is defined to be 1? Use the formula $L = 10 \log \frac{I}{I_0}$ given at the beginning of the lesson.

$$L = 10 \log \frac{I}{I_0}$$

$$66.6 = 10 \log \frac{I}{I_0}$$

$$6.66 = \log I$$

$$\text{antilog } 6.66 = \text{antilog } (\log I)$$

$$10^{6.66} = I$$

$$4{,}570{,}882 \approx I$$

The sound several miles away from the rock concert was approximately 4,570,000 times the minimum intensity of sound detectable by the human ear.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Describe** the function you are performing when you take the antilogarithm of a value, such as x.
2. **Name** the base used by the calculator LOG key. What are these logarithms called?
3. When a number is expressed in scientific notation, its exponent of 10 corresponds to what part of its common logarithm?

MATH JOURNAL

4. The word logarithm is actually a contraction of "logical arithmetic."
   a. **Explain** why using logarithms was once considered a logical way to multiply or divide some numbers.
   b. **Explain** why logarithms are no longer used to multiply or divide numbers.
   c. **Name** some uses for common logarithms today.

Guided Practice

If log 875 = 2.9420, find each number.

5. characteristic of log 875
6. log 8.75

Use a scientific calculator to find the logarithm for each number rounded to four decimal places. Then state the mantissa and characteristic.

7. 13.7
8. 0.056

Use a scientific calculator to find the antilogarithm of each logarithm rounded to four decimal places.

9. 0.4573
10. −2.1477

11. **Astronomy** The *parallax* of a star is the difference in direction of the star as seen from two widely separate points. The brightness of a star as observed from Earth is its apparent magnitude. Interstellar space is measured in parsecs. One parsec is about 19.2 trillion miles. The absolute magnitude of a star is the magnitude that a star would have if it were 10 parsecs from Earth. For stars more than 30 parsecs from Earth, the formula relating the parallax p, the absolute magnitude M, and the apparent magnitude m is $M = m + 5 + 5 \log p$. The star M35 in the constellation Gemini has an apparent magnitude of 5.3 and a parallax of about 0.018. Find the absolute magnitude of star M35.

EXERCISES

Practice

If log 6500 = 3.8129, find each number.

12. mantissa of log 6500
13. characteristic of log 6500
14. antilog 3.8129
15. log 6.5
16. $10^{3.8129}$
17. mantissa of log 0.065

Use a scientific calculator to find the logarithm for each number rounded to four decimal places. Then state the mantissa and characteristic.

18. 64.7
19. 900.4
20. 0.047
21. 6.377
22. 0.0035
23. 0.0007

Use a scientific calculator to find the antilogarithm for each logarithm rounded to four decimal places.

24. 0.3142
25. 2.1495
26. −0.2615
27. −1.8143
28. 0.5734 − 3
29. 7.1394 − 10

Critical Thinking

30. Try to find $(-3)^3$ on your calculator. Some calculators will say "*ERROR*," even though $(-3)^3 = -27$. Can you think of a reason why they might do this? Explain.

Application and Problem Solving

Real World

31. **Acoustics** Mary Esperanza had a new muffler installed on her car. As a result, the noise level of the engine of her car dropped from 85 decibels to 73 decibels.
    a. How many times the minimum intensity of sound detectable by the human ear was the car with the old muffler if I_0 is defined to be 1?
    b. How many times the minimum intensity of sound detectable by the human ear is the car with the new muffler?
    c. Find the percent of decrease of the intensity of the sound with the new muffler.

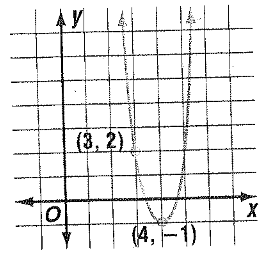
32. **Geology** As you know, the Richter scale is a logarithmic scale. An earthquake that measures 6 on the Richter scale is 10^6 times as intense as the weakest earthquake perceptible by a seismograph.
    a. How much more intense is an earthquake that measures 5.3 on the Richter scale than the weakest perceptible earthquake?
    b. How much more intense was the San Francisco earthquake of 1989, a 7.1 on the Richter scale, than its strongest aftershock, a 4.3 on the Richter scale?

Mixed Review

33. Solve $2 \log_6 3 + 3 \log_6 2 = \log_6 x$. (Lesson 10–3)

34. Determine whether $f(x) = \frac{x-1}{2}$ and $g(x) = 2x + 1$ are inverse functions. (Lesson 8–8)

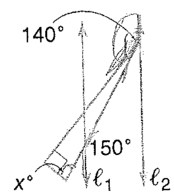
35. Write the equation of the parabola shown at the right. (Lesson 6–6)



36. **Astronomy** When a solar flare occurs on the sun, it sends out light waves that travel through space at a speed of 1.08×10^9 kilometers per hour. If a satellite in space detects the flare 2 hours after its occurrence, how far is the satellite from the sun? (Lesson 5–1)

37. Find $3\begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix} + 2\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} - 5\begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$. (Lesson 4–2)

38. **SAT Practice** If ℓ_1 is parallel to ℓ_2 in the figure, what is the value of x?



For Extra Practice, see page 899.

A 30 B 40 C 70 D 80 E 90

SELF TEST

Solve each equation. (Lesson 10–1)

1. $5^{3y+4} = 5^y$
2. $2^{x+3} = \frac{1}{16}$

Evaluate each expression. (Lesson 10–2)

3. $\log_{10} 10{,}000$
4. $\log_3 \frac{1}{243}$
5. $\log_{25} 5$
6. **Seismology** The earthquake that occurred in southern Peru in 1991 registered a 5.6 on the Richter scale. In 1990, an earthquake occurred in northern Peru that registered a 6.4 on the Richter scale. How much more intense was the 1990 earthquake in northern Peru that the 1991 earthquake in southern Peru? (Lesson 10–2)

Solve each equation. (Lesson 10–3)

7. $2 \log_6 4 - \frac{1}{3} \log_6 8 = \log_6 y$
8. $\log_2 (9t + 5) - \log_2 (t^2 - 1) = 2$

Use a scientific calculator to find the logarithm for each number rounded to four decimal places. Then state the mantissa and the characteristic. (Lesson 10–4)

9. 600.6
10. 0.189

10-5

Natural Logarithms

What YOU'LL LEARN

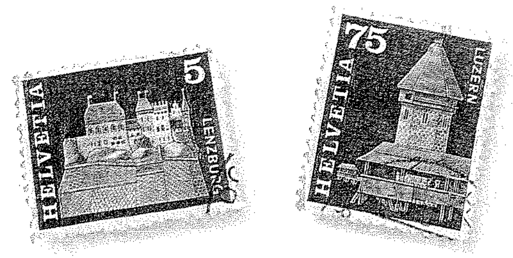
- To find natural logarithms of numbers.

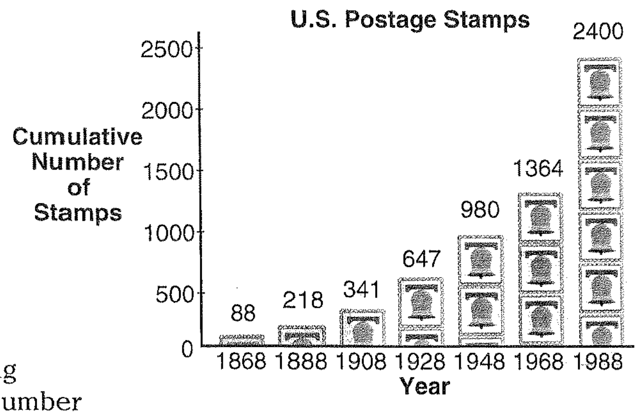
Why IT'S IMPORTANT

You can use natural logarithms to solve problems involving sales and physics.

Real World APPLICATION

Postal Service





The United States Post Office took 77 years to issue its first 600 stamps, 37 years to issue the next 600, approximately half as long to issue the third 600, and so on.

These figures suggest that the time needed to issue a fixed number of stamps has been decreasing exponentially. In fact, the number of stamps issued is growing exponentially and is approximately modeled by the formula $S = 83e^{0.024t}$, where S represents the cumulative number of stamps issued, t represents the number of years since 1848, and e is a special irrational number.

In 2008, 160 years after the first stamp was issued, about how many different U.S. postage stamps will have been issued? *This problem will be solved in Example 3.*

The number e used in this **exponential growth** problem is used extensively in science and mathematics. It is an irrational number whose value is approximately 2.718. e is the base for the **natural logarithms,** which are abbreviated ln. The natural logarithm of e is 1. All properties of logarithms that you have learned apply to the natural logarithms as well. The key marked [LN] on your calculator is the natural logarithm key.

Example 1 **Use a scientific calculator to find ln 3.925.**

Enter: 3.925 [LN] *1.367366351*

The natural logarithm of 3.925 is approximately 1.3674.

You can take antilogarithms of natural logarithms as well. The symbol for the antilogarithm of x is antiln x.

Example 2

a. Find x if $\ln x \approx 3.4825$

$\ln x \approx 3.4825$

$x \approx \text{antiln } 3.4825$

Enter: 3.4825 [2nd] [e^x] *32.5409729*

So x is approximately 32.5410.

b. Find e if $\ln e = 1$.

$\ln e = 1$

$e = \text{antiln } 1$

Enter: 1 [2nd] [e^x] *2.718281828*

So e is approximately 2.7183.

Equations involving e are easier to solve using natural logarithms, rather than using common logarithms, since $\ln e = 1$.

Example 3

Real World APPLICATION

Postal Service

Use the formula $S = 83e^{0.024t}$ and natural logarithms to determine about how many different U.S. postage stamps will have been issued from 1848 to 2008.

$t = 2008 - 1848$ or 160 years

$S = 83e^{0.024t}$	
$S = 83e^{0.024(160)}$	*$t = 160$*
$S = 83e^{3.84}$	*Simplify.*
$\ln S = \ln(83e^{3.84})$	*Take the natural logarithm of each side.*
$\ln S = \ln 83 + 3.84 \ln e$	*Power and product properties of logarithms*
$\ln S = \ln 83 + 3.84$	*Since e is the base for natural logarithms, ln e = 1.*
$\ln S \approx 8.258841$	
$\text{antiln}(\ln S) \approx \text{antiln } 8.258841$	*Take the antilogarithm of each side.*
$S \approx 3862$	

So, approximately 3862 stamps will have been issued by 2008.

TECHNOLOGY Tip

If your scientific calculator has no e^x key, use the INV key and then the LN key.

When interest is compounded *continuously*, the amount of money A in an account after t years is found using the formula $A = Pe^{rt}$, where P represents the amount of the principal and r represents the annual interest rate.

Example 4

Real World APPLICATION

Finance

Mr. and Mrs. Franco are planning to take a cruise for their twenty-fifth wedding anniversary. They have six years to save \$3500 for the cruise. If the six-year certificate of deposit they buy now pays 8% interest compounded continuously, how much should they invest now in order to have at least \$3500 for the cruise?

$Pe^{rt} \geq A$	*Write as inequality.*
$Pe^{(0.08)(6)} \geq 3500$	*Replace A with 3500, r with 0.08, and t with 6.*
$Pe^{0.48} \geq 3500$	*Simplify.*
$\ln(Pe^{0.48}) \geq \ln 3500$	*Take the natural logarithm of each side.*
$\ln P + (0.48)\ln e \geq \ln 3500$	*Power and product properties of logarithms*
$\ln P + 0.48 \geq \ln 3500$	*Since e is the base for natural logarithms, ln e = 1.*
$\ln P \geq 7.680518$	
$\text{antiln}(\ln P) \geq \text{antiln } 7.680518$	*Take the antilogarithm of each side.*
$P \geq 2165.74$	

Mr. and Mrs. Franco should invest at least \$2165.74 to earn enough for the trip.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Name** the base of natural logarithms.
2. **Describe** a situation for which you should choose to use natural logarithms instead of common logarithms to solve a problem.
3. **Describe** a situation that could be represented by the equation $A = 2500e^{0.08t}$.

Guided Practice

Use a scientific calculator to find each value rounded to four decimal places.

4. ln 3.12
5. ln 0.045
6. ln 0.772
7. antiln 0.2594
8. antiln 2.0175
9. antiln −1.5771

10. **Finance** Atepa's grandparents opened a savings account for her when she was born. They placed $1000 in an account that paid $6\frac{1}{2}\%$ interest compounded continuously. Atepa is now 16 years old and would like to buy a used car that costs $2500. Does she have enough money in her account to buy the car? Explain.

EXERCISES

Practice

Use a scientific calculator to find each value rounded to four decimal places.

11. ln 7.95
12. ln 1.34
13. ln 57.3
14. ln 0.958
15. ln 2.7183
16. ln 10,000
17. ln 0.005
18. ln 1.002
19. ln 0.01
20. antiln 0.782
21. antiln 0
22. antiln 2.6049
23. antiln −0.112
24. antiln −4.567
25. antiln 1.005
26. antiln −2.003
27. antiln 1.55
28. antiln −1.679

Graph each pair of equations on the same axis.

29. $y = \ln x$ and $y = e^x$
30. $y = \log x$ and $y = \ln x$
31. Compare and contrast the graphs in Exercise 29.
32. Compare and contrast the graphs in Exercise 30.

Critical Thinking

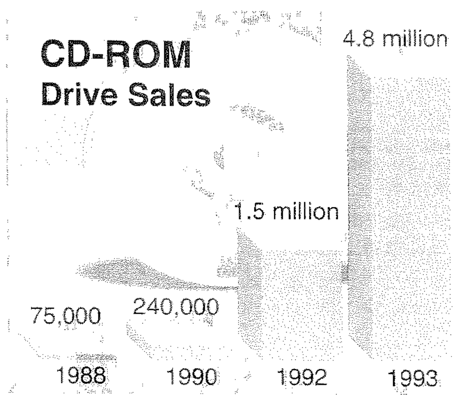
33. The great Swiss mathematician Leonhard Euler, for whom the number e is named, defined e as the sum of the series $1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{1\cdot 2\cdot 3\cdot 4}+\ldots$.
    a. Calculate the value of e using six terms in the series.
    b. Calculate the value of e using eight terms in the series.
    c. Which value is more accurate?
    d. Find the percent of the change in the values.

Applications and Problem Solving

Real World

34. **Finance** Kang is saving money to go on a trip to Europe after his college graduation. He will finish college five years from now. If the five-year certificate of deposit he buys pays 7.25% interest compounded continuously, how much should he invest now in order to have at least $3000 for the trip?

35. **Sales** The sales of CD-ROM drives have been increasing since 1984. Suppose the number of CD-ROM drives sold S in a given year is approximated by the formula $S = 75{,}000e^{0.83t}$, where t is the number of years since 1988.
    a. Estimate the number of these drives that were sold in the year 2000.
    b. Draw a graph of $S = 75{,}000e^{0.83t}$. How does your graph compare to the graph at the right?



interNET CONNECTION

Data Update For more information on technology sales, visit: **www.algebra2.glencoe.com**

36. **Physics** The intensity of light decreases as it passes through sea water. The equation $\ln \frac{I_0}{I} = 0.014d$ relates the intensity of light I at the depth of d centimeters with the intensity of light I_0 in the atmosphere. Find the depth of the water where the intensity of the light is half the intensity of the light in the atmosphere.

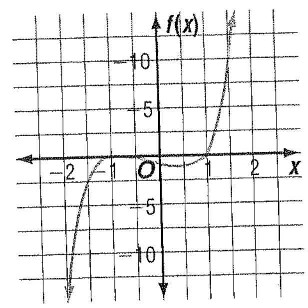
Mixed Review

37. Use a calculator to find each value, rounded to four decimal places. (Lesson 10–4)

a. log 77.3 b. log 0.0056 c. antilog 3.5567 d. antilog (6.7891 – 10)

38. Simplify $\dfrac{\frac{x^2}{x^2 - 25y^2}}{\frac{x}{5y - x}}$. (Lesson 9–3)

39. Use the graph of the polynomial function $f(x) = x^5 + x^4 - x - 1$ at the right to determine at least one of the binomial factors of the polynomial. Then find all factors of the polynomial. (Lesson 8–2)



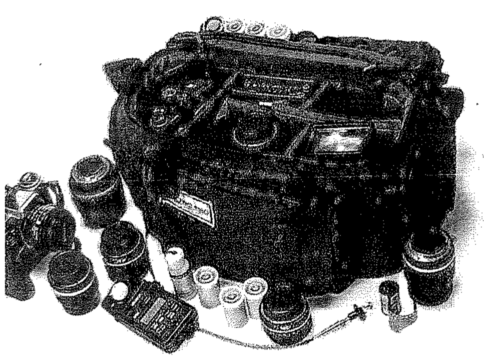
40. **SAT Practice Quantitative Comparison**

$cx \neq 0$

Column A	Column B
$-2cx$	$(x-c)^2$

A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

41. **Photography** Shina Murakami is a professional photographer. She has a photograph that is 4 inches wide and 6 inches long. She wishes to make a print of the photograph for a competition. The area of the new print is to be five times the area of the original. If Ms. Murakami is going to add the same amount to the length and the width of the photograph, what will the dimensions of the new print be? (Lesson 6–4)



42. Solve $\begin{vmatrix} x & 5 & 2 \\ -6 & 4 & 1 \\ 3 & 1 & x \end{vmatrix} = x^2 + 22x - 1$. (Lesson 4–4)

43. Graph the system of inequalities. Name the coordinates of the vertices of the polygon formed. Find the maximum and minimum values of the function. (Lesson 3–5)

$x \geq 0$

$y \geq 0$

$y \leq 3 - x$

$3x + y \leq 6$

$f(x, y) = 2x + 4y$

For Extra Practice, see page 900.

10-6 Solving Exponential Equations

What YOU'LL LEARN

- To solve equations with variable exponents by using logarithms,
- to evaluate expressions involving logarithms with different bases, and
- to solve problems by using estimation.

Why IT'S IMPORTANT

You can use exponential equations to solve problems involving sports and demographics.

Real World APPLICATION

Demographics

The population of the United States is continually growing. In 1999, the population in the United States was 274 million, and the exponential growth rate was 0.9% per year. An equation of the form $P(t) = P_0e^{kt}$ can be used to model the growth pattern of many things, ranging from growing investments to growing populations. In this case, P_0 is the population at time 0, $P(t)$ represents the population at time t, and k is a positive constant that depends on the situation. The constant k is often called the **exponential growth rate.**

Equations of the type described above are called **exponential equations.** Exponential equations are equations in which the variables appear as exponents. These equations can be solved using the property of equality for logarithmic functions. **Use estimation** to make sure you're on the right track.

Example 1

PROBLEM SOLVING
Use Estimation

Suppose $6^x = 42$.

a. Estimate the value of x.

b. Solve the equation.

a. Since $6^2 = 36$ and $6^3 = 216$, the value of x is between 2 and 3. The value of x should be much closer to 2 than 3.

b.

$$6^x = 42$$

$$\log 6^x = \log 42 \quad \textit{Property of equality for logarithmic functions}$$

$$x \log 6 = \log 42 \quad \textit{Power property of logarithms}$$

$$x = \frac{\log 42}{\log 6} \quad \textit{Divide each side by log 6.}$$

$$x \approx \frac{1.6232}{0.7782}$$

$$x \approx 2.086$$

The solution is approximately 2.086, which is consistent with the estimate.

Check: Use a scientific calculator to find the value of $6^{2.086}$.

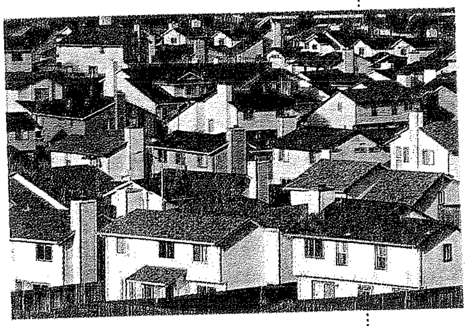
Enter: 6 [x^y] 2.086 [=] 41.99750672

The answer is reasonable.

Example 2 Refer to the application at the beginning of the lesson. How many years after 1999 will it take for the U.S. population to reach 300 million if the exponential growth rate remains at 0.9%?

Real World APPLICATION

Demographics



$$P(t) = P_0e^{kt}$$

$$300 = 274e^{0.009t} \quad \textit{P_0 = 274, k = 0.009, and P(t) = 300}$$

$$\frac{300}{274} = e^{0.009t} \quad \textit{Divide each side by 274.}$$

$$\ln \frac{300}{274} = \ln e^{0.009t} \quad \textit{Take the natural logarithm of each side.}$$

$$\ln \frac{300}{274} = 0.009t \quad \textit{Definition of natural logarithm}$$

$$\ln 300 - \ln 274 = 0.009t \quad \textit{Quotient property of logarithms}$$

$$5.7038 - 5.6131 \approx 0.009t$$

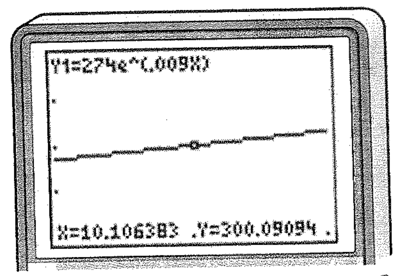
$$0.0907 \approx 0.009t$$

$$10.1 \approx t$$

The population in the United States will reach 300,000,000 about 10 years after 1999, or in 2009.

Check: Use a graphing calculator to graph the equation $y = 274e^{0.009x}$. Use TRACE to find the value of x when $y = 300$.

The value of x is about 10.1. The answer is correct.



In some equations, variables are found in more than one exponent.

Example 3 Solve $8^{2x-5} = 5^{x+1}$.

$$8^{2x-5} = 5^{x+1}$$

$$\log 8^{2x-5} = \log 5^{x+1} \quad \textit{Property of equality for logarithmic functions}$$

$$(2x - 5) \log 8 = (x + 1) \log 5 \quad \textit{Power property of logarithms}$$

$$2x \log 8 - 5 \log 8 = x \log 5 + \log 5 \quad \textit{Distributive property}$$

$$2x \log 8 - x \log 5 = \log 5 + 5 \log 8$$

$$x(2 \log 8 - \log 5) = \log 5 + 5 \log 8 \quad \textit{Distributive property}$$

$$x = \frac{\log 5 + 5 \log 8}{2 \log 8 - \log 5}$$

$$x \approx \frac{0.6990 + 5(0.9031)}{2(0.9031) - 0.6990}$$

$$x \approx 4.7095$$

The solution is approximately 4.7095. *Check this result.*

It is possible to evaluate expressions involving logarithms with different bases. Since calculators are usually not programmed with all possible bases for logarithms, the **change of base formula** can be very helpful.

Change of Base Formula	**For all positive numbers *a*, *b*, and *n*, where $a \neq 1$ and $b \neq 1$,** $\log_a n = \frac{\log_b n}{\log_b a}$.

Example 4 **Express each logarithm in terms of common logarithms. Then approximate its value to three decimal places.**

a. $\log_8 77$

$\log_a n = \frac{\log_b n}{\log_b a}$ *Change of base formula*

$\log_8 77 = \frac{\log 77}{\log 8}$ *$a = 8, n = 77, b = 10$*

≈ 2.0889

The value of $\log_8 77$ is approximately 2.089.

b. $\log_{16} 64$

$\log_a n = \frac{\log_b n}{\log_b a}$ *Change of base formula*

$\log_{16} 64 = \frac{\log 64}{\log 16}$ *$a = 16, n = 64, b = 10$*

$= 1.5$

The value of $\log_{16} 64$ is 1.5. *Why is this an exact value?*

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **You Decide** Karen sees the exponent in the equation $36 = x^5$ and decides to use logarithms to solve the equation. Tisha tells her that this is not an exponential equation and she does not need logarithms to solve the equation. Who is correct? Explain.
2. **Describe** a situation when you might use the change of base formula.
3. Could you use the change of base formula to express a logarithm in terms of natural logarithms? Explain.

Guided Practice

Find the value of each logarithm to three decimal places.

4. $\log_4 22$
5. $\log_{12} 95$

Use logarithms to solve each equation. Round to three decimal places.

6. $5^x = 52$
7. $8^{2a} = 124$
8. $2.1^{t-5} = 9.32$
9. $y = \log_4 125$
10. $2^{2x+3} = 3^{3x}$
11. $2^n = \sqrt{3^{n-2}}$

12. **Finance** If \$1500 is placed in an account that pays 6.5% interest compounded continuously, how long will it take for the money in the account to double? Use the formula $A = Pe^{rt}$.

EXERCISES

Practice

Find the value of each logarithm to three decimal places.

13. $\log_5 16$
14. $\log_6 82$
15. $\log_3 125$
16. $\log_2 100$
17. $\log_{12} 25$
18. $\log_4 48$

Use logarithms to solve each equation. Round to three decimal places.

19. $9^b = 45$
20. $2^x = 30$
21. $5^p = 34$
22. $3.1^{a-3} = 9.42$
23. $6^{x+2} = 17.2$
24. $8.2^{n-3} = 42.5$
25. $x = \log_5 61.4$
26. $8^{y-2} = 7.28$
27. $t = \log_8 200$
28. $5^{s+2} = 15.3$
29. $9^{z-4} = 6.28$
30. $7.6^{a-2} = 41.7$
31. $3.5^{3x+1} = 65.4$
32. $20^{x^2} = 70$
33. $8^{x^2-2} = 32$
34. $5.8^{x^2-3} = 82.9$
35. $9^a = 2^a$
36. $5^{x-1} = 3^x$
37. $7^{t-2} = 5^t$
38. $16^{d-4} = 3^{3-d}$
39. $8^{x-2} = 5^x$
40. $5^{3y} = 8^{y-1}$
41. $5^{5a-2} = 2^{2a+1}$
42. $8^{2y} = 52^{4y+3}$
43. $40^{3x} = 5^{2x+1}$
44. $4^n = \sqrt{5^{n-2}}$
45. $\sqrt[3]{2^{x-1}} = 8^{x-2}$

Critical Thinking

46. Let x be any real number and let a, b, and n be positive real numbers with $a \neq 1$ and $b \neq 1$. Show that if $x = \log_a n$, then $x = \frac{\log_b n}{\log_b a}$.

Applications and Problem Solving

47. **Demographics** The population of Antlers, Oklahoma, is about 2500. Suppose it is growing at an exponential rate of 3%. Use the formula $P(t) = P_0e^{kt}$ to determine approximately how long it will take the town's population to double.

48. **Olympics** Since the modern Olympics began in 1896 with 42 events, the number of events has continued to grow. Suppose the exponential growth rate of the number of events is 1.9%.

Summer Olympics Events

Year	City	Number
1968	Mexico City	172
1972	Munich	196
1976	Montreal	199
1980	Moscow	200
1984	Los Angeles	223
1988	Seoul	237
1992	Barcelona	257
1996	Atlanta	271

Source: Atlanta Committee for the Olympic Games

a. Use the formula $E = 42e^{kt}$, where E is the number of events, k is the exponential growth rate, and t is the time in years since 1896, to determine how long after the first modern Olympics we can expect there to be 350 Summer Olympic events.

b. Draw the graph of $E = 42e^{0.019t}$. How does your graph compare to the chart above?

Mixed Review

49. Use a calculator to find antiln -6.083, rounded to four decimal places. (Lesson 10–5)

50. **Construction** A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job in 30 days. How long would it have taken the painter to do the job alone? (Lesson 9–5)

51. Solve $b^4 - 5b^2 + 4 = 0$. (Lesson 8–6)

52. State whether the graph of $4y^2 - x^2 - 24y + 6x = 11$ is a parabola, a circle, an ellipse, or a hyperbola. (Lesson 7–6)

53. Solve $(x + 2)(x + 9) > 0$. (Lesson 6–7)

54. **ACT Practice** $\frac{x^0(yz)^2}{y^2z^3} =$

A $\frac{1}{yz^2}$ B $\frac{x}{z}$ C x D $\frac{1}{z}$ E $\frac{x}{yz^2}$

55. Find the values of x and y for which the sentence $2x + 5yi = 4 + 15i$ is true. (Lesson 5–9)

56. Write the system of equations as a matrix equation. Then solve the system.

$6x + 5y = 8$ (Lesson 4–6)

$3x - y = 7$

57. Write an equation in standard form for the line that passes through the point at (4, 6) and is perpendicular to the line whose equation is $y = \frac{2}{3}x + 5$. (Lesson 2–4)

For Extra Practice, see page 900.

WORKING ON THE Investigation

Refer to the Investigation on pages 590–591.

Your manager has asked you to explore the maximum spring potential of the bungee. She is interested in finding the longest distance a jumper would bounce back for any length. She believes that the new bungee product would dominate the market if the spring was greater than that of other bungee equipment. The jumper would therefore experience a greater bounce at the end of his or her drop.

You need to analyze the spring of the rubber bands you tested. Define the *elastic potential* as the ratio of the distance bounced to the length of the stretch. In this manner, you can find the point at which the rubber band has the greatest spring in relationship to the amount it is stretched.

1 Examine one of your graphs. How can the elastic potential be illustrated on that graph using your definition? Is there a numerical value that describes the elastic potential? Explain.

2 Draw new graphs for all three rubber bands using the ratio on the vertical axis instead of the distance bounced. Connect the points in order with line segments. Determine the slope of each segment. Record the slope. Describe the slope in terms of a ratio. What does the ratio indicate?

3 What is the maximum elastic potential for each of the rubber bands? At the maximum elastic potential, what is the distance stretched and the resulting shot distance for each rubber band?

4 Now make three graphs with the horizontal axis representing the distance stretched and the vertical axis representing the elastic potential. Are the graphs logarithmic or exponential in shape? Do each of the graphs verify your predictions for the maximum elastic potentials?

5 Make a recommendation regarding the spring of the rubber bands. Indicate the optimum stretch length for each rubber band you tested. Justify your recommendation.

Add the results of your work to your Investigation Folder.

10-7 Growth and Decay

What YOU'LL LEARN

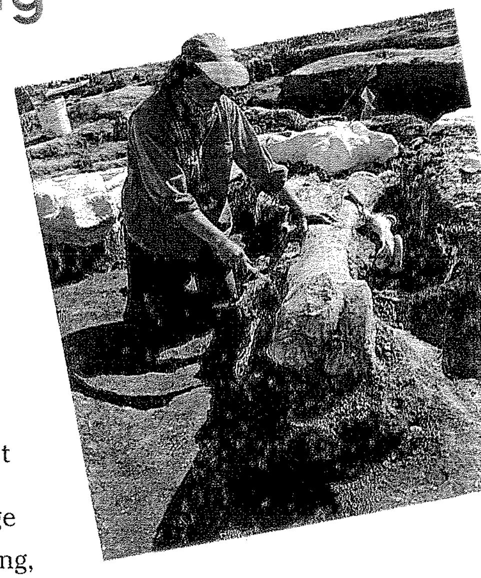
- To use logarithms to solve problems involving growth and decay.

Why IT'S IMPORTANT

You can solve growth and decay problems to learn more about paleontology and communication.

Real World APPLICATION

Paleontology



Paleontologists study life of past geological periods by studying fossil remains. They use carbon-14 (C^{14}) to estimate the age of fossils. Carbon-14 decays with time. In 5760 years, just half of the mass of this substance will remain. This period is called its *half-life.* To find the age of a fossil with just $\frac{1}{5}$ of the carbon-14 remaining, paleontologists must use the decay formula for this substance.

The **general formula for growth and decay** is $y = ne^{kt}$, where y is the final amount, n is the initial amount, k is a constant, and t is the time. To determine the decay formula for carbon-14, assume that the initial amount n is represented by 2 units. Then, after 5760 years, the final amount y must be 1 unit. Substitute these values into the general formula and solve for the constant k.

$$\begin{aligned}
y &= ne^{kt} && \textit{General formula for growth and decay}\\
1 &= 2e^{k(5760)} && y = 1,\ n = 2,\ t = 5760\\
0.5 &= e^{5760k} &&\\
\ln 0.5 &= \ln e^{5760k} && \textit{Take the natural logarithm of each side.}\\
\ln 0.5 &= 5760k \ln e && \textit{Power property of logarithms}\\
\ln 0.5 &= 5760k && \ln e = 1\\
\frac{\ln 0.5}{5760} &= k &&\\
-0.00012 &\approx k &&
\end{aligned}$$

The equation for the decay of carbon-14 is $y = ne^{-0.00012t}$, where t is given in years.

Now use this formula to determine the age of a fossil that has $\frac{1}{5}$ of its carbon-14 remaining. Assume that the initial amount is 5 units and the final amount is 1 unit. Then, solve the equation for t.

$$\begin{aligned}
y &= ne^{-0.00012t} && \textit{Formula for the decay of carbon-14}\\
1 &= 5e^{-0.00012t} && y = 1,\ n = 5\\
0.2 &= e^{-0.00012t} &&\\
\ln 0.2 &= \ln e^{-0.00012t} && \textit{Take the natural logarithm of each side.}\\
\ln 0.2 &= -0.00012t \ln e && \textit{Power property of logarithm}\\
\ln 0.2 &= -0.00012t && \ln e = 1\\
\frac{\ln 0.2}{-0.00012} &= t &&\\
13{,}412 &\approx t &&
\end{aligned}$$

The fossil is about 13,400 years old.

The general formula for growth and decay also describes materials that grow exponentially. In this case, the value of k will be positive.

fabulous FIRSTS

Willard F. Libby (1908–1980)

W.F. Libby of the University of Chicago won the Nobel Prize for Chemistry in 1960 for his method to use carbon-14 for age determination in archaeology. In 1947, he was the first scientist to explain the formation of carbon-14 in the atmosphere and develop a radioactive test to date organic deposits.

Example 1 CONNECTION **Biology**

Bacteria usually reproduce by a process known as *binary fission*. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal conditions, some bacteria can reproduce every 20 minutes. Find the constant k for the growth of these types of bacteria under ideal conditions and write the growth equation.

$y = ne^{kt}$

$2 = 1e^{k(20)}$ *One bacterium can produce two in 20 minutes.*

$2 = e^{20k}$

$\ln 2 = \ln e^{20k}$ *Take the natural logarithm of each side.*

$\ln 2 = 20k \ln e$ *Power property of logarithms*

$\ln 2 = 20k$ *$\ln e = 1$*

$\frac{\ln 2}{20} = k$

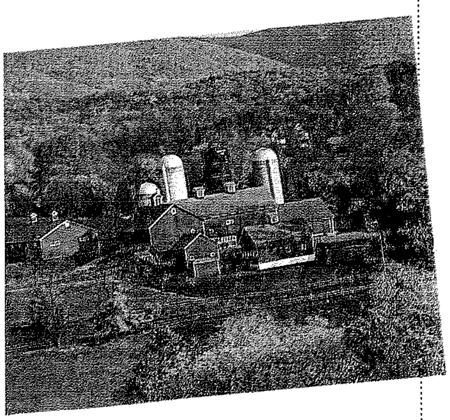
$0.0347 \approx k$

The value of the constant for the bacteria is approximately 0.0347. The growth equation is $y = ne^{0.0347t}$, where t is given in minutes.

Certain assets, such as cars, houses, and business equipment, appreciate or depreciate, that is, increase or decrease in value, with time. The formula $V_n = P(1 + r)^n$, where V_n is the new value, P is the initial value, r is the fixed rate of appreciation or depreciation, and n is the number of years, can be used to compute the value of an asset. The value of r for a depreciating asset will be negative, and the value of r for an appreciating asset will be positive.

Example 2 Real World APPLICATION **Agriculture**

The Diller family includes several generations of farmers. They have an opportunity to buy 50 acres adjacent to their farm for \$800 per acre. In the past, the price of farmland has gone up 3% a year. If this continues, how long will it be before the land is worth at least \$1000 per acre?



Explore Read the problem. The problem gives the value of the land now and the annual percent of increase in the price of land. It asks you to find when the land will be worth at least \$1000 an acre.

Plan Write the formula as the inequality $P(1 + r)^n \geq V_n$. Then substitute 1000 for V_n, 800 for P, and 0.03 for r and solve for n.

Solve

$P(1 + r)^n \geq V_n$

$800(1 + 0.03)^n \geq 1000$ *$V_n = 1000, P = 800, r = 0.03$*

$1.03^n \geq 1.25$

$\log 1.03^n \geq \log 1.25$ *Take the common logarithm of each side.*

$n \log 1.03 \geq \log 1.25$ *Product property of logarithms*

$n \geq \frac{\log 1.25}{\log 1.03}$

$n \geq 7.549$

The land will be worth at least \$1000 per acre in about $7\frac{1}{2}$ years.

Examine Use the formula to find the value of the land in 7.5 years.

$P(1 + r)^n = V_n$

$800(1 + 0.03)^{7.5} = V_n$ *$P = 800, r = 0.03, n = 7.5$*

$999 \approx V_n$ *Use a calculator to evaluate the expression.*

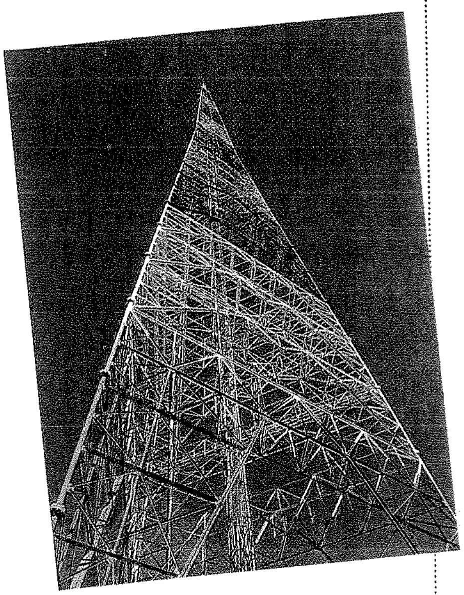
The value of the V_n is about \$1000, so the answer seems reasonable.

Logarithms can be used to solve problems involving an exponential function $y = ab^x$.

Example 3

Real World APPLICATION

Broadcasting



The FM frequencies of radio transmissions range from 88 to 108 megahertz. However, these frequencies are not marked uniformly along the display of a radio receiver. Assume that the frequencies are an exponential function of the distance from the left end of the display. Write an equation for the FM frequencies along a display that is 15 centimeters long.

Let x represent the distance from the left end, and let y represent the frequency. Since 88 megahertz is 0 centimeters from the left end and 108 megahertz is 15 centimeters from the left end, the ordered pairs (0, 88) and (15, 108) are solutions to the function $y = ab^x$. Use (0, 88) to solve for a.

$$y = ab^x$$
$$88 = ab^0 \quad \textit{x = 0, y = 88}$$
$$88 = a(1) \quad \textit{b}^0 = 1$$
$$88 = a$$

Then use (15, 108) to solve for b.

$$y = ab^x$$
$$108 = 88b^{15} \quad \textit{a = 88, x = 15, and y = 108}$$
$$\log 108 = \log (88b^{15}) \quad \textit{Take the common logarithm of each side.}$$
$$\log 108 = \log 88 + \log b^{15} \quad \textit{Product property of logarithms}$$
$$\log 108 = \log 88 + 15 \log b \quad \textit{Power properties of logarithms}$$
$$\log 108 - \log 88 = 15 \log b$$
$$\frac{\log 108 - \log 88}{15} = \log b$$
$$0.0059 \approx \log b$$
$$\text{antilog } 0.0059 \approx \text{antilog } (\log b) \quad \textit{Take the antilogarithm of each side.}$$
$$1.0137 \approx b$$

The relationship of the frequencies and the distance from the left side of the display is approximated by the equation $y = 88(1.0137)^x$, where y is the frequency in megahertz and x is the distance in centimeters.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Describe** a situation in which the constant k in the formula for growth and decay is positive and one in which k is negative. Describe the situation if the value of k is zero.
2. **Explain** why the natural logarithm was used in Example 1 and the common logarithm was used in Examples 2 and 3. Could you solve Example 1 using common logarithms? Could you solve Examples 2 and 3 by using natural logarithms? Explain.

Guided Practice

3. **Broadcasting** Refer to Example 3.
   a. Find the FM frequency that corresponds to the point that is 3.5 centimeters from the left side of the display.
   b. Find the location on the radio display that corresponds to the FM frequency of 100 megahertz.

4. **Business** Zeller Industries bought a computer for $4600. It is expected to depreciate at a steady rate of 20% a year. When will the value have depreciated to $2000?

5. **Chemistry** The half life of radium (Ra^{226}) is 1620 years.
   a. Find the constant k in the formula $y = ne^{kt}$ for radium (Ra^{226}) when t is given in years.
   b. Write the equation for the decay of radium (Ra^{226})
   c. Suppose a 20-gram sample of radium (Ra^{226}) is sealed in a box. Find the mass of the radium after 5000 years.
   d. When will a sample of radium (Ra^{226}) be one fourth of its original mass?
   e. When will a 20-gram sample of radium (Ra^{226}) be completely gone? Explain.

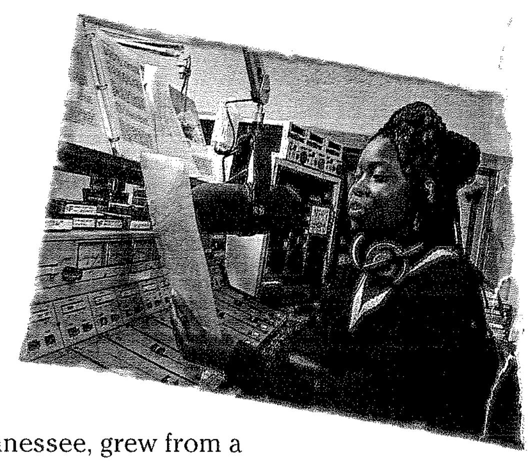
EXERCISES

Applications and Problem Solving

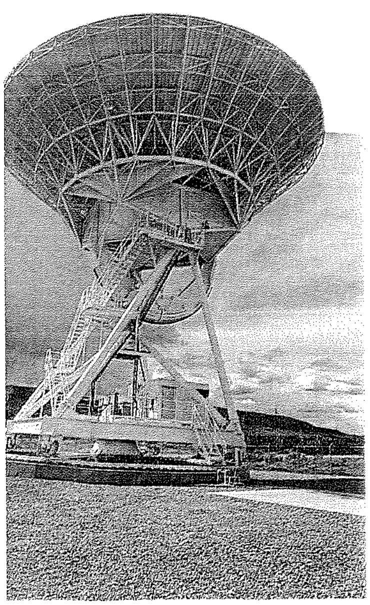
6. **Real Estate** Mr. and Mrs. Sawyer bought a condominium for $75,000. Assuming that its value will appreciate at most 6% a year, how much will the condo be worth in five years when the Sawyers are ready to move?

7. **Medicine** Radioactive iodine is used to determine the health of the thyroid gland. It decays according to the equation $y = ne^{-0.0856t}$, where t is in days. Find the half-life of this substance.

8. **Broadcasting** The AM frequencies of radio transmissions range from 535 to 1705 kilohertz. Assume that the frequencies are an exponential function of the distance from the left of the display on a radio.
   a. Write an equation for the AM frequencies along a display that is 15 centimeters long.
   b. Find the AM frequency that corresponds to the point that is 4.5 centimeters from the left side of the display.
   c. Find the location on the radio display that corresponds to the AM frequency of 1000 kilohertz.



9. **Population Growth** The city of Knoxville, Tennessee, grew from a population of 546,488 in 1980 to a population of 585,960 in 1990.
   a. Use this information to write a growth equation for Knoxville, where t is the number of years after 1980.
   b. Use your equation to predict the population of Knoxville in 2010.
   c. What factors might affect the population growth of a city such as Knoxville?



10. **Space Travel** A radioisotope is used as a power source for a satellite. The power output is given by the equation $P = 50e^{-\frac{t}{250}}$, where P is the power in watts and t is the time in days.
    a. Find the power available after 100 days.
    b. Ten watts of power are required to operate the equipment in the satellite. How long can the satellite continue to operate?

11. **Paleontology** A paleontologist finds a bone that might be a dinosaur bone. In the laboratory, she finds that the radiocarbon found in this bone is $\frac{1}{10}$ of that found in living bone tissue. Could this bone have belonged to a dinosaur? Explain. (*Hint:* The dinosaurs lived from 220 million years ago to 63 million years ago.)

12. **Real Estate** The Diaz family bought a new house 10 years ago for $80,000. The house is now worth $140,000. Assuming a steady rate of growth, what was the yearly rate of appreciation?

13. **Cooking** The amount of time needed to cook scrambled eggs in the microwave depends on the number of eggs being cooked. The chart at the right shows the suggested times for cooking eggs in a certain microwave.

Number of Eggs	Cooking Time (min)
2	$1\frac{3}{4}$
4	3

a. Assume that the number of minutes is a function of some power of the number of eggs. Write a general equation of the form $t = an^b$, where t is the time in minutes, n is the number of eggs, and a and b are constants. (*Hint:* Use a system of equations to solve for the constants.)

b. Find the amount of time needed to cook 3 eggs and to cook 5 eggs.

14. **Communication** A rumor can spread very quickly. The number of people H who have heard a rumor can be approximated by the equation $H = \frac{P}{1 + (P - S)e^{-0.4t}}$ where P is the total population, S is the number of people who start the rumor, and t is the time in minutes. During lunch period, two students decide to start a rumor that the principal will let the students out of school one hour early that day. If there are 1600 students in the school, how much time will pass before half of the students have heard the rumor?

Programming

15. The graphing calculator program at the right uses the formula $S = R\left[\frac{(1 + I)^N - 1}{I}\right]$ to find how many payments are needed to accumulate a given amount of money when payments are made at regular intervals and interest is compounded at the end of each payment period. In the formula, S represents the money accumulated after the last payment, R represents the amount of each payment, I represents the interest rate per payment period (represented as a decimal), and N represents the number of payments per year. In the program, Y represents the number of payments per year, and A represents the annual interest rate.

```
PROGRAM: PAYMENTS
: Prompt S,R,A,Y
: A/Y→I
: log(SI/R+1)/(log(1+I))→N
: If N=int (N+1)
: Then
: Goto 1
: End
: int(N+1)→N
: Lbl 1
: Disp "PAYMENTS NEEDED:",N
```

Use the program to determine the number of payments needed to accumulate the indicated amount of money, given the amount of each payment, the annual interest rate, and the number of payments each year.

a. $4000, $300, 9%, 2

b. $7500, $400, 8.5%, 4

c. $8995, $156, 9.25%, 12

d. $14,600, $195, 8.75%, 12

e. $96,000, $850, 9.65%, 12

f. $90,000, $425, 9.65%, 26

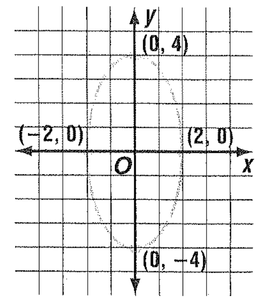
Critical Thinking

16. Compare the formulas for exponential growth, $y = ne^{kt}$, and for continuously compounded interest, $A = Pe^{rt}$. Explain how the formulas are related.

Mixed Review

17. Solve $7^{x-2} = 5^{3-x}$. (Lesson 10–6)

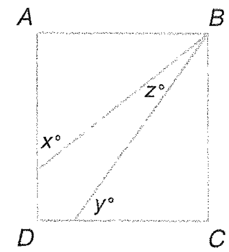
18. Find the equation for the ellipse shown at the right. Then give the coordinates of the foci. (Lesson 7–4)



19. **SAT Practice** In rectangle $ABCD$, what is the sum $x + y$ in terms of z?

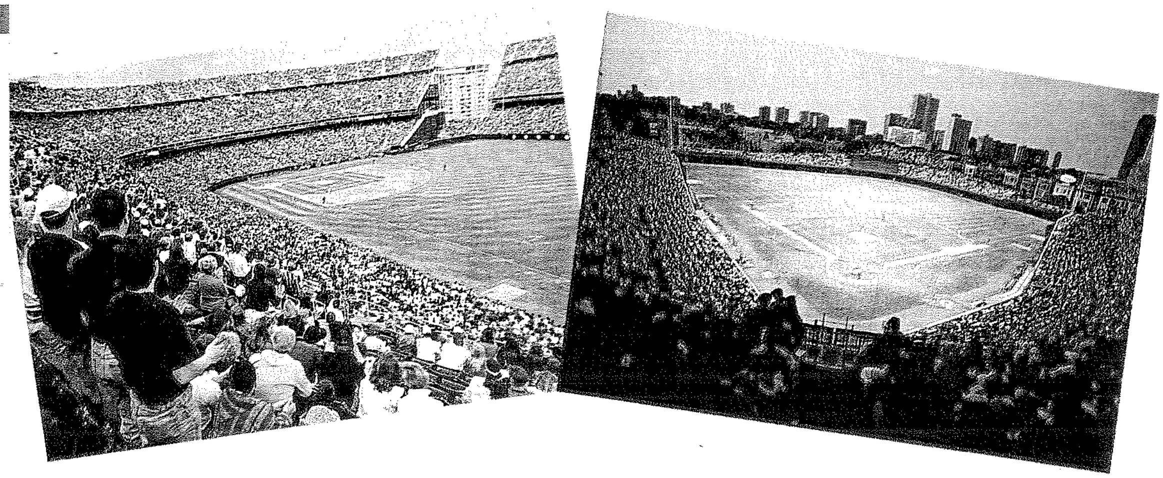
A $90 + z$ **B** $190 - z$ **C** $180 + z$

D $270 - z$ **E** $360 - z$



FYI

The baseball stadium with largest seating capacity is Mile High Stadium, home of the Colorado Rockies. Wrigley Field, home of the Chicago Cubs, has the smallest seating capacity of the major league fields.



20. **Baseball** The seating capacities of the major league baseball stadiums are listed below. (Lesson 4–8)

34,142	38,710	40,625	42,400	43,739	44,702	47,313	48,000
48,041	49,292	50,516	52,003	52,416	52,952	53,192	54,816
55,601	55,883	56,000	56,227	57,545	58,727	59,002	59,702
62,000	62,382	64,593	76,100				

a. Find the range of the data.
b. Find the quartiles of the data.
c. Find the interquartile range of the data.
d. Find any outliers in the data.
e. Draw a box-and-whisker plot for the data.

21. **Geometry** The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$, where A represents the measure of the area, h represents the measure of the altitude, and b_1 and b_2 represent the measures of the bases. Find the measure of the area of the trapezoid whose height is 8 centimeters and whose bases measure 12 centimeters and 20 centimeters.

For Extra Practice, see page 900.

CHAPTER 10 HIGHLIGHTS

INTERNET CONNECTION

Chapter Review For additional lesson-by-lesson review, visit: **www.algebra2.glencoe.com**

VOCABULARY

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Algebra

antilogarithm (p. 618)
change of base formula (pp. 594, 628)
characteristic (p. 618)
common logarithms (p. 617)
exponential equations (p. 626)
exponential function (pp. 594, 596)
exponential growth (p. 622)
exponential growth rate (p. 626)
general formula for growth and decay (p. 631)
logarithm (p. 605)
logarithmic function (pp. 594, 607)
mantissa (p. 618)
natural logarithms (p. 622)
power property of logarithms (p. 613)
product property of logarithms (p. 611)
property of equality for exponential functions (p. 599)
property of equality for logarithmic functions (p. 608)
quotient property of logarithms (p. 612)

Problem Solving

use estimation (p. 626)

UNDERSTANDING AND USING THE VOCABULARY

Choose the letter that best answers each question.

1. To solve $\log_8 (2y + 3) = \log_8 (y - 4)$, what property would you use?
2. What do the following have in common? $\log 6500$ and $\log 6.5$
3. What kind of function is $y = \frac{1}{2}(5)^x$?
4. Name the property shown in each example.
   a. $3 \log_2 5 = \log_2 5^3$
   b. $\log_4 2x = \log_4 2 + \log_4 x$
   c. $\log_3 \frac{4}{5} = \log_3 4 - \log_3 5$
5. What do the following have in common? $\log 0.28$ and $\log \frac{3}{4}$
6. What kind of function is $y = \log_2 x$?
7. To solve $9^{2p} = 27^{p-1}$, what property would you use?

a. characteristic
b. exponential function
c. logarithmic function
d. mantissa
e. power property of logarithms
f. product property of logarithms
g. property of equality for exponential functions
h. property of equality for logarithmic functions
i. quotient property of logarithms

STUDY GUIDE AND ASSESSMENT

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Upon completing this chapter, you should be able to:

- simplify expressions and solve equations involving real exponents (Lesson 10–1)

Simplify $16^{\sqrt{12}} \div 8^{\sqrt{3}}$.

$16^{\sqrt{12}} \div 8^{\sqrt{3}} = (2^4)^{\sqrt{12}} \div (2^3)^{\sqrt{3}}$

$= 2^{8\sqrt{3}} \div 2^{3\sqrt{3}}$

$= 2^{8\sqrt{3} - 3\sqrt{3}}$

$= 2^{5\sqrt{3}}$

REVIEW EXERCISES

Use these exercises to review and prepare for the chapter test.

Simplify each expression.

8. $3^{\sqrt{2}} \cdot 3^{\sqrt{2}}$
9. $\left(x^{\sqrt{5}}\right)^{\sqrt{20}}$
10. $\frac{49^{\sqrt{2}}}{7^{\sqrt{12}}}$
11. $\left(8^{\sqrt{3}}\right)\left(8^{-2\sqrt{3}}\right)\left(8^{4\sqrt{3}}\right)$

Solve each equation or inequality. Check your solution.

12. $2^{6x} = 4^{5x+2}$
13. $49^{3p+1} = 7^{2p-5}$
14. $9^{x^2} \leq 27^{x^2-2}$
15. $9^x = \frac{1}{81}$

- write exponential equations in logarithmic form and vice versa (Lesson 10–2)

Write $3^3 = 27$ in logarithmic form.

$3^3 = 27$

$3 = \log_3 27$

Write $\log_4 64 = 3$ in exponential form.

$\log_4 64 = 3$

$64 = 4^3$

Write each equation in logarithmic form.

16. $7^3 = 343$
17. $5^{-2} = \frac{1}{25}$
18. $4^0 = 1$
19. $4^{\frac{3}{2}} = 8$

Write each equation in exponential form.

20. $\log_4 64 = 3$
21. $\log_8 2 = \frac{1}{3}$
22. $\log_6 \frac{1}{36} = -2$
23. $\log_6 1 = 0$

- evaluate logarithmic expressions (Lesson 10–2)

Evaluate $\log_3 3^5$.

$\log_3 3^5 = x$

$3^x = 3^5$

$x = 5$

Evaluate each expression.

24. $6^{\log_6 7}$
25. $\log_{10} 10^{-3}$
26. $\log_{64} 8$
27. $\log_{12} 144$

- solve equations involving logarithmic functions (Lesson 10–2)

Solve $\log_b 16 = 4$.

$\log_b 16 = 4$

$b^4 = 16$

$b^4 = 2^4$

$b = 2$

Solve $\log_3 10 = \log_3 (2x)$.

$\log_3 10 = \log_3 (2x)$

$10 = 2x$

$5 = x$

Solve each equation or inequality. Check your solution.

28. $\log_b 9 = 2$
29. $\log_4 x = \frac{1}{2}$
30. $\log_3 x = -3$
31. $\log_7 2401 = x$
32. $\log_7 (x^2 + x) = \log_7 12$
33. $\log_6 12 = \log_6 (5x - 3)$
34. $\log_8 (3y - 1) > \log_8 (y + 4)$
35. $\log_2 (x^2 + 6x) = \log_2 (x - 4)$

OBJECTIVES AND EXAMPLES

- simplify and evaluate expressions using properties of logarithms (Lesson 10–3)

Use $\log_{12} 9 \approx 0.884$ and $\log_{12} 18 \approx 1.163$ to evaluate $\log_{12} 2$.

$$\begin{aligned}\log_{12} 2 &= \log_{12}\left(\frac{18}{9}\right)\\ &= \log_{12} 18 - \log_{12} 9\\ &\approx 1.163 - 0.884\\ &\approx 0.279\end{aligned}$$

REVIEW EXERCISES

Use $\log_9 7 \approx 0.8856$ and $\log_9 4 \approx 0.6309$ to evaluate each expression.

36. $\log_9 28$
37. $\log_9 49$
38. $\log_9 144$
39. $\log_9 15.75$

- solve equations involving logarithms (Lesson 10–3)

Solve $2 \log_5 6 - \frac{1}{3} \log_5 27 = \log_5 x$.

$$\begin{aligned}2 \log_5 6 - \tfrac{1}{3} \log_5 27 &= \log_5 x\\ \log_5 6^2 - \log_5 27^{\frac{1}{3}} &= \log_5 x\\ \log_5 36 - \log_5 3 &= \log_5 x\\ \log_5 \frac{36}{3} &= \log_5 x\\ \log_5 12 &= \log_5 x\\ 12 &= x\end{aligned}$$

Solve each equation.

40. $\log_3 x - \log_3 4 = \log_3 12$
41. $\log_2 y = \frac{1}{3}\log_2 27$
42. $\log_5 7 + \frac{1}{2}\log_5 4 = \log_5 x$
43. $2 \log_2 x - \log_2 (x + 3) = 2$
44. $\log_7 m = \frac{1}{3} \log_7 64 + \frac{1}{2} \log_7 121$

- find common logarithms and antilogarithms (Lesson 10–4)

Use a scientific calculator to find the logarithm of 56.4.

Use the LOG key.

Enter: 56.4 LOG 1.751279104

Use a scientific calculator to find the antilogarithm of 2.738.

Use the 10^x key.

Enter: 2.738 2nd 10^x 547.0159629

Use a scientific calculator to find the logarithm for each number rounded to four decimal places.

45. 46.56
46. 678.1
47. 0.00468
48. 0.183

Use a scientific calculator to find the antilogarithm for each number rounded to four decimal places.

49. 2.75
50. 1.999
51. −0.567
52. −3.47

- find natural logarithms of numbers (Lesson 10–5)

Use a scientific calculator to find the natural logarithm of 56.4.

Use the LN key.

Enter: 56.4 LN 4.032469158

Use a scientific calculator to find the natural antilogarithm of 2.738.

Use the e^x key.

Enter: 2.738 2nd e^x 15.45604208

Use a scientific calculator to find each value rounded to four decimal places.

53. ln 2.3
54. ln 9.25
55. ln 50
56. ln 0.05
57. antiln 1.9755
58. antiln 2.246
59. antiln −0.489
60. antiln −4.5

OBJECTIVES AND EXAMPLES

- evaluate expressions involving logarithms with different bases (Lesson 10–6)

Approximate the value of $\log_8 72$ to three decimal places.

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_8 72 = \frac{\log 72}{\log 8}$$

$$= \frac{1.8573}{0.9031}$$

$$\approx 2.057$$

REVIEW EXERCISES

Approximate the value of each logarithm to three decimal places.

61. $\log_5 15$
62. $\log_4 100$
63. $\log_{12} 15$
64. $\log_2 36$
65. $\log_9 108$
66. $\log_{11} 104$

- solve equations with variable exponents by using logarithms (Lesson 10–6)

Solve $3^{x-4} = 5^{x-1}$.

$$3^{x-4} = 5^{x-1}$$

$$\log 3^{x-4} = \log 5^{x-1}$$

$$(x - 4) \log 3 = (x - 1)\log 5$$

$$x \log 3 - 4 \log 3 = x \log 5 - \log 5$$

$$x \log 3 - x \log 5 = 4 \log 3 - \log 5$$

$$x (\log 3 - \log 5) = 4 \log 3 - \log 5$$

$$x = \frac{4 \log 3 - \log 5}{\log 3 - \log 5}$$

$$x \approx -5.4520$$

Use logarithms to solve each equation.

67. $2^x = 53$
68. $\log_4 11.2 = x$
69. $2.3^{x^2} = 66.6$
70. $3^{4x-7} = 4^{2x+3}$
71. $6^{3y} = 8^{y-3}$
72. $x = \log_{20} 1000$
73. $12^{x-4} = 4^{2-x}$
74. $2.1^{x-5} = 9.32$

APPLICATIONS AND PROBLEM SOLVING

75. **Atmospheric Pressure** Atmosphere pressure can be determined by using the equation $P = 14.7(10)^{-0.02h}$, where P is the atmospheric pressure in pounds per square inch and h is the altitude above sea level in miles. Find the atmospheric pressure at an altitude of 3 miles above sea level. (Lesson 10–1)

76. **Finance** Mr. and Mrs. Grauser invested \$500 at 6.5% compounded continuously. (Lesson 10–5)
    a. Find the value of the investment after at most 7 years.
    b. When will the Grausers' investment triple?

77. **Biology** For a certain strain of bacteria, k is 0.872 when t is measured in days. How long will it take 9 bacteria to increase to 738 bacteria? (Lesson 10–7)

78. **Chemistry** Radium-226 decomposes radioactively. Its half-life, the time it takes for half of the sample to decompose, is 1800 years. Find the constant k in the decay formula for this compound. (Lesson 10–7)

A practice test for Chapter 10 is provided on page 921.

ALTERNATIVE ASSESSMENT

COOPERATIVE LEARNING PROJECT

Buying a House Exponential equations in the form of formulas enable people to make major financial decisions.

In this project, you will develop a report for a married couple that is buying their first house. Jenna and Jarod had a meeting with a mortgage banker. He told them that the rule of thumb for borrowing money for a mortgage is that their monthly payment should be no more than 28% of their gross monthly income. Jenna makes \$35,000 a year as a nurse, and Jarod makes \$25,000 a year as a teacher. They are thinking about waiting at least three more years to start a family. At that time, Jenna hopes to take a year off work and then resume her work part-time.

Use the monthly payment formula

$$MP = P\left[\frac{\frac{r}{12}\left(1+\frac{r}{12}\right)^{12T}}{\left(1+\frac{r}{12}\right)^{12T-1}}\right],$$

where P = amount of mortgage, r = interest rate, and T = term of mortgage in years, to write a report for Jenna and Jarod about whether to use a 15-year or 30-year mortgage and how much they should plan on borrowing, after putting 20% down, based on their salaries, jobs, future plans, and term of the mortgage.

Consider these ideas while accomplishing your task.

- Calculate how much their monthly payment should be based on the rule of thumb and their circumstances.
- Analyze what calculations will be needed.
- Prepare a spreadsheet or chart to organize the various calculations.
- Determine the price range of a house for them considering a 15-year mortgage.
- Determine the price range of a house for them considering a 30-year mortgage.
- Write a report explaining all of their options.
- Give a recommendation and explain why.

THINKING CRITICALLY

- Show algebraically and graphically that $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions.
- On some calculators, $\log x$ and 10^x are on the same key position. Why is this? Give examples of other functions that share this relationship.

PORTFOLIO

Evaluating what you don't understand about a concept is important in achieving a comprehensive understanding of that subject. Think about the concept you found most difficult to understand in this chapter. Write about why you found that concept difficult to understand and explain how you managed to understand it. What methods did you use in order to gain understanding and did they work? Place this in your portfolio.

STANDARDIZED TEST PRACTICE

CHAPTERS 1–10

Section One: MULTIPLE CHOICE

There are eight multiple-choice questions in this section. After working each problem, write the letter of the correct answer on your paper.

1. Which is a quadratic equation?

   A. $x^3 + 2x + 3 = 0$
   B. $3x - 7 = 0$
   C. $2x^2 - 9x + 7 = 0$
   D. $5x^2 + 3xy - 1 = 0$

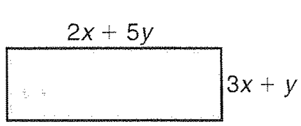
2. **Geometry** A mathematics professor called the hardware store to order fencing to outline his pentagon-shaped garden. He gave the store manager the measures of each side as $\frac{1}{x}$, $\frac{2}{x-2}$, $\frac{3}{x}$, $\frac{4}{x-2}$, and $\frac{x}{x-2}$. The manager panicked until he found an algebra student to help him out. What was the perimeter of the professor's garden?

   A. $\frac{9+x}{x(x-2)}$ B. $\frac{x^2+10x-8}{x(x-2)}$
   C. $\frac{x^2+9x-6}{x}$ D. $\frac{x+2}{x-2}$

3. Find the second row of $\begin{bmatrix} 0 & 9 \\ 4 & 3 \\ -2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & -2 \\ 0 & 8 & -5 \end{bmatrix}$.

   A. $[0\ \ 72\ \ -45]$ B. $[-4\ \ 58\ \ -31]$
   C. $[8\ \ 20\ \ -23]$ D. $[72\ \ 20\ \ 58]$

4. **Geometry** Find the area of the figure below.

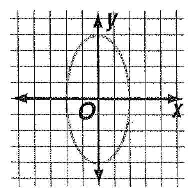


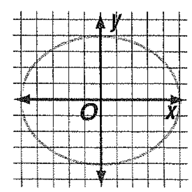
   A. $(5x^2 + 10xy + 5y^2)$ square units
   B. $(6x^2 + 17xy + 5y^2)$ square units
   C. $(21x^2y^2)$ square units
   D. $(11x^2 + 17xy + y^2)$ square units

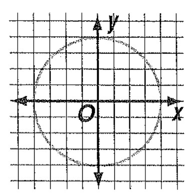
5. Sally's office has a system to let people know when the department will have a meeting. Sally calls three people, then those three people each call three other people, and so on, until the whole department is notified. If it takes 10 minutes for a person to call three people and the whole department is notified within 30 minutes, how many people will be notified in the last round?

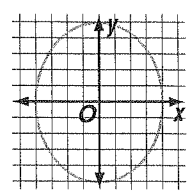
   A. 3 people   B. 30 people
   C. 9 people   D. 27 people

6. Choose the graph of $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

   A.


   B.


   C.


   D.


7. In planning a trip, the distance you travel varies jointly as the time and rate of speed. LaDonna Metcalf must travel 396 miles in 8 hours to meet a client. She travels 6 hours at 55 mph. She stops for a half an hour to rest and eat lunch. What is the minimum speed at which she must travel to meet her appointment?

   A. 66 mph   B. 99 mph
   C. 44 mph   D. 50 mph

8. **Geometry** The perimeter of Mr. Baxter's back yard is 152 feet. He plans to use wood fencing along one length of the yard, and wire fencing along the other three boundaries. If the length exceeds twice the width by 7 feet, how much wood fencing will Mr. Baxter require?

   A. 53 feet   B. 23 feet
   C. 99 feet   D. 138 feet