Focus On

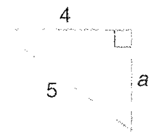
PREREQUISITE SKILLS

To be successful in this chapter, you'll need to understand and apply these concepts. Refer to the examples to help you complete these problems.

Use the Pythagorean Theorem.

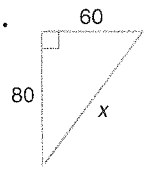
Example Find the value of a in the right triangle.

$$a^2 + b^2 = c^2$$
$$a^2 + 4^2 = 5^2$$
$$a^2 + 16 = 25$$
$$a^2 = 9$$
$$a = 3$$

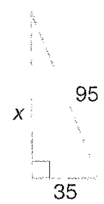


Find the value of x. Round to the nearest tenth, if necessary.

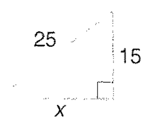
1.



2.



3.



Use the FOIL method to multiply two binomials.

Example Use the FOIL method to multiply $(x - 4)(x + 7)$.

$$(x - 4)(x + 7) = (x)(x) + (x)(7) + (-4)(x) + (-4)(7)$$
$$= x^2 + 7x - 4x - 28$$
$$= x^2 + 3x - 28$$

Find each product.

4. $(x + 3)(x - 8)$ **5.** $(2a - 5b)(3a + b)$ **6.** $(z + 1)(z - 1)$ **7.** $(c + d)^2$

Focus On

READING SKILLS

In this chapter, you learn about ***equations*** and ***inequalities***. The word *equation* contains the word *equal*. An equation is a mathematical sentence in which the value of one side is equal to the value of the other side. The word *inequality* contains the prefix "in" which can mean "not." An inequality is a mathematical sentence in which the value of one side is not necessarily equal to the value of the other side.

1–1A Graphing Technology Expressions

A Preview of Lesson 1–1

In the Graphing Technology lessons in this text, you will be introduced to keying sequences that will allow you to perform mathematical computations, graph equations, and utilize other features of a graphing calculator.

Remember that, as with any scientific calculator, graphing calculators follow the order of operations.

Example 1 **Evaluate $39 - 3^3 \cdot 2 + \frac{5^2 + 9}{4}$.**

Enter: 39 [−] 3 [^] 3 [×] 2 [+] [(] 5 [x^2] [+] 9 [)] [÷] 4 [ENTER] −6.5

You may enter 5^2 as 5 [^] 2 or as 5 [x^2].

Use parentheses for all grouping symbols on a graphing calculator.

Occasionally, you will need to evaluate an algebraic expression for several different values of a variable. This can be done efficiently using the [STO▸] and [ENTRY] features.

Example 2 **Evaluate $6x(14 - x)^2$ for $x = -6$, 11, and 1.25.**

Enter: [(−)] 6 [STO▸] [X,T,θ,n] *Stores −6 as x.*

[2nd] [:] 6 [X,T,θ,n] [(] 14 *Evaluates the expression.*

[−] [X,T,θ,n] [)] [x^2] [ENTER] −14400

Now press [2nd] [ENTRY] and use the editing features of the calculator to change the value for x. Move the cursor with the arrow keys to the −6 and change it to 11. The new value is 594 and is computed when you press [ENTER]. Repeat for $x = 1.25$.

EXERCISES

Use a graphing calculator to evaluate each expression. Round each answer to the nearest hundredth.

1. $5 \times \frac{4 + 7}{6}$
2. 231.5^6
3. $(52 \times 4)^3 + \frac{76}{6}$
4. $3\left[\frac{4 + \frac{3(4 + 6)}{5}}{4}\right] + 11$
5. $112 - 2\{16 + 2[14 - 2(7 + 1)]\} - 4^2$
6. $[(-4 + 11)^2 \div 4]3$
7. $(1.12 \times 10^4)(3.65 \times 10^{-3})$
8. $(5.24 \times 10^{-6})(3.24 \times 10^4)$
9. Evaluate $\frac{5}{9}(F - 32)$ for $F = 50$, 32, 0, −5, and 80.
10. Evaluate $x^2 - y^2$ for $x = 15$, $y = 13$ and $x = 0.8$, $y = 0.4$.

1–1 Expressions and Formulas

What YOU'LL LEARN

- To use the order of operations to evaluate expressions, and
- to use formulas.

Why IT'S IMPORTANT

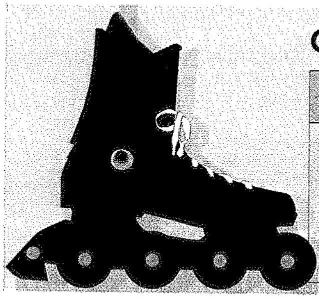
You can use expressions to solve problems involving exercise, baseball, and geometry.

Real World APPLICATION

Exercise

Chris Edwards of Escondido, California, earned his nickname "Airman" by soaring 13 feet off of a ramp while on in-line skates.

According to a recent study by the University of Massachusetts, you burn as many Calories in-line skating at moderate speeds or faster as you do running. The chart below shows the Calories burned per minute for various body weights and skating speeds.



Calories Burned per Minute While In-Line Skating

Weight	8 mph	9 mph	10 mph	11 mph	12 mph	13 mph	14 mph	15 mph
120 lb	4.2	5.8	7.4	8.9	10.5	12.1	13.6	15.2
140 lb	5.1	6.7	8.3	9.9	11.4	13.0	14.6	16.1
160 lb	6.1	7.7	9.2	10.8	12.4	13.9	15.5	17.1
180 lb	7.0	8.6	10.2	11.7	13.3	14.9	16.4	18.0
200 lb	7.9	9.5	11.2	12.6	14.2	15.8	17.3	18.9

To determine your skating speed in miles per hour, divide 60 (the number of minutes in an hour) by the number of minutes you skated. Then multiply that number by the number of miles you skated. You can represent this by the expression $(60 \div t) \times d$, where t represents the time skated in minutes and d represents the distance traveled in miles.

Maya works out each day by in-line skating. She skates about three miles in 20 minutes, and she weighs 140 pounds. About how many Calories does she burn during a workout?

First, find how fast Maya was skating.

$$(60 \div t) \times d = (60 \div 20) \times 3 \quad \textit{t = 20 and d = 3}$$

How do you evaluate $(60 \div 20) \times 3$? What do you do first? Do you multiply or divide? A numerical expression must have exactly one value. In order to find that value, you must follow an **order of operations.**

Order of Operations

1. **Simplify the expressions inside grouping symbols, such as parentheses, brackets, braces, and fraction bars.**
2. **Evaluate all powers.**
3. **Do all multiplications and divisions from left to right.**
4. **Do all additions and subtractions from left to right.**

Using the rules above, evaluate the expression.

$$(60 \div 20) \times 3 = 3 \times 3 \quad \textit{Perform the operation inside the parentheses first.}$$

$$= 9 \quad \textit{Multiply.}$$

Maya skated at an average speed of 9 mph during her 20-minute workout.

To determine how many Calories Maya burns, find her weight on the chart. Then go across to the column with her average speed of 9 mph. Maya burns 6.7 Calories per minute during her 20-minute workout. So, she burns a total of 20×6.7 or 134 Calories.

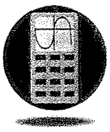
As in the expression $(60 \div 20) \times 3$, grouping symbols can be used to change or clarify the order of operations. Frequently-used grouping symbols are parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in $\frac{2+4}{3}$. When calculating the value of an expression, begin with the operation in the innermost set of grouping symbols.

Example 1 **Find the value of $[(3 + 6)^2 \div 3] \cdot 4$.**

$[(3 + 6)^2 \div 3] \cdot 4$	$= [(9)^2 \div 3] \cdot 4$	*First add 3 and 6.*
	$= (81 \div 3) \cdot 4$	*Then find 9^2.*
	$= 27 \cdot 4$	*Divide 81 by 3.*
	$= 108$	*Multiply 27 by 4.*

The value is 108.

Scientific calculators follow the order of operations.



EXPLORATION SCIENTIFIC CALCULATORS

Your Turn

a. Simplify $3 + 5 \times 2 - 1$ using a scientific calculator.
b. Explain how your calculator arrived at the answer.
c. Suppose someone concluded that $3 + 5 \times 2 - 1 = 15$. How is this possible?
d. Evaluate $2045 - (18^2 + 711)$ using your calculator. Explain how the answer was calculated.
e. If you remove the parentheses in part d above, would the solution remain the same? Explain.
f. Write a set of directions for someone to find the value of $16^2 - (135 + 10 \times 8)$ without a scientific calculator.

Algebraic expressions contain at least one variable. You can evaluate an algebraic expression by replacing each variable with a value and then applying the rules for the order of operations.

Example 2 **a. Evaluate $a[b^2(b + a)]$ if $a = 12$ and $b = 0.5$.**

$a[b^2(b+a)]$	$= 12[(0.5)^2\ (0.5 + 12)]$	*Replace a with 12 and b with 0.5.*
	$= 12[0.25(0.5 + 12)]$	*Find $(0.5)^2$.*
	$= 12[0.25(12.5)]$	*Add 0.5 and 12.*
	$= 12[3.125]$	*Multiply 0.25 by 12.5.*
	$= 37.5$	*Multiply 12 by 3.125.*

The value is 37.5.

b. Evaluate $\frac{y^3}{3ab + 2}$ if $y = 4$, $a = -2$, and $b = -5$.

The fraction bar, which indicates division, is a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

$$\frac{y^3}{3ab+2} = \frac{4^3}{3(-2)(-5)+2} \quad \textit{Replace y with 4, a with −2, and b with −5.}$$

$$= \frac{64}{3(10)+2} \quad \textit{Evaluate the numerator and denominator separately.}$$

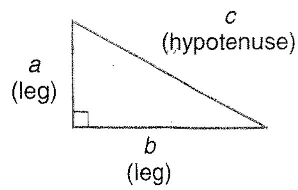
$$= \frac{64}{32} \quad \textit{Divide.}$$

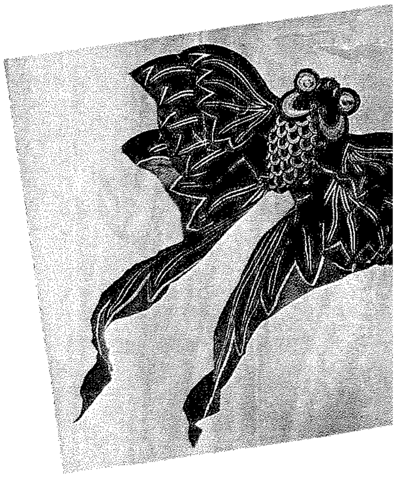
$$= 2$$

The value is 2.

A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know a value for every variable in the formula except one, you can find the value of the remaining variable.

One formula you may use is the **Pythagorean theorem,** which was developed by the Greek mathematician Pythagoras. In a right triangle, the side opposite the right angle is called the **hypotenuse.** This side is always the longest side of a right triangle. The other two sides are called the **legs** of the right triangle. The Pythagorean theorem states that in a right triangle, if a and b are the measures of the legs and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.





Example 3

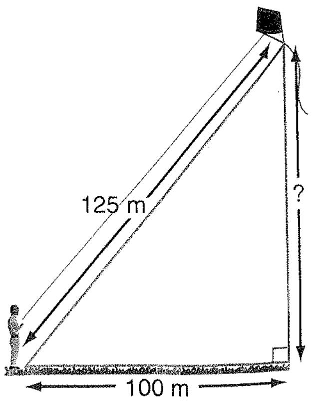
INTEGRATION
Geometry

FYI

The fastest recorded speed for a kite is 120 mph. It was flown by Pete DiGiacomo at Ocean City, MD, on September 22, 1989.

Suppose the distance from a person flying a kite to the ground directly below the kite is 100 meters, and the length of the string from the person to the kite is 125 meters. Use the Pythagorean theorem to find the height of the kite.

You can think of the kite and the ground as parts of a right triangle as shown at the right.



$$c^2 = a^2 + b^2$$

$$125^2 = 100^2 + b^2 \quad \textit{Substitute.}$$

$$15{,}625 = 10{,}000 + b^2 \quad \textit{Evaluate all powers.}$$

$$b^2 = 5625 \quad \textit{Subtract.}$$

$$b = \sqrt{5625} \quad \textit{Take the square root of each side.}$$

$$b = 75$$

The kite is 75 meters high.

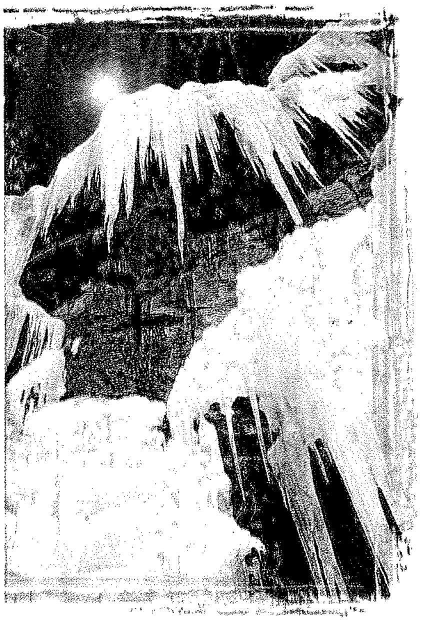
CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Explain** which operation to perform first when evaluating an expression that has both brackets and parentheses.
2. **Describe** how you would evaluate the expression $\frac{1}{3} - \frac{12(77-11)}{2}$.
3. **List** three formulas that you have used before.
4. **Write** three examples of algebraic expressions.
5. **You Decide** To find the volume of a cone, you multiply the area of the base by the height, then divide by 3. Andrea developed the formula $V = \frac{1}{3}\pi r^2 h$ to find the volume of a cone while Lorenzo developed the formula $V = \frac{\pi r^2 h}{3}$. The cone has a height of 16.3 centimeters, and its base has a radius of 14.7 centimeters. Explain which formula will work, and why.

Guided Practice



Find the value of each expression.

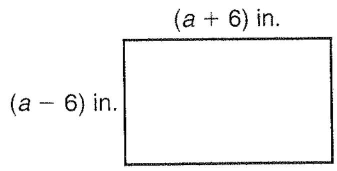
6. $7 - 6 \div 3$
7. $9(4 + 2)$
8. $18 \cdot 6 + 12$
9. $25 \cdot 2 - 3$
10. $\frac{45(4 + 32)}{10}$
11. $(4 - 3)^5 \cdot 9$

Evaluate each expression if $a = 3$, $b = -4$, and $c = 5$.

12. $a + b - c$
13. $a + c^2$
14. $a(b + c)$

The relationship between Celsius temperature C and Fahrenheit temperature F is given by $C = \frac{5(F - 32)}{9}$. Find the Celsius temperature for each Fahrenheit temperature.

15. normal body temperature, 98.6°
16. freezing point of water, 32°
17. **Geometry** Write a formula to represent the area A of the rectangle shown at the right.



EXERCISES

Practice

Find the value of each expression.

18. $4(3^2 + 3)$
19. $2(9 + 2) - 3$
20. $(7 + 5)3 - 3$
21. $4 + 2^2 - 15 + 4$
22. $10 + 16 \div 4 + 8$
23. $5 + 9(3) \div 3 - 8$
24. $7 - [4 + (6 \cdot 5)]$
25. $[21 - (9 - 2)] \div 2$
26. $4 + [9 \div (9 - 2(4))]$
27. $[(-7 + 4) \times 5 - 2] \div 6$

28. $\frac{1}{2}(5^2 + 3)$

29. $\frac{14(8 - 15)}{2}$

30. $-3(2^2 + 3)$

31. $4 + (49 \div 7) \times 8 \div 2$

32. $0.4(0.6 + 3.2) \div 2$

33. $0.5(2.3 + 25) \div 1.5$

34. $3 + [8 \div (9 + 2(-4))]$

35. $\frac{1}{3} - \frac{12(77 \div 11)}{9}$

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$.

36. $d(3 + c)$

37. $a + b + d$

38. $a + 2b - c$

39. $a + 10 \div c$

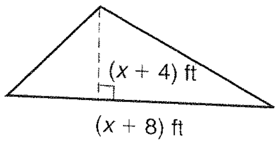
40. $2^d + a$

41. $\frac{3ab}{cd}$

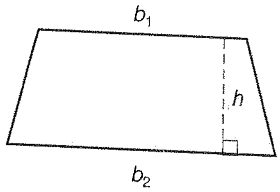
42. $\frac{3a + 4c}{2c}$

43. $(a + c)^2 - bd$

44. **Geometry** The formula for the area A of a triangle is $A = \frac{1}{2}bh$ where b is the measure of the base and h is the measure of the height. Write an expression to represent the area of the triangle at the right.



The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$. A represents the area, h represents the measure of the altitude, and b_1 and b_2 represent the measures of the bases. Find the area of each trapezoid given the following values.



45. $h = 6, b_1 = 22, b_2 = 17$

46. $h = 10, b_2 = 17, b_1 = 52$

47. $b_2 = 6.25, b_1 = 12.50, h = 4$

48. $h = \frac{5}{8}, b_1 = \frac{3}{4}, b_2 = \frac{1}{2}$

Simple interest is the amount paid or earned for the use of money for a unit of time. It is calculated using the formula $I = prt$ where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the simple interest I given each of the following values.

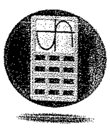
49. $p = \$1500, r = 6.5\%, t = 3$ years

50. $p = \$2500, r = 7.25\%, t = 2$ years

51. $p = \$20{,}005, r = 7.9\%, t = 2$ years, 3 months

52. $p = \$65{,}283.21, r = 9.32\%, t = 78$ months

Programming



53. The graphing calculator program at the right finds the area of a trapezoid once the height and lengths of the two bases are entered.

Run the program to find the area of each trapezoid.

a. $h = 8, b_1 = 112, b_2 = 20$

b. $h = 7, b_1 = 4, b_2 = 11$

c. $h = 4.8, b_1 = 5.6, b_2 = 6.4$

```
PROGRAM:AREA
: Disp "HEIGHT =?"
: Input H
: Disp "B1 ="
: Input B
: Disp "B2 = "
: Input C
: Disp "AREA =",
  H(B+C)/2
```

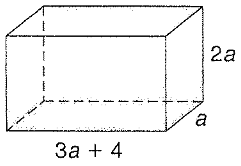
Alter the program to find the area of a triangle if the formula is $A = \frac{1}{2}bh$, where b is the measure of the base (in cm) and h is the measure of the height (in cm). Find the area of each triangle.

d. $b = 6.7, h = 13.8$ e. $b = 127.2, h = 82.6$

Critical Thinking

54. Insert grouping symbols as needed to make each statement true.
   a. $1 + 3 \cdot 2^2 = 16$
   b. $1 + 3 \cdot 2^2 = 49$
   c. $1 + 3 \cdot 2^2 = 13$
   d. $1 + 3 \cdot 2^2 = 37$

55. **Geometry** Study the rectangular prism at the right.



   a. Create a formula that will determine the total surface area of this rectangular prism.
   b. What is the surface area if $a = 4$?
   c. What is the surface area if $a = 6.2$?

Applications and Problem Solving

Real World

56. **Medicine** Dosages of medicine are based upon the weight and/or age of a patient. Suppose an asthma patient must take a medicine that is dispensed in 100-milligram tablets. The dosage is 5 milligrams per kilogram of body weight and is given every six hours.

   a. If the patient weighs 20 kilograms, what dosage should he receive?
   b. How many tablets would be needed for a 30-day supply?

57. **Number Theory** The sum of the factors of 6 that are less than 6 is $1 + 2 + 3$, or 6. Thus, 6 is called a **perfect number** because the number 6 equals the sum of its factors, excluding itself. 28 is also a perfect number because the sum of the factors, $1 + 2 + 4 + 7 + 14$, is equal to 28. The formula for generating perfect numbers is $p = (2^{n-1})(2^n - 1)$. In order for the formula to work, both n and $(2^n - 1)$ must be prime numbers.
   a. List the first 25 prime numbers. Recall that a prime number is an integer greater than 1 whose only positive factors are 1 and itself.
   b. Find four prime numbers n where $2^n - 1$ is also prime.
   c. Find the next two perfect numbers after 28.

58. **SAT Practice** **Quantitative Comparison**

Column A	Column B
$\dfrac{\frac{3}{5}}{\left(\frac{3}{5}\right)^2}$	$\frac{5}{3}$

   A if the quantity in Column A is greater
   B if the quantity in Column B is greater
   C if the two quantities are equal
   D if the relationship cannot be determined from the information given

For Extra Practice, see page 876.

1-2 Properties of Real Numbers

What YOU'LL LEARN

- To determine the sets of numbers to which a given number belongs, and
- to use the properties of real numbers to simplify expressions.

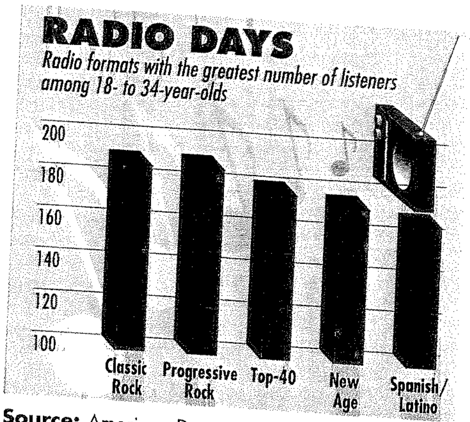
Why IT'S IMPORTANT

You can use the properties of real numbers to evaluate expressions and solve equations.

INTEGRATION

Statistics

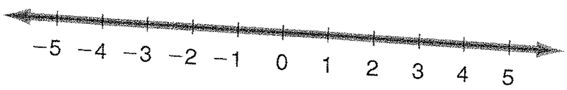
Tables, charts, and graphs are used to represent data in a manner that is easy to read and understand.



Source: American Demographics

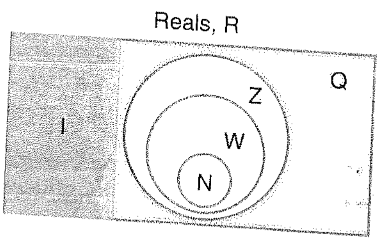
Source: National Sporting Goods Association

Both of the figures above use numbers to convey their message to the reader. All of the numbers that you use in everyday life are **real numbers.** Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.



Every real number can be classified as either **rational** or **irrational.** A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal. Some examples of rational numbers are $\frac{1}{3}$, $1.\overline{34}$, 5.8, −6, and 0. Any real number that is not rational is irrational. $\sqrt{2}$, π, and $\sqrt{7}$ are irrational numbers.

The sets of natural numbers, {1, 2, 3, 4, 5, . . .}, whole numbers, {0, 1, 2, 3, 4, . . .}, and integers, {. . . , −3, −2, −1, 0, 1, 2, . . .} are all subsets of the rational numbers.



The Venn diagram at the left shows the relationship among these sets of numbers.

R = reals
I = irrationals
W = wholes
Q = rationals
Z = integers
N = naturals

Example 1 **Find the value of each expression. Then name the sets of numbers to which each value belongs.**

a. $\sqrt{17}$

$\sqrt{17} = 4.1231056\ldots$ reals (R), irrationals (I)

b. $8 \div 4$

$8 \div 4 = 2$ reals (R), rationals (Q), integers (Z), whole numbers (W), natural numbers (N)

c. 0.25×0

$0.25 \times 0 = 0$ reals (R), rationals (Q), integers (Z), whole numbers (W)

d. $10 - 25$

$10 - 25 = -15$ reals (R), rationals (Q), integers (Z)

e. $6 \div 10$

$6 \div 10 = 0.6$ or $\frac{3}{5}$ reals (R), rationals (Q)

Operations with real numbers have several important properties. The chart below summarizes the properties of real numbers for addition and multiplication.

$-a$ is read "the opposite of a".

For any real numbers *a*, *b*, and *c*:		
	Addition	**Multiplication**
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Example 2 **Name the property illustrated by each equation.**

a. $(3 + 4a)2 = 2(3 + 4a)$

commutative property of multiplication

The commutative property says that the order in which you multiply does not change the product.

b. $62 + (38 + 75) = (62 + 38) + 75$

associative property of addition

The associative property says that the way you group three numbers when adding does not change their sum.

Example 3 **Name the additive inverse and multiplicative inverse for each number.**

a. $\frac{3}{8}$

Since $\frac{3}{8} + \left(-\frac{3}{8}\right) = 0$, the additive inverse of $\frac{3}{8}$ is $-\frac{3}{8}$.

Since $\left(\frac{3}{8}\right)\left(\frac{8}{3}\right) = 1$, the multiplicative inverse of $\frac{3}{8}$ is $\frac{8}{3}$.

b. -2.5

Since $-2.5 + 2.5 = 0$, the additive inverse of -2.5 is 2.5.

To find the multiplicative inverse of -2.5, find $\frac{1}{-2.5}$.

Enter: 1 [÷] 2.5 [+/−] [=] −0.4

The multiplicative inverse of -2.5 is -0.4.

You can use properties to simplify algebraic expressions.

Example 4 **Simplify $4(2b - 6c) + 2(3b + c)$.**

$4(2b - 6c) + 2(3b + c)$

$= 4(2b) - 4(6c) + 2(3b) + 2(c)$ *Use the distributive property.*

$= 8b - 24c + 6b + 2c$ *Multiply.*

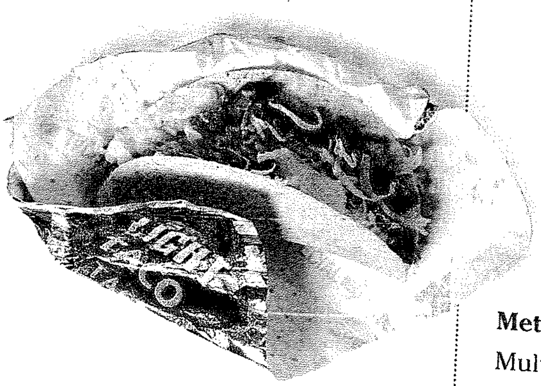
$= 14b - 22c$ *Combine like terms.*

The distributive property is often used in real-world applications.

Example 5

Real World APPLICATION

Consumerism



A Mexican restaurant is offering a new light taco supreme for a special introductory price of 99¢ to attract customers who would like a healthier choice. The number of light taco supremes sold each day for 5 days in one restaurant are given in the table below. What was the average income from the sale of the tacos during each day?

Monday	Tuesday	Wednesday	Thursday	Friday
182	341	246	303	378

An average is calculated by adding to find the total amount, then dividing the total amount by the number of items. In this case, it would be the total dollar amount divided by the number of days, 5.

There are two ways to find the total dollar amount.

Method 1

Multiply each daily amount by 0.99 and then add.

$T = 0.99(182) + 0.99(341) + 0.99(246) + 0.99(303) + 0.99(378)$

$= 180.18 + 337.59 + 243.54 + 299.97 + 374.22$

$= 1435.50$

(continued on the next page)

Method 2

Add the daily amounts and then multiply the total by 0.99.

$T = 0.99(182 + 341 + 246 + 303 + 378)$

$= 0.99(1450)$

$= 1435.50$

Now find the mean or average by dividing the total by 5.

$1435.50 \div 5 = 287.10$

The average daily income from light taco supremes was $287.10.

The two methods of calculating the average income illustrate the distributive property of multiplication over addition.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Explain** how you know π is an irrational number given that the digits of π never end and never repeat a pattern.
2. **Examine** the Venn diagram on page 13 that shows the relationships among all real numbers. Explain why natural numbers are enclosed by whole numbers, integers, and rationals.
3. **List** five rational numbers and five irrational numbers.
4. **Explain** the difference between the commutative and associative properties.

MODELING MATHEMATICS

5. You can model algebraic expressions with algebra tiles. The unit tile ☐ represents 1. The x tile ▭ represents x.
   a. Model $x + 1$ with algebra tiles.
   b. Model $2(x + 1)$ with algebra tiles.
   c. Use algebra tiles to show that $2(x + 1) = 2x + 2$.

Guided Practice

Find the value of each expression. Then name the sets of numbers to which each value belongs.

6. $7 - 6$ 7. $6 - 7$ 8. $6 \div 2^2$ 9. $\sqrt{49 + 8}$

Determine whether each statement is *true* or *false*. If *false*, give an example of a number that shows the statement is false.

10. Every integer is a whole number.
11. Every whole number is an integer.

Name the property illustrated by each equation.

12. $8(4) = 4(8)$ 13. $2 + (-2) = 0$ 14. $6 + (0 + 4) = 6 + (4 + 0)$

Name the additive inverse and the multiplicative inverse of each number.

15. 7 16. $-\frac{2}{3}$

Simplify each expression.

17. $2(4c + 5d) + 6(2c - d)$ 18. $3x + 5y + 7x - 3y$

19. **Baby-sitting** Maria baby-sat for 5 hours on Friday night and 7 hours on Saturday night to earn money for band camp. She charges $3 per hour. How much money did Maria earn for band camp?

EXERCISES

Practice

Find the value of each expression. Then name the sets of numbers to which each value belongs.

20. $2.9 + 3.7$
21. $-56 \div 8$
22. $58 \div 100$
23. -4.2×10
24. $1 - 5$
25. $\sqrt{25} - 6$
26. $3^3 + 2^2$
27. $1\frac{1}{2} + \frac{3}{4}$
28. $10 \times (-3.9)$
29. $4 \div 2^3$
30. $-81 \div (-9)$
31. $\sqrt{64 + 3}$

Determine whether each statement is *true* or *false*. If *false*, give an example of a number that shows the statement is false.

32. Every real number is irrational.
33. Every integer is a rational number.
34. Every rational number is an integer.
35. Every irrational number is a real number.
36. Every natural number is an integer.
37. Every real number is either a rational number or an irrational number.

Name the property illustrated by each equation.

38. $m(4 - 3) = m \cdot 4 - m \cdot 3$
39. $(5 + 9) + 13 = 13 + (5 + 9)$
40. $s + t + 0 = s + t$
41. $(a + b) + [-(a + b)] = 0$
42. $(3 + 9) \cdot 5 = 3(5) + 9(5)$
43. $(48)3 = 3(48)$
44. $\frac{4}{5}(1) = \frac{4}{5}$
45. $\left(\frac{1}{4}\right)4 = 1$

Name the additive inverse and the multiplicative inverse of each number.

46. 8
47. 0.2
48. -1.25
49. -1
50. $\frac{5}{6}$
51. $-3\frac{5}{7}$

Simplify each expression.

52. $6x - 2y - 3x + 2y$
53. $4(14c - 10d) - 6(d + 4c)$
54. $\frac{1}{2}(17 - 4x) - \frac{3}{4}(6 - 16x)$
55. $\frac{3}{4}(2x - 5y) + \frac{1}{2}\left(\frac{2}{3}x + 4y\right)$
56. $\frac{1}{4}(12 + 20a) + \frac{3}{4}(12 + 20a)$
57. $7(0.2m + 0.3n) + 5(0.6m - n)$

Critical Thinking

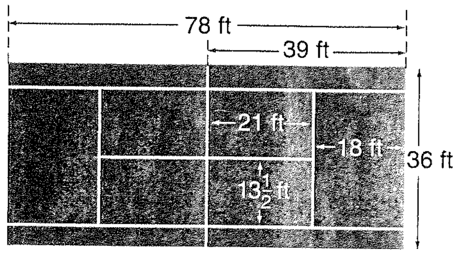
58. Use the properties of real numbers to answer these questions.
   a. If $a + b = a$, what is the value of b?
   b. If $ab = 1$, what is the value of b? What is b called?
   c. If $ab = a$, what is the value of b?

Applications and Problem Solving

Real World

59. **Accounting** A number is divisible by 9 if the sum of its digits is divisible by 9. This fact is used by accountants to check figures in double entry books. If the totals of the credit and debit columns do not match, and the difference between the totals is divisible by 9, then the error was probably made when two digits were reversed in one of the entries. Explain whether the errors in the following might have come from reversing the digits of an entry.
   a. credit = \$638, debit = \$577
   b. credit = \$1050, debit = \$1095
   c. Why does this error check work?

60. **Geometry** Find the area of the tennis court at the right in two different ways.



Mixed Review

Find the value of each expression. (Lesson 1–1)

61. $0.2(0.5 + 2.2) \div 6$
62. $8 - [21 - (3 \cdot 5)]$
63. $3(13 - 7) + (7 - 5)2$
64. $17 - [22 \div (21 - 2(5))]$

65. Evaluate the expression $\frac{3a + 4c}{b}$ if $a = -3$, $b = 2$, and $c = 0.5$. (Lesson 1–1)

66. Evaluate the expression $d - [c \div (a - b(a))]$ if $a = -1$, $b = 5$, $c = 12$, and $d = 17$. (Lesson 1–1)

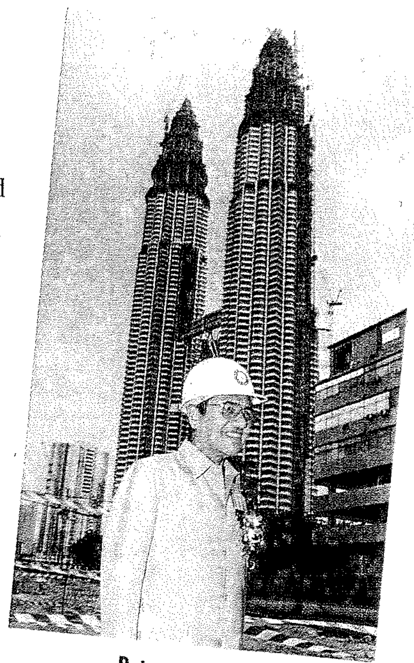
67. **ACT Practice** The ratio of girls to boys in a class is 4 to 3. If there are a total of 35 students in the class, how many girls are in the class?
   A 4 **B** 7 **C** 15
   D 16 **E** 20

68. **Geometry** The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$. Find the area of a trapezoid with height 3 and bases 4 and 8. (Lesson 1–1)

69. Use the formula $I = prt$ to find the simple interest if $p = \$2500$, $r = 7.37\%$, and $t = 4$ years. (Lesson 1–1)

70. **Buildings** The Sears Tower in Chicago is the tallest building in the United States. It is 1454 feet tall, only 22 feet shorter than the Petronas Towers in Kuala Lumpur, Malaysia, the tallest buildings in the world. Use the formula $Y = \frac{F}{3}$ to find the height of both buildings in yards if Y is the height in yards and F is the height in feet. (Lesson 1–1)

71. Evaluate $12 + 18 \div 6 + 7$. (Lesson 1–1)


Petronas Towers

For Extra Practice, see page 876.

1-3

Integration: Statistics
Graphs and Measures of Central Tendency

What YOU'LL LEARN

- To represent and interpret data using line plots and stem-and-leaf plots, and
- to find and use the median, mode, and mean to interpret data.

Why IT'S IMPORTANT

Measures of central tendency can help you easily describe a set of data.

Real World APPLICATION

Education

The average salaries for teachers in 1997–98 are listed in the table below. How do teacher salaries in your state compare with other states?

Average Teacher Salaries (1997–98)							
State	**Salary**	**State**	**Salary**	**State**	**Salary**	**State**	**Salary**
AL	$32,799	IN	$39,752	NE	$32,668	SC	$33,608
AK	$48,275	IA	$34,084	NV	$40,572	SD	$27,839
AZ	$34,071	KS	$33,800	NH	$36,663	TN	$34,584
AR	$32,119	KY	$34,453	NJ	$50,284	TX	$33,537
CA	$44,585	LA	$30,090	NM	$30,309	UT	$32,981
CO	$37,240	ME	$34,349	NY	$48,712	VT	$36,299
CT	$51,727	MD	$41,404	NC	$33,123	VA	$37,024
DE	$42,439	MA	$44,285	ND	$28,231	WA	$38,755
FL	$34,473	MI	$48,361	OH	$39,099	WV	$33,396
GA	$37,412	MN	$39,104	OK	$30,940	WI	$38,179
HI	$36,598	MS	$28,691	OR	$42,301	WY	$32,022
ID	$32,834	MO	$34,001	PA	$47,542		
IL	$43,707	MT	$30,617	RI	$44,506		

Source: American Federation of Teachers

CAREER CHOICES

Statisticians collect, analyze, and present data for surveys and experiments. Their work is very important in the fields of economics, biology, engineering, medicine, and psychology.

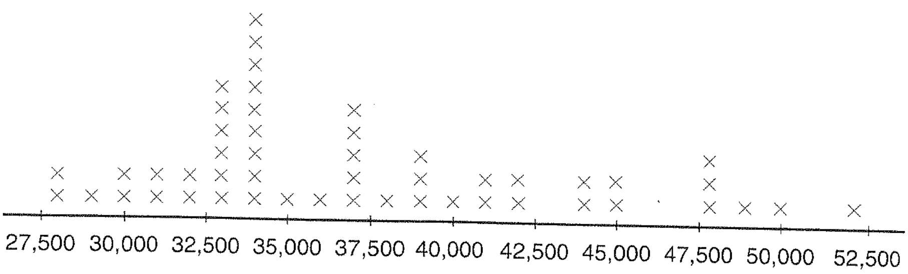
A bachelor's degree with a major in statistics or mathematics is required to enter the field.

For more information, contact:

American Statistics Association
1429 Duke St.
Alexandria, VA 22314

The answer to this question is not immediately obvious from the table. You are familiar with statistical graphs such as *bar graphs, line graphs,* and *circle graphs* that are used to organize and illustrate data. Another way to display statistical data such as that provided in the table above is on a number line called a **line plot.** Like other statistical graphs, line plots can help you see patterns and variability in data.

To make a line plot, determine a scale that includes all of the data and appropriate intervals. Then plot each number using a symbol to represent the data. The teacher salaries data ranges from 27,839 to 51,727. Round each salary to the nearest thousand. Let's use a scale of 27,500 to 52,500 with intervals of 2500 and denote each salary with an "×."



Locate your state's data on the line plot. Now it should be easy to see how teachers' salaries in your state compare with those in other states.

A **stem-and-leaf plot** can also be used to display data in a compact way. When making a stem-and-leaf plot, each item of data is separated into two parts. The *stems* usually consist of the digits in the greatest common place value of each item of data. The *leaves* contain the other digits of each item of data. For example, if the greatest common place value is thousands, then the stem for 1573 is 1, and the leaf is 573.

If data values have many digits, you may want to round each item of data before you plot it in a stem-and-leaf plot. This way each leaf will have only one digit. If 1573 is rounded to 1600, then the stem is 1 and the leaf is 6.

Example 1

Real World APPLICATION

Education

Refer to the application at the beginning of the lesson. Make a stem-and-leaf plot of the rounded salaries.

Round each item of data to the nearest thousand.
So, using rounded data, 3 | 0 = 29,500 to 30,499, inclusive.

Stem	Leaf
2	
·	8 8 9
3	0 0 1 1 2 2 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4
·	5 6 7 7 7 7 7 8 9 9 9
4	0 1 1 2 2 4 4
·	5 5 8 8 8 9
5	0 2

To make the plot easier to interpret, the stems have been broken into two parts. 4 | = $40,000 to $44,000 and · | = $45,000 to $49,000

A **back-to-back stem-and-leaf plot** is sometimes used to compare two sets of data or rounded values of the same set of data. In a back-to-back plot, the same stem is used for the leaves of both plots.

Example 2

CONNECTION

Civics

The number of Republicans and Democrats in the U.S. Senate from 1965–1997 is given in the table below.
a. Make a back-to-back stem-and-leaf plot of the data.
b. Compare the number of Republican and Democratic senators since 1965.

Political Divisions of the U.S. Senate (1965–1997)

Years	Number of Democrats	Number of Republicans	Years	Number of Democrats	Number of Republicans
1965–67	68	32	1981–83	46	53
1967–69	64	36	1983–85	46	54
1969–71	58	42	1985–87	47	53
1971–73	54	44	1987–89	54	46
1973–75	56	42	1989–91	57	43
1975–77	61	37	1991–93	57	43
1977–79	61	38	1993–95	56	44
1979–81	58	41	1995–97	47	53

Source: *World Almanac*

a.

Democrats	Stem	Republicans
	3	2 6 7 8
6 6 7 7	4	1 2 2 3 3 4 4 6
4 4 6 6 7 7 8 8	5	3 3 3 4
1 1 4 8	6	

***fabulous* FIRSTS**

Nydia M. Velazquez (1953–)

In 1992, Nydia Velazquez became the first Puerto Rican woman elected to the U.S. House of Representatives. She earned a master's degree from New York University and served on the City Council of New York City.

b. The stem-and-leaf plot shows that there have usually been more Democratic senators than Republican senators. The number of Democratic senators ranged from 46 to 68, while the number of Republican senators ranged from 32 to 54.

Sometimes it is convenient to have one number that describes a set of data. This number is called a **measure of central tendency** because it represents the center or middle of the data. The most commonly-used measures of central tendency are the **median, mode,** and **mean.**

Definition of Median, Mode, and Mean	
Median:	**The median of a set of data is the middle value. If there are two middle values, it is the mean of the two middle values.**
Mode:	**The mode of a set of data is the most frequent value. Some sets of data have multiple modes and others have no mode.**
Mean:	**The mean of a set of data is the sum of all the values divided by the number of values.**

Example 3

Real World APPLICATION

Consumerism

Over the years, jeans have become very popular with consumers. Some prices for these jeans are listed below. Find the median, mode, and mean prices.

\$44.99	**\$39.99**	**\$39.99**	**\$27.99**	**\$32.00**	**\$34.99**
\$29.99	**\$32.99**	**\$44.99**	**\$29.99**	**\$24.99**	**\$44.99**

Median: To find the median of the jean data, arrange the dollar values in order.

44.99 44.99 44.99 39.99 39.99 34.99 32.99 32.00 29.99 29.99 27.99 24.99

If there were an odd number of dollar values, the middle one would be the median. However, since there is an even number of dollar values, the median is the average of the two middle values, 34.99 and 32.99.

$$\text{median} = \frac{34.99 + 32.99}{2} = \frac{67.98}{2} = 33.99$$

The median is \$33.99. Notice that the number of values that are greater than the median is the same as the number of values that are less than the median.

Mode: To find the mode of the jean data, look for the number that occurs most often. In this set, \$44.99 appears three times. Thus, the mode of the data is \$44.99.

Mean: To find the mean of the jean data, find the sum of the dollar values and divide by 12, the number of values in the set.

$$\text{mean} = \frac{44.99 + 44.99 + 44.99 + \ldots + 24.99}{12} = \frac{427.89}{12} = 35.6575$$

The mean of the data is approximately \$35.66.

As you can see, the median, mode, and mean are not always the same number. In Example 3, the mode is greater than both the median and the mean. This means that more of the jeans cost $44.99 than any other price. In this example, the mean is the most representative average of the prices. If you buy any pair of jeans in the set of data, you will pay an average of $35.66.

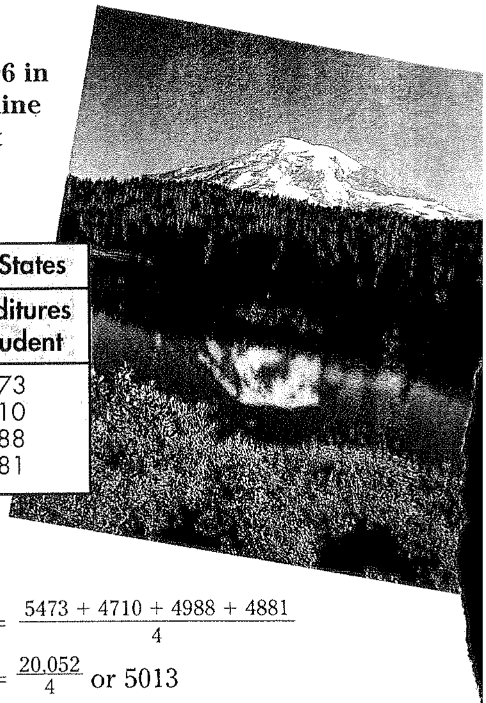
Extreme values are those data values that vary greatly from the central group of data values. Every data value affects the value of the mean, so when extreme values are included in a set of data, the mean may become less representative of the set. However, the values of the median and the mode are not affected by extreme values in the set.

Example 4

Real World APPLICATION

Business

The amounts of money spent per student in 1996 in two regions of the U.S. are listed below. Determine the mean of the values in each column. To what extent is each mean representative of the data?



Pacific States		Southwest Central States	
State	Expenditures Per Student	State	Expenditures Per Student
Alaska	$9012	Texas	$5473
California	5108	Arkansas	4710
Washington	6044	Louisiana	4988
Oregon	6615	Oklahoma	4881

Source: U.S. Dept. of Ed.

$$\text{mean} = \frac{9012 + 5108 + 6044 + 6615}{4}$$

$$= \frac{26{,}779}{4} \text{ or } 6694.75$$

$9012 is an extreme value. In this case, the mean is not representative of the data.

$$\text{mean} = \frac{5473 + 4710 + 4988 + 4881}{4}$$

$$= \frac{20{,}052}{4} \text{ or } 5013$$

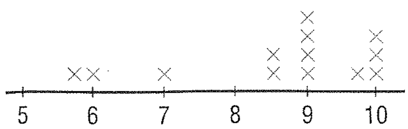
There are no extreme values in this set. In this case, the mean is representative of the data.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Describe** what the line plot below tells you about the overall scores for the floor exercises of the Buckeye Gymnastics team members.



2. **Explain** how the number 98.6 would be plotted on a stem-and-leaf plot if 54.6 is plotted using stem 5 and leaf 4.
3. **Explain** what an extreme value does to the median.
4. **Tell** which measure, the median, mode, or mean, must be a member of the set of data.

MATH JOURNAL

5. **Assess Yourself** List the main advantages and disadvantages for using the median, mode, and mean to describe a set of data.

Guided Practice

Find the median, mode, and mean for each set of data.

6. 0, 2, 2, 3, 4

7. 4, 5, 8, 10, 12

8. 7, 7, 7, 7, 7, 7

internet CONNECTION

Data Update For more information on salaries of college graduates, visit: **www.algebra2.glencoe.com**

9. The table shows the starting salaries of graduating college students with bachelor's degrees in various fields of study in 1997.
   a. Make a stem-and-leaf plot of the data rounded to the nearest thousand.
   b. What is the mean salary?
   c. What is the median salary?
   d. Which is the mode salary?
   e. Write three questions that can be answered using this data.

Field of Study	Bachelor's Degree
Accounting	$30,393
Business, general	28,506
Marketing	28,658
Engineering:	
Civil	32,170
Chemical	42,758
Electrical	39,811
Industrial	37,732
Mechanical	39,852
Packaging	35,353
Chemistry	31,261
Mathematics	32,055
Physics	31,972
Education	25,742
Social sciences	24,232
Computer science	36,964

Source: Michigan State University Collegiate Employment Research Institute

EXERCISES

Practice

Find the median, mode, and mean for each set of data.

10. 298, 256, 399, 388, 276

11. 3, 75, 58, 7, 34

12. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1

13. 80, 50, 65, 55, 70, 65, 75, 50

14. 61, 89, 93, 102, 45, 89

15. 13.3, 15.4, 12.5, 10.7

16. 101, 192, 121, 153, 101

17. 43, 43, 55, 43, 54, 42, 51

18. 2301, 2324, 2000, 1999, 2738, 1947, 1989, 2004, 2938

19. Find the median, mode, and mean for the stem-and-leaf plot at the right. Round to the nearest whole number if necessary.

Stem	Leaf
5	5 5
6	0 4 5 6 7 8
7	0 1 1 1 2 2 5 6 6
8	0 5 6 8 8 8

$8|5 = 85$

20. In Mrs. Elizondo's algebra class, report card averages are based on 100 points. Tests and quizzes account for $\frac{2}{3}$ of the final average. Homework, classwork, and journals account for $\frac{1}{3}$ of the final average. Jennifer's test and quiz scores are 100, 100, 88, 76, 95, 88, and 93. What is Jennifer's average on tests and quizzes?

21. The Millersburg school board is negotiating a pay raise with the teacher's union. Three of the administrators have salaries of $80,000 each. However, a majority of the teachers have salaries of about $35,000 per year.
   a. You are a member of the school board and would like to show that the current salaries are reasonable. Would you quote the median, mode, or mean as the "average" salary to justify your claim? Explain.
   b. You are the head of the teacher's union and maintain that a pay raise is in order. Which of the median, mode, or mean would you quote to justify your claim? Explain your reasoning.

Graphing Calculator

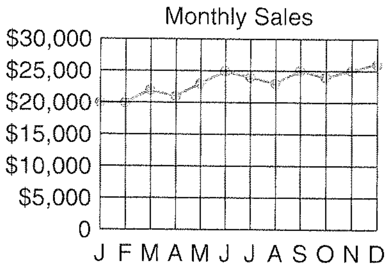
22. Use a graphing calculator to find the mean and median for each set of data below. Access the LIST MATH function by pressing [2nd] [LIST] [▶]. Then choose 3, for mean or 4, for median. Enter the numbers by first pressing [2nd] [{] and ending with [2nd] [}], a right parenthesis, and [ENTER].

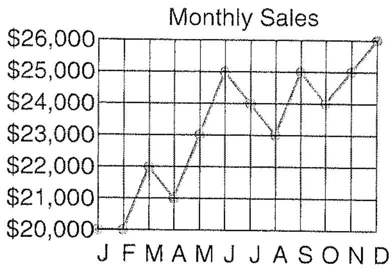
   a. 3, 5, 7, 5, 3, 8, 2

   b. 45.7, 64.8, 33.2, 66.1, 54.4, 64.5

Critical Thinking

23. Statistics can sometimes be misleading if an incorrect representation of the data leads to wrong conclusions. Study the two graphs below.





   a. Explain why the graphs made from the same data look different.

   b. Explain a situation where graph A might be used.

   c. Explain a situation where graph B might be used.

24. Choose five whole numbers from 1 to 10 for each situation below. Numbers may be used more than once.

   a. Construct a set of data in which the median, mode, and mean are the same number.

   b. Find the median of a set of data with the greatest possible mean.

   c. Construct a set of data in which the median and mean are the same number and there is no mode.

Applications and Problem Solving

Real World

25. **Employment** The table below lists the employment figures for 11 states during a recent month.

Employment, Selected States

State	Employed	Unemployed
California	14,411,000	1,197,000
Florida	6,384,000	445,000
Illinois	5,672,000	378,000
Massachusetts	2,979,000	205,000
Michigan	4,570,000	247,000
New Jersey	3,830,000	277,000
New York	8,048,000	561,000
North Carolina	3,443,000	180,000
Ohio	5,282,000	274,000
Pennsylvania	5,428,000	344,000
Texas	8,842,000	555,000

Source: *USA Today*

   a. Round each number to the nearest hundred thousand. Make a back-to-back stem-and-leaf plot of the data.

   b. What observations can you make from the data?

   c. How could you calculate the unemployment rate for each state?

   d. Calculate the approximate unemployment rate for each state.

26. **Advertising** A camera store placed an ad in the newspaper showing five videocameras for sale. The ad says, "Our videocameras average $695." The prices of the videocameras are $1200, $999, $1499, $895, $695, $1100, $1300, and $695.

a. Find the median, mode, and mean of the prices.

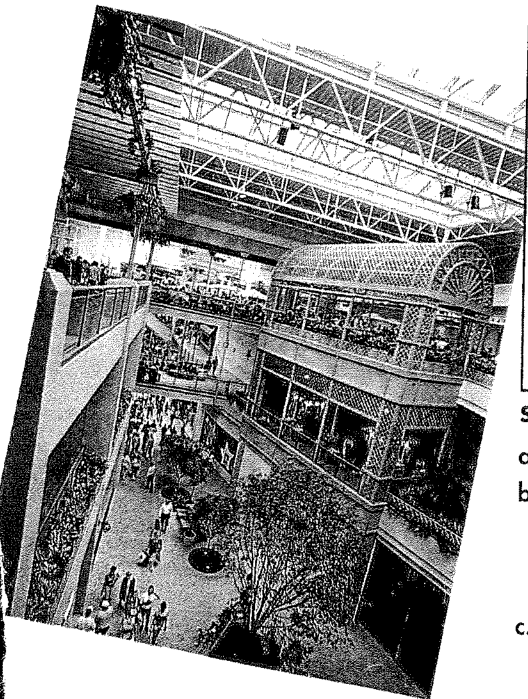
b. Which measure was the store using in its ad? Why did they choose this measure?

c. As a consumer, which measure would you want to see advertised? Explain your reasoning.

27. **Business** The minimum start-up costs for some of the fastest-growing franchises in a recent year are listed below. Find the median, mode, and mean for the costs. Which average is the most representative of the data? Explain.

Business	Minimum Start-up Cost
convenience stores	$12,500
submarine sandwiches	38,900
retail hardware	12,600
retail hardware	42,500
carpet, upholstery, and drapery services	3,550
pizza	170,000
hamburgers	73,000
commercial cleaning services	3,250
postal and business services	28,180
commercial cleaning services	425
commercial cleaning services	2,500
donuts	175,000

28. **Shopping Malls** The table below lists the areas of some of the largest shopping malls in the United States.



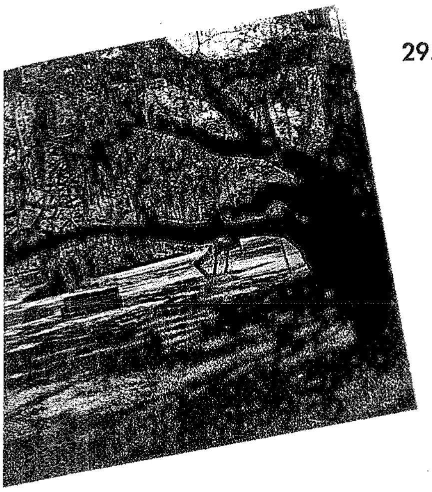
	Mall	Gross Leasable Area (sq ft)
1	Del Amo Fashion Center, Torrance, California	3,000,000
2	South Coast Plaza/Crystal Court, Costa Mesa, California	2,918,236
3	Mall of America, Bloomington, Minnesota	2,472,500
4	Lakewood Center Mall, Lakewood, California	2,390,000
5	Roosevelt Field Mall, Garden City, New York	2,300,000
6	Gurnee Mills, Gurnee, Illinois	2,200,000
7	The Galleria, Houston, Texas	2,100,000
8	Randall Park Mall, North Randall, Ohio	2,097,416
9	Oakbrook Shopping Center, Oak Brook, Illinois	2,006,688
10	Sawgrass Mills, Sunrise, Florida	2,000,000
10	The Woodlands Mall, The Woodlands, Texas	2,000,000
10	Woodfield, Schaumburg, Illinois	2,000,000

Source: Blackburn Marketing Service

a. Find the median, mode, and mean of the gross leasable area.

b. You are a realtor who is trying to lease mall space in different areas of the country to a large retailer. Which measure would you talk about if the customer felt that the price per square foot was expensive to lease? Explain.

c. Which measure would you talk about if the customer had lots of inventory to display? Explain.



29. **Weather** The average monthly temperatures for Mobile, Alabama, are listed below.

January, 59.7°	May, 84.6°	September, 86.9°
February, 63.6°	June, 90.0°	October, 79.5°
March, 70.9°	July, 91.3°	November, 70.3°
April, 78.5°	August, 90.5°	December, 62.9°

a. Make a line plot. Round each temperature to the nearest degree.
b. Use the line plot to find the median, mode, and mean.
c. Suppose each of the average monthly temperatures rose 5°. Make a line plot of these new temperatures.
d. How do the two line plots compare? Are the median, mode, and mean the same or different? Explain.
e. How could you make the new line plot from the original line plot?

Mixed Review

30. Name the property illustrated by $5 + (2 + x) = (2 + x) + 5$. (Lesson 1-2)
31. Simplify $a(3 + 5) - 6(3a - 1)$. (Lesson 1-2)
32. Simplify $\frac{2}{3}(6a - 18) + 3(2a - 9)$. (Lesson 1-2)
33. **SAT Practice Grid-in** Twenty-five bottles contain a total of 15 liters of lemonade. If each bottle contains the same amount of lemonade, how much lemonade (in liters) is in each bottle?
34. Evaluate $(r + 7) \div s$ if $r = 20$ and $s = 3$. (Lesson 1-1)
35. **Chemistry** The boiling point of the metal zinc is 787.1°F. Use the formula $C = \frac{5(F - 32)}{9}$ to find the equivalent Celsius temperature. (Lesson 1-1)

For Extra Practice, see page 876.

Mathematics and SOCIETY

What, No Census?

The article below appeared in *Business Week* on November 28, 1994.

SINCE 1790, THE GOVERNMENT HAS FAITHfully tried to count every resident once a decade. Now, a panel of statisticians has a blunt message for the Census Bureau: Give up. For 2000, a National Academy of Sciences committee says a "fundamentally redesigned" census should rely heavily on statistical sampling and estimation. The committee says that these methods can produce a more accurate—and far less costly—count of residents who don't respond to the census' mail-in questionnaire, especially minority groups. Congress, angered by the inaccuracy and expense of the 1990 census, is likely to embrace the committee's proposal. ■

1. Each census compiles large amounts of data about us, including age, sex, race, education, home address, household size and composition, and income level. What uses could the government make of the census data? Who else might be interested in the data?
2. Why would a census using statistical sampling and estimation be less expensive than the past method of trying to count everyone by using questionnaires?
3. Describe how you might use statistical sampling to study a group of people that you want to learn more about. What factors would you want to consider in planning your procedures?

1-4

Solving Equations

What YOU'LL LEARN

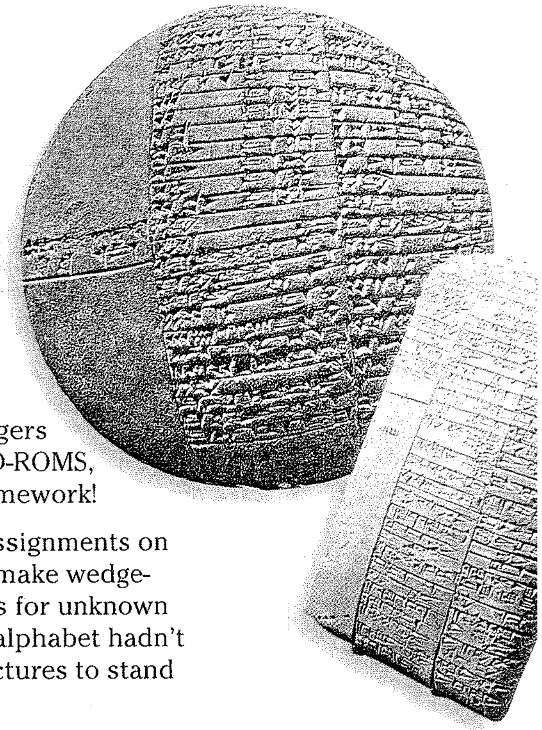
- To translate verbal expressions and sentences into algebraic expressions and equations,
- to solve equations by using the properties of equality, and
- to solve equations for a specific variable.

Why IT'S IMPORTANT

You can use equations to solve problems involving history, astronomy, and travel.

CONNECTION

History



Have you ever thought it would have been great to live 4500 years ago in Babylon because you wouldn't have had algebra homework? Those teenagers may not have had shopping malls, CD-ROMS, or TVs, but they did have algebra homework!

Babylonian students wrote their assignments on clay tablets with little sticks used to make wedge-shaped marks. They didn't use letters for unknown values in equations because today's alphabet hadn't been invented. Instead, they drew pictures to stand for the unknowns.

A Greek mathematician named Diophantus introduced algebraic symbols to write equations. The chart below shows examples of his equations along with today's algebraic form.

Diophantine Equation	Modern Meaning
ζισβ	$x = 2$
ζγισθ	$x + 3 = 9$
ζγβισθ	$3x + 2 = 9$
ζθΛγισβ	$9x - 3 = 2$
ζβΛθισζγ	$2x - 9 = x + 3$

GLOBAL CONNECTIONS

Diophantus of Alexandria, who lived about A.D. 250, was famous for his work in algebra. His main work was titled *Arithmetica* and introduced symbolism to Greek algebra as well as propositions in number theory and polygonal numbers.

The language of today's algebra provides a powerful way to translate word expressions into algebraic or mathematical expressions. **Variables** are used to represent numbers that are not known. Any letter can be used as a variable.

Verbal Expression	Algebraic Expression
• *a number* increased by 4	$x + 4$
• twice the cube of *a number*	$2n^3$
• the square of *a number* decreased by the cube of the *same number*	$c^2 - c^3$
• three times the sum of *a number* and 6	$3(b + 6)$

Sentences with variables to be replaced, such as $4x - 8 = 32$ and $2x + 4 > 9$, are called **open sentences.** An open sentence that states that two mathematical expressions are equal is called an **equation.** Equations can be used to represent verbal mathematical sentences.

Verbal Sentence	Equation
• Nine is equal to five plus four.	$9 = 5 + 4$
• *A number* decreased by 6 is −3.	$m - 6 = -3$
• *A number* divided by 3 is equal to $\frac{3}{4}$.	$\frac{x}{3} = \frac{3}{4}$

To solve an equation, you find replacements for the variables that make the equation true. Each of these replacements is called a **solution** of the equation.

We can use certain properties of equality to solve equations or open sentences. Some of those properties are listed below.

Reflexive Property of Equality	**For any real number a, $a = a$.**
Symmetric Property of Equality	**For all real numbers a and b, if $a = b$, then $b = a$.**
Transitive Property of Equality	**For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.**
Substitution Property of Equality	**If $a = b$, then a may be replaced by b.**

Example 1 **Name the property illustrated by each statement.**

a. If 1.5(7.5) = 11.25, then 11.25 = 1.5(7.5).

symmetric property of equality

b. If 7 = 1 + 2 + 4 and 1 + 2 + 4 = 4 + 3, then 7 = 4 + 3.

transitive property of equality

Sometimes an equation can be solved by adding or subtracting the same number on each side.

Addition and Subtraction Properties of Equality	**For any numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.**

Example 2 **Solve $x + 54.57 = 78$.** *Estimate: 80 − 55 = 25*

$$x + 54.57 = 78$$

$$x + 54.57 - 54.57 = 78 - 54.57 \quad \text{Subtract 54.57 from each side.}$$

$$x = 23.43$$

Check:

$$x + 54.57 = 78$$

$$23.43 + 54.57 \stackrel{?}{=} 78 \quad \text{Replace } x \text{ with } 23.43.$$

$$78 = 78 \quad \checkmark$$

The solution is 23.43.

Some equations may be solved by multiplying or dividing each side by the same number.

Multiplication and Division Properties of Equality	**For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.**

Example 3 **Solve each equation.**

a. $4x = -12$

$$4x = -12$$
$$\frac{1}{4}(4x) = \frac{1}{4}(-12) \quad \textit{Multiply each side by } \frac{1}{4}\textit{, the reciprocal of 4.}$$
$$x = -3$$

The solution is -3.

Check:
$$4x = -12$$
$$4(-3) \stackrel{?}{=} -12$$
$$-12 = -12 \checkmark$$

This equation could also be solved by dividing each side by 4.

b. $-\frac{3}{4}t = 15$

$$-\frac{3}{4}t = 15$$
$$-\frac{4}{3}\left(-\frac{3}{4}\right)t = \left(-\frac{4}{3}\right)(15) \quad \textit{Multiply each side by } -\frac{4}{3}\textit{, the reciprocal of } -\frac{3}{4}.$$
$$t = -20$$

The solution is -20.

Check:
$$-\frac{3}{4}t = 15$$
$$-\frac{3}{4}(-20) \stackrel{?}{=} 15$$
$$15 = 15 \checkmark$$

In order to solve some equations, it may be necessary to apply more than one property.

Example 4 **Solve $3(2a + 25) - 2(a - 1) = 78$.**

$$3(2a + 25) - 2(a - 1) = 78$$
$$6a + 75 - 2a + 2 = 78 \quad \textit{Distributive and substitution properties}$$
$$4a + 77 = 78 \quad \textit{Commutative, distributive, and substitution properties}$$
$$4a = 1 \quad \textit{Subtraction and substitution properties}$$
$$a = \frac{1}{4} \quad \textit{Division and substitution properties}$$

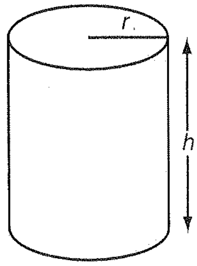
The solution is $\frac{1}{4}$. *Check this result.*

Sometimes you need to solve an equation or formula for a variable.

Example 5

INTEGRATION
Geometry

The formula for the volume of a cylinder is $V = \pi r^2 h$, where V is the volume, r represents the radius of the circular base and top and h represents the height of the cylinder. Solve the formula for h.



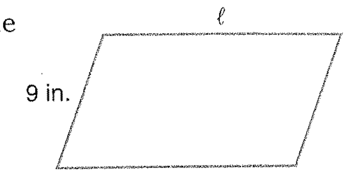
$$V = \pi r^2 h$$
$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad \textit{Divide each side by } \pi r^2.$$
$$\frac{V}{\pi r^2} = h$$

You can use a four-step **problem-solving plan** to help you solve problems.

Problem-Solving Plan	**1. Explore the problem.** **2. Plan the solution.** **3. Solve the problem.** **4. Examine the solution.**

Example 6 INTEGRATION Geometry

The perimeter of a parallelogram is 48 inches. What is the length of the longer side if the shorter side measures 9 inches?



Explore Draw a diagram and let ℓ represent the measure of the longer side.

Plan The perimeter equals the sum of the lengths of the sides. So, we can write the following equation.

$$2(9) + 2(\ell) = 48$$

Solve

$$2(9) + 2(\ell) = 48$$
$$18 + 2\ell = 48$$
$$\ell = 15$$

The length of the longer side is 15 inches.

Examine If one of the two longer sides has length 15 inches and one of the shorter sides has length 9 inches, the perimeter is $15 + 15 + 9 + 9 = 48$ inches. Thus, the answer is correct.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Write** two verbal expressions and two verbal sentences containing unknown quantities. Write each as an algebraic expression or equation.
2. **Summarize** the properties you studied in this lesson. Exchange summaries with another student. Discuss your summaries and make any necessary revisions.
3. **Explain** the difference between an equation and an expression.
4. **Write** an equation to find the length of a side of a regular pentagon if its perimeter is 250 inches.
5. **You Decide** Was this equation solved correctly? If not, explain what error was made. Then show how to solve the equation correctly.

$$4(a + 5) - 2(a + 6) = a + 16$$
$$4a + 20 - 2a - 12 = a + 16$$
$$2a + 8 = a + 16$$
$$a = 8$$

Guided Practice

Write an algebraic expression to represent each verbal expression.

6. three decreased by twice a number
7. five times a number decreased by three

Name the property illustrated by each statement.

8. If $r + 2 = 8$, then $r = 6$.
9. If $4x = 16$, then $12x = 48$.

Solve each equation.

10. $10 + 5x = 110$
11. $-2(a + 4) = 2$
12. $3b + 4b + 5b = 30$
13. $7 + 5n = -58$
14. $-1.4t + 3 = -7.5$
15. $-\frac{2}{3}k = 14$

Solve each equation or formula for the variable specified.

16. $2x - 3m = 6$, for x
17. $V = \frac{1}{3}\pi r^2 h$, for h

Define a variable, write an equation, and solve the problem. Then check your solution.

18. Marisa is 16 years old. Her parents are both the same age. The three of them have lived a total of 100 years. How old are Marisa's parents?

EXERCISES

Practice

Write an algebraic expression to represent each verbal expression.

19. fourteen decreased by the square of a number
20. twice the sum of a number and 11
21. four times the sum of a number and its square
22. the product of the square of a number and five
23. the sum of 7 and three times a number
24. the square of the sum of a number and 13

Name the property illustrated by each statement.

25. $(5 + 6) + 7 = (5 + 6) + 7$
26. If $4 + 8 = 12$, then $12 = 4 + 8$.
27. If $3x = 10$, then $3x + 6 = 10 + 6$.
28. If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.
29. If $7 + s = 21$, then $s = 14$.
30. If $q + (8 + 5) = 32$, then $q + 13 = 32$.

Solve each equation.

31. $3y + 16 = 22$
32. $14 - x = -7$
33. $34 - 10w = 6w + 2$
34. $t + 2t + 3t + 4t + 5t = 45$
35. $\frac{1}{8} - \frac{3}{4}x = \frac{1}{16}$
36. $\frac{3}{4} - \frac{3}{5}x = \frac{2}{5}x + \frac{2}{4}$
37. $5 = -5(y + 3)$
38. $2d + 5 = 8d + 2$
39. $280 - 26f = 1098$
40. $3(4 - 5k) = 2k - 4$
41. $4m - 9 = 5m + 7$
42. $32g + 245 = 3829$
43. $12x - 24 = -14x + 28$
44. $18 = -6(p + 5)$
45. $4.5(b + 1) - 2 = 4(b + 3)$
46. $2.3n + 1 = 1.3n + 7$
47. $\frac{5}{7}x - 4 = \frac{3}{7}x + 1$
48. $\frac{3}{4}n - 2 = \frac{1}{2}n + 7$

Solve each equation or formula for the variable specified.

49. $x(y + 2) = z$, for y

50. $I = prt$, for t

51. $5a - 6b = 9$, for b

52. $de - 4f = 5g$, for e

53. $F = G\frac{Mm}{r^2}$, for M

54. $qr + s = t$, for q

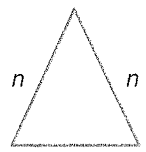
Define a variable, write an equation, and solve the problem. Then check your solution.

55. The high school principal has given Mrs. Diaz $420 to buy tickets to a Broadway play for her English class and chaperones. The school requires that there be one adult chaperone for every five students on such a trip. How many $15.00 student tickets and $30.00 adult tickets can she order?

56. Hearthstone Cars' price for a new sedan is $18,999. Sheila Rayburn offered the dealership a price of $17,099. Sheila's price is what percent of the dealership's price?

57. You have $32 to spend on supplies for your science fair project. If you buy two plants for experiments, you will have $18 left for other supplies. How much is each plant?

58. **Geometry** The perimeter of an isosceles triangle is 116 centimeters. The length of the base is 36 cm. What is the length of one of the equal sides?



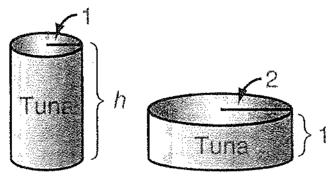
Critical Thinking

59. Write a verbal expression to represent the algebraic expression $2x(x + 4) + 2(x + 6)$.

Applications and Problem Solving

Real World

60. **Packaging** Two designs for a tuna can are shown at the right. If each can holds the same amount of tuna, what is the height of can A?

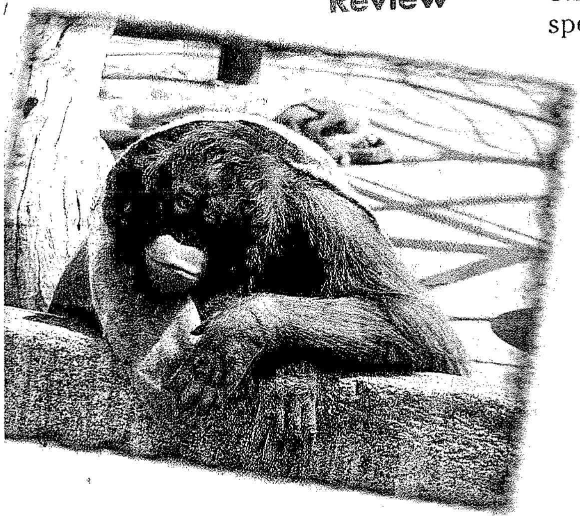


61. **Transportation** The U.S. Department of Transportation regulations prohibit a truck driver from driving more than 70 hours in any 8-day period. Over the past 7 days, a driver has accumulated 54.75 hours of driving time. Does he have enough hours available on the eighth day to deliver freight if he must drive 510 miles at a speed of 50 mph? Explain.

62. **Astronomy** Earth is about 93,000,000 miles from the sun. When Venus is on the opposite side of the sun from Earth, it is about 69,000,000 from the sun. What is the distance from Earth to Venus?

63. **Travel** The distance by water from New York City to San Francisco by way of Cape Horn is about 13,200 miles. By going through the Panama Canal, the distance is only 5280 miles. How many miles does a ship save by going through the Panama Canal?

Mixed Review



64. **Animals** The table at the right lists major U.S. zoological parks and the number of species at each park. (Lesson 1–3)

  a. Make a stem-and-leaf plot of the data. Round each number to the nearest ten.

  b. Find the median, mode, and mean of the data.

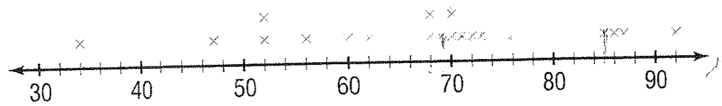
Zoo	Species
Bronx	670
Cleveland	563
Dallas	329
Detroit	413
Houston	596
Memphis	445
Minnesota	311
Philadelphia	560
Phoenix	324
San Antonio	700
San Francisco	270

65. **Statistics** Find the median, mode, and mean for the following set of data. (Lesson 1–3)

216, 399, 219, 179, 180, 399

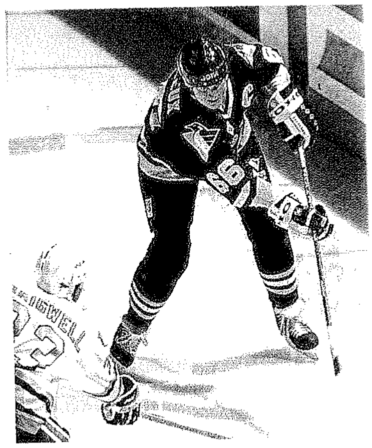
66. **Hockey** Use the line plot to answer each question. (Lesson 1–3)

Most NHL Goals in a Season



  a. What is the greatest number of goals scored in a season?

  b. In the 1995–96 season, Mario Lemieux scored 69 goals. In the 1988–89 season, he scored 85 goals. How many scores were greater than Mario's in 1995–96 but less than Mario's in 1988–89?



Mario Lemieux

67. **Weather** The average monthly temperatures in selected cities for January and July are given in the table below. Make a back-to-back stem-and-leaf plot of the temperatures of the cities rounded to the nearest degree. (Lesson 1–3)

City	January Temperature	July Temperature
Baton Rouge, LA	50.8	82.1
Caribou, ME	10.7	65.1
Charlotte, NC	40.5	78.5
Chicago, IL	21.4	73.0
Dallas, TX	44.0	86.3
Denver, CO	29.5	73.4
Indianapolis, IN	26.0	75.1
Jacksonville, FL	53.2	81.3
Juneau, AK	21.8	55.7
Roswell, NM	41.4	77.7
San Diego, CA	56.8	70.3
Tulsa, OK	35.2	83.2

Source: U.S. National Oceanic and Atmospheric Administration

68. **Statistics** Find the median, mode, and mean for the following set of data. (Lesson 1–3)

11, 10, 13, 12, 12, 13, 15

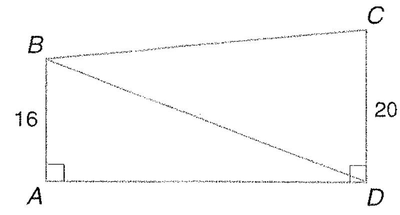
69. Simplify $2(9a - 2) - 3(5 + a)$. (Lesson 1–2)

Name the property illustrated by each equation. (Lesson 1–2)

70. $11(3a + 2b) = 11(2b + 3a)$

71. $a + b + 0 = a + b$

72. **SAT Practice** In the figure, if $AB = 16$, $CD = 20$, and the area of $\triangle ABD = 280$, what is the area of polygon $ABCD$?



A 560 **B** 630 **C** 700 **D** 840 **E** 980

73. **Banking** Find the interest earned in 6 years on a savings account containing \$20,000 if the interest rate is 14.5%. (Lesson 1–1)

74. Evaluate $3^b - a + c$ if $a = 13$, $b = 2$ and $c = 9$. (Lesson 1–1)

75. Evaluate $4 - [32 \div (16 - 12)]$. (Lesson 1–1)

76. Evaluate $[18 - (5 + 22)] \times 2$. (Lesson 1–1)

For Extra Practice, see page 877.

SELF TEST

Evaluate each expression if $m = 2$, $n = -3$, and $p = 4$. (Lesson 1–1)

1. $m + n - p$

2. $m(n + p)$

Find the value of each expression. Then name the sets of numbers to which each value belongs. (Lesson 1–2)

3. $7 - 8$

4. $3.9 + 2.6$

5. $\sqrt{36} + 5$

6. **Consumerism** The Super Shoes catalog contains 29 styles of shoes that can be ordered through the mail. The prices are \$53, \$42, \$49, \$38, \$39, \$48, \$37, \$48, \$37, \$39, \$58, \$59, \$32, \$50, \$59, \$37, \$36, \$30, \$40, \$33, \$30, \$45, \$40, \$30, \$35, \$48, \$37, \$48, and \$50. (Lesson 1–3)
   a. Make a stem-and-leaf plot of the shoe prices.
   b. Find the median, mode, and mean of the prices.

Solve each equation. (Lesson 1–4)

7. $4.5 - 3.9m = 20.1$

8. $9 = 16d + 51$

9. $2y - 8 = 14 - 9y$

10. $285 - 38x = 2033$

1–5A Graphing Technology
Using Tables to Estimate Solutions

A Preview of Lesson 1–5

You can use a graphing calculator to estimate solutions to equations by building tables of values.

Example 1 **Estimate the solution of $12x - 3 = 5$ to the nearest hundredth.**

Rewrite the equation in an equivalent form to get 0 on one side.

$12x - 3 = 5 \quad \rightarrow \quad 12x - 8 = 0$

To estimate the solution means to find a value for x so that $12x - 8$ is very close to 0. Let $y = 12x - 8$ and make a table of values for x and y. First, enter $y = 12x - 8$.

Enter: Y= 12 X,T,θ,n − 8

Then, set up a table of values for x and y. You must enter a starting value for x and an increment for successive values. Let's start with $x = 0$ and use increments of 1.

Enter: 2nd TblSet 0 ENTER

1 2nd Table

X	Y1
0	−8
1	4
2	16
3	28
4	40
5	52
6	64

X=0

We need to find the value of x when $y = 0$. From the table, we can see that x is a value between 0 and 1. To get a better approximation for x, create a new table that starts at $x = 0$ and use increments of 0.1.

Enter: 2nd TblSet 0 ENTER

.1 2nd Table

X	Y1
.2	−5.6
.3	−4.4
.4	−3.2
.5	−2
.6	−.8
.7	.4
.8	1.6

X=.7

Use the arrow keys to scroll down the table to determine where $y = 0$. The solution is between 0.6 and 0.7. Continue to adjust the estimated value for x.

Enter: 2nd TblSet .6 ENTER

.01 2nd Table

X	Y1
.62	−.56
.63	−.44
.64	−.32
.65	−.2
.66	−.08
.67	.04
.68	.16

X=.66

Repeating this process one more time, we see that the solution is between 0.666 and 0.667. Thus, the solution is about 0.67.

Example 2 involves an absolute value equation.

Example 2 **Estimate the solutions of $|1.5x - 3| = 0.7$ to the nearest hundredth.**

Rewrite the equation as $|1.5x - 3| - 0.7 = 0$. Then, enter the equation and set up the table. Let's start with $x = 0$ and then use increments of 1 for the x values.

Enter: [Y=] [2nd] [ABS] [(] 1.5 [X,T,θ,n] [−] 3 [)] [−] .7 [2nd] [TblSet] 0 [ENTER] 1 [2nd] [Table]

X	Y1
0	2.3
1	.8
2	−.7
3	.8
4	2.3
5	3.8
6	5.3

X=1

We need to find the value of x when $y = 0$. From the table, we can see that x is a value between 1 and 2 and also between 2 and 3. Begin to adjust your estimate by repeating the process.

Enter: [2nd] [TblSet] 1 [ENTER] .1 [2nd] [Table]

X	Y1
1	.8
1.1	.65
1.2	.5
1.3	.35
1.4	.2
1.5	.05
1.6	−.1

X=1

The solution is between 1.5 and 1.6. Continue the process to estimate the first solution. Then adjust your estimate for the second solution.

Enter: [2nd] [TblSet] 2 [ENTER] .1 [2nd] [Table]

X	Y1
2	−.7
2.1	−.55
2.2	−.4
2.3	−.25
2.4	−.1
2.5	.05
2.6	.2

X=2

The solution is between 2.4 and 2.5. Continue the process to estimate the second solution.

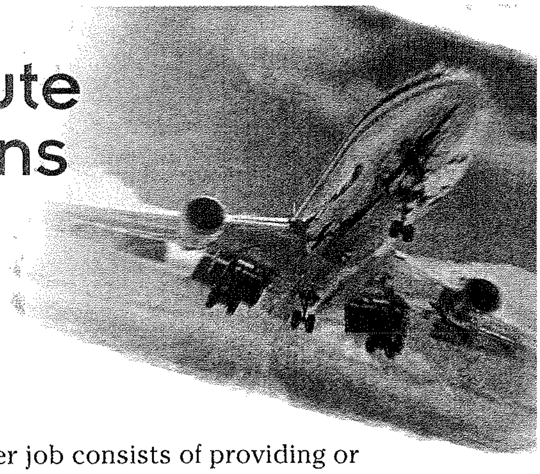
The solutions are about 1.53 and 2.47.

EXERCISES

Use the TABLE feature to estimate the solution(s) of each equation to the nearest hundredth.

1. $4x + 6 = 9$
2. $3.5x + 7 = 11$
3. $-1.25 - 0.3x = 8$
4. $5(x - 3) = -2$
5. $2x + 1 = 12 - x$
6. $|6x + 4| - 7 = 2$
7. $|3 - x| = 5$
8. $|2.21 + 0.55x| = 1.75$
9. $\left|\frac{1}{2}x - 5\right| = 17$
10. $40 = \frac{5}{9}(x - 32)$

1–5 Solving Absolute Value Equations



What YOU'LL LEARN

- To solve equations containing absolute value, and
- to solve problems by making lists.

Why IT'S IMPORTANT

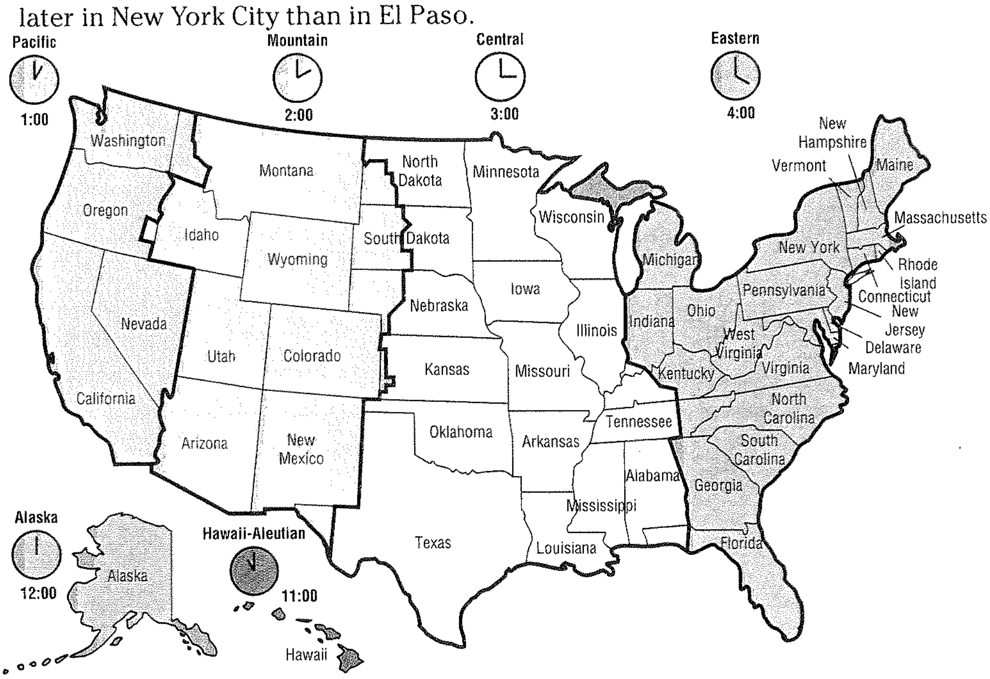
You can use absolute value equations to solve problems involving travel and manufacturing.

Real World APPLICATION

Time Zones

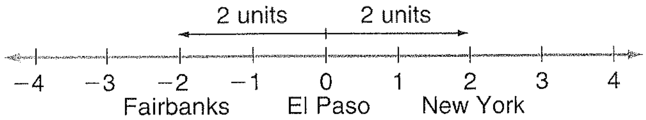
Joan Haghiri works as a consultant. Her job consists of providing or developing training for businesses or organizations to help their employees perform better. Joan travels frequently as a part of her job and often goes from one time zone to another.

Joan lives in El Paso, Texas. One week she flew to New York City to give a presentation. She called home the first night to talk to her 6-year-old son. She called at 10:00 P.M. to be in time for his 8:00 P.M. bedtime. Since New York is in the Eastern time zone and El Paso is in the Mountain time zone, it is two hours later in New York City than in El Paso.



While in Fairbanks, Alaska, the next week, Joan had to remember to call her son at 6:00 P.M. Alaska time. This is because it is two hours later in El Paso.

On a number line, the time zone for El Paso is the starting point and corresponds to 0. New York City is at $+2$, while Fairbanks is at -2.



Certainly -2 and 2 are quite different, but they do have something in common. They are the same distance from 0 on the number line.

We say that -2 and 2 have the same **absolute value.** The absolute value of a number is the number of units it is from 0 on the number line. We use the symbol $|x|$ to represent the absolute value of a number x.

The absolute value of -2 is 2. $|-2| = 2$

The absolute value of 2 is 2. $|2| = 2$

We can also define absolute value in the following way.

Absolute Value	**For any real number a:** **if $a \geq 0$, then $\lvert a \rvert = a$;** **if $a < 0$, then $\lvert a \rvert = -a$.**

The symbol ≥ means "is greater than or equal to."

Example 1 **Find each absolute value.**

a. $|9|$

$|9| = 9$

b. $|-14|$

$|-14| = -(-14)$ or 14

You will solve certain kinds of problems by using the strategy **list possibilities**. List possibilities is one of many *problem-solving strategies* that you can use to solve problems. Here are some other problem-solving strategies.

Problem-Solving Strategies	
draw a diagram	solve a simpler (or a similar) problem
make a table or chart	eliminate the possibilities
make a model	look for a pattern
guess and check	act it out
check for hidden assumptions	work backward
use a graph	identify subgoals

Example 2

PROBLEM SOLVING
List Possibilities

Rosa forgot the personal identification number (PIN) for her automatic teller bank card. She remembered that her PIN is the rearranged digits of her house number. If her house number is 1256, what PINs should she try in the automatic teller machine?

List the possible PINs.

Possible PINs starting with 1:

1265 1526 1562
1625 1652

Why isn't 1256 listed?

Possible PINs starting with 2:

2156 2165 2516
2561 2615 2651

Possible PINs starting with 5:

5126 5162 5216
5261 5612 5621

Possible PINs starting with 6:

6125 6152 6215
6251 6512 6521

These are the 23 possible PINs that Rosa should try in the automatic teller machine.

You can list the possibilities to help you find absolute values.

Example 3 **Find the absolute value of $x - 15$.**

Make a list of the possible cases.

Case 1: If x is 15 or greater, then $x - 15 \geq 0$.
So, $|x - 15| = x - 15$.

Case 2: If x is less than 15, then $x - 15 < 0$.
So, $|x - 15| = -(x - 15)$ or $15 - x$.

You can evaluate expressions that contain absolute values. The absolute value bars can be grouping symbols.

Example 4 **Evaluate $|3x - 6| + 3.2$ if $x = -2$.**

$$
\begin{aligned}
|3x - 6| + 3.2 &= |3(-2) - 6| + 3.2 && \textit{Replace } x \textit{ with } -2. \\
&= |-6 - 6| + 3.2 && \textit{Simplify within absolute value bars first.} \\
&= |-12| + 3.2 \\
&= 12 + 3.2 \\
&= 15.2
\end{aligned}
$$

The value is 15.2.

Some equations contain absolute value expressions. The definition of absolute value is used in solving the equations. When an equation has more than one solution, the solutions are often written as a set, $\{a, b\}$.

Example 5 **Solve $|x - 25| = 17$. Check each solution.**

$|x - 25| = 17$ means $x - 25 = 17$ or $-(x - 25) = 17$.

$$
\begin{aligned}
x - 25 &= 17 \\
x - 25 + 25 &= 17 + 25 \\
x &= 42
\end{aligned}
\qquad \text{or} \qquad
\begin{aligned}
-(x - 25) &= 17 \\
x - 25 &= -17 \\
x - 25 + 25 &= -17 + 25 \\
x &= 8
\end{aligned}
$$

Check:

$$
\begin{aligned}
|x - 25| &= 17 \\
|42 - 25| &\stackrel{?}{=} 17 \\
|17| &\stackrel{?}{=} 17 \\
17 &= 17 \checkmark
\end{aligned}
\qquad \text{or} \qquad
\begin{aligned}
|x - 25| &= 17 \\
|8 - 25| &\stackrel{?}{=} 17 \\
|-17| &\stackrel{?}{=} 17 \\
17 &= 17 \checkmark
\end{aligned}
$$

The solutions are 42 and 8. Thus, the solution set is $\{42, 8\}$.

Example 6

CONNECTION

Health

***Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both are potentially dangerous conditions and occur when a person's body temperature is more than 8° above or below the normal body temperature of 98.6°F. At what temperatures do these conditions begin to occur?**

Let b = normal body temperature. Then solve $|b - 98.6| = 8$.

$$|b - 98.6| = 8$$

$$
\begin{aligned}
b - 98.6 &= 8 \\
b &= 106.6
\end{aligned}
\qquad \text{or} \qquad
\begin{aligned}
b - 98.6 &= -8 \\
b &= 90.6
\end{aligned}
$$

The solutions are 106.6 and 90.6. Hypothermia occurs when the body temperature is below 90.6°F. Hyperthermia occurs when the body temperature is above 106.6°F.

Sometimes an equation has no solution. For example, $|x| = -6$ is never true. Since the absolute value of a number is always positive or zero, there is no replacement for x that will make that sentence true. The solution set has no members. It is called the **empty set** and is symbolized by { } or $\varnothing$.

Example 7 **Solve $|2x + 7| + 5 = 0$.**

$$|2x + 7| + 5 = 0$$
$$|2x + 7| = -5$$

This sentence is *never* true, so the solution set is $\varnothing$.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equations is used, the answers may not be actual solutions to the original equation.

Example 8 **Solve $|x - 2| = 2x - 10$.**

$|x - 2| = 2x - 10$ means $x - 2 = 2x - 10$ or $x - 2 = -(2x - 10)$.

$x - 2 = 2x - 10$	or	$x - 2 = -(2x - 10)$
$-2 = x - 10$		$x - 2 = -2x + 10$
$8 = x$		$3x = 12$
		$x = 4$

Check:	$|x - 2| = 2x - 10$		$|x - 2| = 2x - 10$
	$|8 - 2| \stackrel{?}{=} 2(8) - 10$	or	$|4 - 2| \stackrel{?}{=} 2(4) - 10$
	$|6| \stackrel{?}{=} 16 - 10$		$|2| \stackrel{?}{=} 8 - 10$
	$6 = 6$ ✓		$2 \neq -2$

The only solution is 8.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Explain** why an absolute value equation can have two solutions.
2. **Write** a convincing argument for why $|x - 7| + 4 = 0$ has no solution.
3. **You Decide** Janelle says that $-a$ is always negative in the definition of absolute value. (If $a < 0$, then $|a| = -a$.) Bobby says that even $-a$ can be positive. Who is correct, and why?
4. **Determine** whether $-x < x$ is always true, sometimes true, or never true. Explain your reasoning.
5. **Write** an absolute value equation that has no solution.

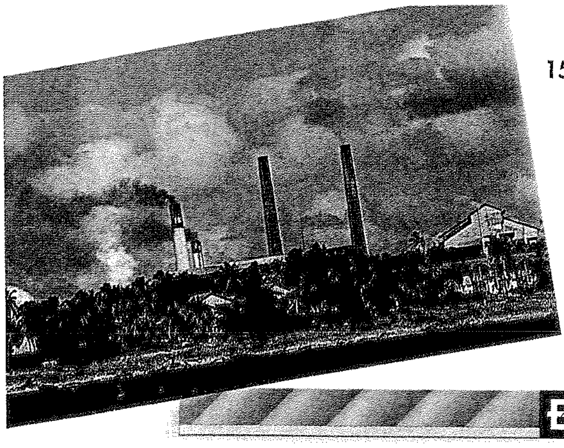
Guided Practice

Evaluate each expression if $x = 2.5$.

6. $|x + 6|$
7. $|-2x|$
8. $-|x + 10|$

Solve each equation.

9. $|x + 5| = 18$
10. $|x + 9| = 25$
11. $|x - 6| = 12$
12. $|x - 3| = 15$
13. $|3 + x| = 45$
14. $6|5x + 2| = 312$



15. **Manufacturing** A machine is to fill each of several boxes with 16 ounces of sugar. After the boxes are filled, another machine weighs the boxes. If the box is more than 0.2 ounces above or below the desired weight, the box is rejected.

a. Write an absolute value equation to find the heaviest and lightest box the machine will approve.

b. Solve the equation.

EXERCISES

Practice

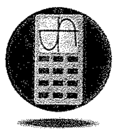
Evaluate each expression if $x = -4$, $y = 5$, and $z = 1.2$.

16. $|-4x|$
17. $|2y - 5|$
18. $|3z|$
19. $|x + 5|$
20. $|-2y|$
21. $-|2z - 4|$
22. $6 - |4y + 10|$
23. $7 - |3z + 10|$
24. $3|x + 4| + |3x|$

Solve each equation.

25. $|x - 3| = 17$
26. $|x + 6| = 18$
27. $|x + 11| = 42$
28. $3|x + 6| = 36$
29. $11|x - 9| = 121$
30. $|2x + 9| = 30$
31. $8|x - 3| = 88$
32. $|2x + 7| = 0$
33. $|4x - 3| = -27$
34. $8|4x - 3| = 64$
35. $3|3x + 2| = 51$
36. $5|x + 4| = 45$
37. $4|6x - 1| = 29$
38. $|3t - 5| = 2t$
39. $|2a + 7| = a - 4$
40. $|x - 3| + 7 = 2$
41. $3|x + 6| = 9x - 6$
42. $5|3x - 4| = x + 1$

Programming



43. The graphing calculator program at the right tests decimal values to estimate the solution for $|x^2 - 2| = 0$. Enter a possible solution for x. The program will test it, give you the value of $|x^2 - 2|$, and tell you if you need to try again. If you guess correctly, the program will give you both solutions.

```
PROGRAM:ABSVALUE
:Prompt X
:Disp "abs(X²-2)=",abs(X²-2)
:If abs(X²-2)=0
:Then
:Goto 1
:End
:Disp "TRY AGAIN"
:Disp "PRESS ENTER"
:Stop
:Lbl 1
:Disp "SOLUTIONS ARE",X
:Disp "AND",-X
```

Use the program to approximate to the nearest tenth the solutions for each equation. You will need to enter the equation into Y1 on the Y= list for each exercise. All solutions are between −10 and 10.

a. $x^2 - 2x - 4 = 0$; 2 solutions
b. $x^3 - 3x = 0$; 3 solutions
c. $|3x - 2| - 4 = 0$; 2 solutions
d. $5x^3 + 3x^2 - 25x - 15 = 0$; 3 solutions

Critical Thinking

44. Solve $|x + 2| = |2x - 4|$ and explain your method of solution.

Applications and Problem Solving

Real World

45. **Contests** A car dealership is having a contest to win a new car. To win a chance at the car, you must guess the number of keys in a jar within 5 of the actual number. The people who are within this range get to try a key in the ignition of the car. Suppose there are 697 keys in the jar.
    a. Write an equation to determine the highest and lowest guesses that will win a chance at the car.
    b. Solve the equation.

46. **Chemistry** For hydrogen to be a liquid, its temperature must be within 2°C of −257°C.
    a. Write an equation to determine the least and greatest temperatures for this substance to remain a liquid.
    b. Solve the equation.

47. **List Possibilities** The telephone number of a local business is 555-1829. They are trying to make a word from the last three digits of their number so that customers will remember it easily. The digit 8 can be T, U, or V; 2 can be A, B, or C; 9 can be W, X, or Y. List the possible combinations of letters that their number can represent.

Mixed Review

48. Write an algebraic expression to represent *four plus three times a number.* (Lesson 1–4)

49. Solve $3 - 2x = 18$. (Lesson 1–4)

50. **Geometry** The perimeter of a square is 42 inches. Find the length of one side of the square. (Lesson 1–4)

51. **Zoology** The population of gorillas at a zoo was decreased when five of them were moved to another zoo. Write an expression to represent the original number of gorillas at the zoo if there are p gorillas there now. (Lesson 1–4)

52. **ACT Practice** A student computed the average of his test scores by adding the 7 scores together and dividing by the number of tests. The average was 87. But he completely forgot one test, which had a score of 79. What is the true average of his tests?

    A 83 **B** 84 **C** 85 **D** 86 **E** 87

53. **Statistics** Find the median, mode, and mean for the stem-and-leaf plot at the right. (Lesson 1–3)

Stem	Leaf
13	0 2 3 3 5 7
14	1 1 1 5 7 8 9 9
15	5 6 7 7 *14|5 = 145*

54. Name the property illustrated by $\frac{3}{8}(1) = \frac{3}{8}$. (Lesson 1–2)

55. Evaluate -2.4×10. Then name the sets of numbers to which it belongs. (Lesson 1–2)

For **Extra Practice**, see page 877.

56. Find the value of $12a^2 + bc$ if $a = 3$, $b = 7$, and $c = -2$. (Lesson 1–1)

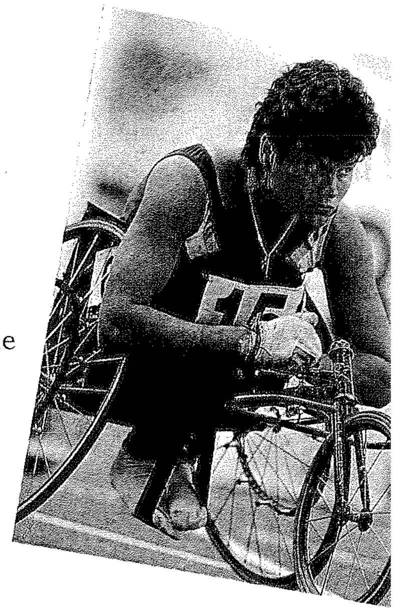
1–6 Solving Inequalities

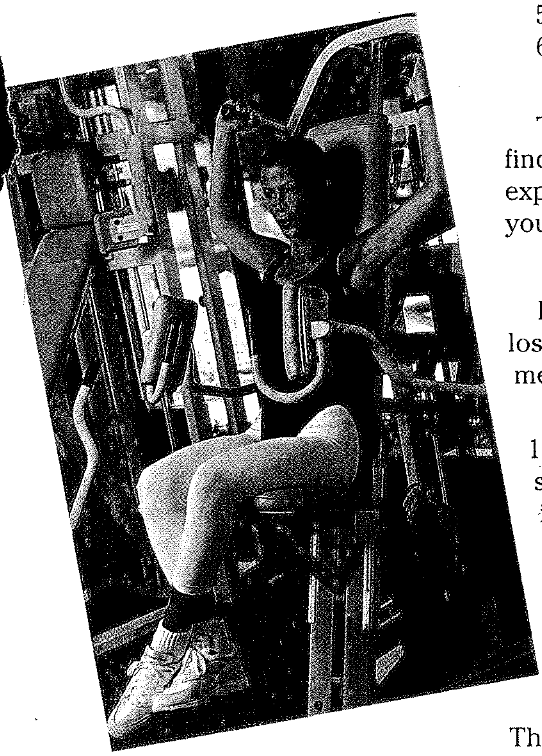
***What* YOU'LL LEARN**

- To solve inequalities and graph the solution sets.

***Why* IT'S IMPORTANT**

You can use inequalities to solve problems involving health, school, and shopping.





CONNECTION

Health

Kristin read in a magazine about an effective way for women to calculate the optimal daily Calorie intake, which is the number of Calories used while at rest. The steps are listed below.

1. Multiply your weight in pounds w by 4.3.
2. Multiply your height in inches h by 4.7.
3. Add the numbers.
4. Add 655 to your result from step 3.
5. Multiply your age a times 4.7.
6. Subtract the product in step 5 from the expression in step 4.

The expression for optimal Calorie intake is $4.3w + 4.7h + 655 - 4.7a$. To find the number of Calories you burn with moderate activity, multiply the expression by 1.3. This gives you the number of Calories per day to maintain your weight.

$$1.3(4.3w + 4.7h + 655 - 4.7a)$$

If you keep your daily Calorie intake between the two numbers, you should lose weight. If you take in fewer Calories than you burn at rest, your metabolism will slow down and you may lose less weight.

A 30-year old woman weighing 130 pounds and 65 inches tall burns about 1400 Calories a day at rest and about 1800 Calories with moderate activity. If she eats 1800 Calories per day, she should maintain her weight. Her actual intake will either be greater than, less than, or equal to her optimal intake of 1800 Calories.

Let x represent her optimal intake and a represent her actual intake. You can compare the optimal and actual intakes using an inequality or an equation.

$$x > a \qquad x < a \qquad x = a$$

This is an illustration of the **trichotomy property.**

Trichotomy Property	**For any two real numbers, a and b, exactly one of the following statements is true.** $a < b \quad a = b \quad a > b$

Adding the same number to each side of an inequality does not change the truth of the inequality.

Addition and Subtraction Properties for Inequalities	**For any real numbers, a, b, and c:** **1. if $a > b$, then $a + c > b + c$ and $a - c > b - c$;** **2. if $a < b$, then $a + c < b + c$ and $a - c < b - c$.**

These properties can be used to solve inequalities. The solution sets of inequalities can then be graphed on number lines.

Example 1 **Solve $6x + 3 > 5x - 2$. Graph the solution set.**

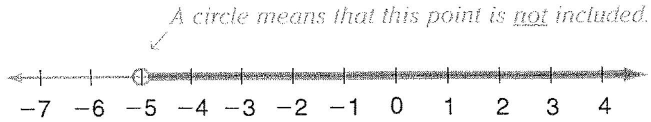
$$6x + 3 > 5x - 2$$

$$-5x + 6x + 3 > -5x + 5x - 2 \quad \textit{Add } -5x \textit{ to each side.}$$

$$x + 3 > -2$$

$$x + 3 + (-3) > -2 + (-3) \quad \textit{Add } -3 \textit{ to each side.}$$

$$x > -5$$



Any real number greater than -5 is a solution.

Check: Substitute -5 for x in $6x + 3 > 5x - 2$. The two sides should be equal. Then substitute a number greater than -5. The inequality should be true.

You know that $12 > -3$ is a true inequality. What happens if you multiply the numbers on each side by a positive number or by a negative number? Is it still true?

Multiply by 6.

$$12 > -3$$

$$6(12) \overset{?}{>} 6(-3)$$

$$72 > -18 \quad \text{true}$$

Multiply the inequality by other positive numbers. Do you think that the inequality will always remain true?

Multiply by $-\frac{1}{3}$.

$$12 > -3$$

$$-\tfrac{1}{3}(12) \overset{?}{>} -\tfrac{1}{3}(-3)$$

$$-4 > 1 \quad \text{false}$$

If you reverse the inequality, the statement is true.

$$-4 < 1 \quad \text{true}$$

Try other negative numbers as multipliers.

This suggests that when you multiply each side of an inequality by a negative number, the order of the inequality must be reversed.

These and other examples suggest the following properties.

Multiplication and Division Properties for Inequalities

For any real numbers *a*, *b*, and *c*:

1. **if *c* is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$;**
2. **if *c* is positive and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$;**
3. **if *c* is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$;**
4. **if *c* is negative and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.**

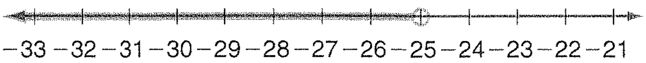
Examples 2 and 3 show how to use these properties when solving inequalities.

Example 2 **Solve $-0.4p > 10$. Graph the solution set.**

$-0.4p > 10$

$\frac{-0.4p}{-0.4} < \frac{10}{-0.4}$ *Reverse the inequality sign because each side is divided by a negative number.*

$p < -25$



Any real number less than −25 is a solution.

The solution in Example 2 can be written using set-builder notation. This solution set can be written as $\{p \mid p < -25\}$. This is read as *the set of all numbers p such that p is less than −25.*

Example 3 **Solve $-y \geq \frac{y+6}{7}$.**

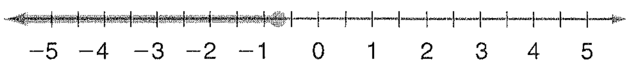
$-y \geq \frac{y+6}{7}$

$-7y \geq y + 6$ *Multiply each side by 7.*

$-8y \geq 6$ *Add $-y$ to each side.*

$y \leq -\frac{3}{4}$ *Divide each side by −8, reversing the inequality sign.*

A dot means that this point is included.



The solution set is $\left\{y \mid y \leq -\frac{3}{4}\right\}$.

You can use a graphing calculator to find the solution of an inequality graphically.

EXPLORATION GRAPHING CALCULATORS

You can use the inequality symbols in the TEST menu on the TI-83 graphing calculator to find the solution to an inequality in one variable. Use the standard viewing window.

Your Turn

a. Clear the Y= list. Enter $8x + 5 < 6x - 3$ as Y1. Press GRAPH. Describe what you see. (The symbol < is item 5 on the TEST menu.)

b. Use the TRACE function to scan the values along the graph. What values of x are on the graph? What do you notice about the values of Y on the graph?

c. Solve the inequality algebraically. How does your solution compare to the pattern you noticed in part b?

Inequalities can be used to solve many verbal problems. You can solve problems with inequalities in the same way you solve problems with equations.

Example 4 **Real World APPLICATION** **School**

Ron's scores on the first three of four 100-point chemistry tests were 90, 96, and 86. What score must he receive on the fourth test to have an average of at least 92 for all the tests?

Explore Let s represent the score needed on the fourth test. *The phrase at least 92 means greater than or equal to 92.*

Plan The average of Ron's test scores is their sum divided by 4. This number must be greater than or equal to 92. Write an inequality. Let s represent the score on the fourth test.

$$\underbrace{\frac{90 + 96 + 86 + s}{4}}_{\text{Ron's average}} \underbrace{\geq}_{\text{is greater than or equal to}} \underbrace{92}_{92.}$$

Solve

$$\frac{90 + 96 + 86 + s}{4} \geq 92$$

$$90 + 96 + 86 + s \geq 368 \quad \textit{Multiply each side by 4.}$$

$$s \geq 96$$

Examine Ron must score at least 96 on the fourth test to average at least 92 for all the tests.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Draw** a graph that shows the solution set $\{x \mid x > -2\}$.
2. **Write** *half of five times a number is less than or equal to 10* as an inequality.
3. **Solve** the following problems.
   a. Choose the correct symbol ($<$, $>$, or $=$) to make each statement true.

      $6 \underline{\ ?\ } 3 \qquad 6 \underline{\ ?\ } -3 \qquad -6 \underline{\ ?\ } 3 \qquad -6 \underline{\ ?\ } -3$

   b. Divide each side of each inequality above by 2. Record each result. Are the inequalities still true?
   c. Divide each side of the original inequalities by -2. Are the inequalities still true? Explain your results.

Guided Practice

Solve each inequality. Graph the solution set.

4. $x > 4.5$
5. $7 \leq 4a$
6. $7 - b \geq 5$
7. $3x + 4 \geq 19$
8. $2c + 15 \geq 3$
9. $\frac{d}{10} - 2 \leq 0$
10. $-0.5y < 6$
11. $\frac{7x + 1}{8} > \frac{7x}{8} + 1$

Define a variable and write an inequality for each problem. Then solve.

12. Four times a number is less than 32.
13. A number plus fifteen is greater than or equal to 27.

EXERCISES

Practice

Solve each inequality. Graph the solution set.

14. $6x < 30$
15. $-5r > 25$
16. $11 - 5y < -77$
17. $0.06 + x < 2$
18. $15 - 5t \geq 55$
19. $6x + 4 \geq 34$
20. $3(4x + 7) < 21$
21. $8x + 5 \geq 10$
22. $40 \leq -6(5r - 7)$
23. $7x - 5 > 3x + 4$
24. $9(2x + 3) > 10$
25. $5(3z - 3) \leq 60$
26. $7 - 2m \geq 0$
27. $2(m - 5) - 3(2m - 5) < 5m + 1$
28. $0.01x - 4.23 \geq 0$
29. $3b - 2(b - 5) < 2(b + 4)$
30. $0.75x - 0.5 < 0$
31. $2.55x - 4.24 \leq 0$
32. $\frac{2x + 3}{5} \leq 0.03$
33. $\frac{3x - 3}{5} < \frac{6(x - 1)}{10}$
34. $\frac{4x + 2}{5} \geq -0.04$
35. $-x \geq \frac{x + 4}{7}$
36. $\frac{x + 8}{4} - 1 > \frac{x}{3}$
37. $20\left(\frac{1}{5} - \frac{w}{4}\right) \geq -2w$

Define a variable and write an inequality for each problem. Then solve.

38. The product of 11 and a number is less than 53.
39. Three fourths of a number decreased by 25 is at least 8.
40. The opposite of five times a number is less than 321.
41. Fifty-seven is greater than one-half a number.
42. Ninety decreased by 5 is greater than or equal to the product of a number and 10.
43. Sixty-two is less than the opposite of 6 times a number.

Graphing Calculator

Use a graphing calculator to solve each inequality.

44. $-49 > 7(2x + 3)$

45. $8r + 3(2 + 7.5) < 25$

46. $2(3k + 4) - 3 \leq 2(-1)$

47. $-5s + 4(3 - 5) < 7$

Critical Thinking

48. Find the set of all numbers that satisfy $3x - 2 \geq 0$ and $5x - 1 \leq 0$.

Applications and Problem Solving

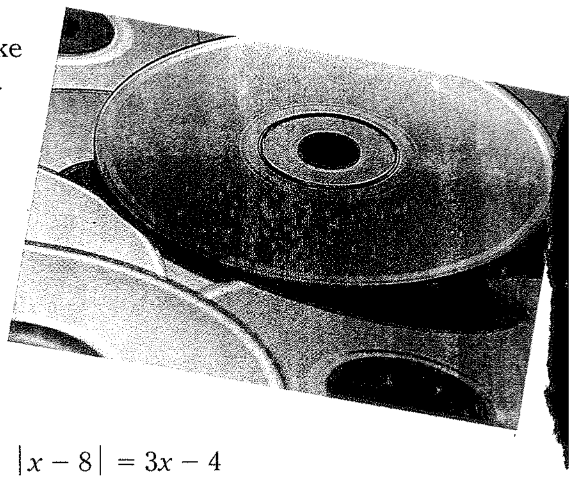
Real World

49. **Health** The National Heart Association recommends that less than 30% of a person's total daily caloric intake come from fat. One gram of fat yields nine Calories. Jason is a healthy 21-year old male whose average daily caloric intake is between 2500 and 3300 Calories.

   a. Write an inequality that represents the suggested fat intake for Jason.

   b. What is the greatest suggested fat intake for Jason?

50. **Consumerism** Tala is buying holiday gifts for her family early this year to take advantage of sales. A music store has a select group of CDs on sale for \$4.99 each. Tala finds 2 jazz CDs for her father, who loves jazz. If she has \$75 to spend on her family, write an inequality that tells how much money Tala has to spend on her other family members.



Mixed Review

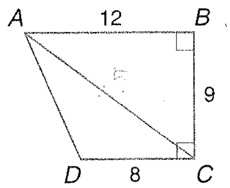
Solve each equation. (Lesson 1–5)

51. $|x - 4| = 11$

52. $|x - 8| = 3x - 4$

53. Evaluate $|2x - 4| + 1.2$ if $x = -3$. (Lesson 1–5)

54. **ACT Practice** In the figure, $\angle B$ and $\angle BCD$ are right angles, BC is 9 units, AB is 12 units, and CD is 8 units. What is the area, in square units, of $\triangle ACD$?



A 36   B 60   C 72   D 135   E 216

55. Use a calculator to solve $68x + 373 = 802$. (Lesson 1–4)

56. **Statistics** Find the median, mode, and mean for the following set of data. 2, 56, 8, 43, 44 (Lesson 1–3)

57. Name the property illustrated by $x(7 - 5) = x \cdot 7 - x \cdot 5$. (Lesson 1–2)

58. Simplify $5(3m - 7n) + 3(4m + n)$. (Lesson 1–2)

59. **Electricity** Find the amount of current I (in amperes) produced if the electromotive force E is 1.5 volts, the circuit resistance R is 2.35 ohms, and the resistance r within a battery is 0.15 ohms, using the formula $I = \frac{E}{R + r}$. (Lesson 1–1)

For Extra Practice, see page 877.

1-7

Solving Absolute Value Inequalities

What YOU'LL LEARN

- To solve compound inequalities using *and* and *or*, and
- to solve inequalities involving absolute value and graph the solutions.

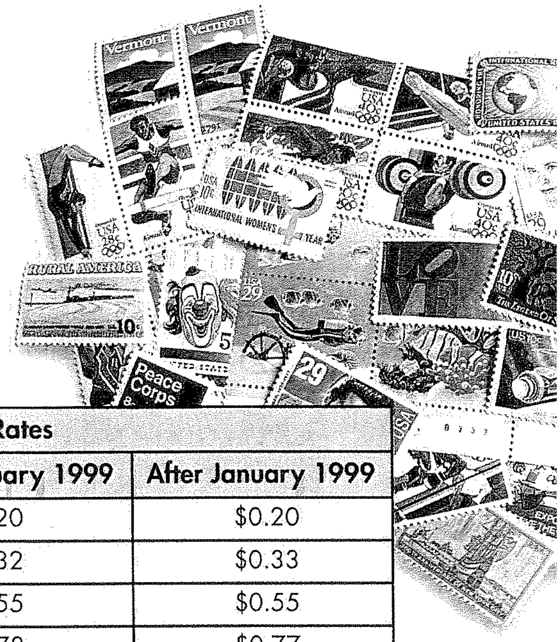
Why IT'S IMPORTANT

You can use absolute value inequalities to solve problems involving entertainment and education.

Real World APPLICATION

Postal Service

In January 1999, the postage for a first-class stamp rose from \$0.32 to \$0.33. The table below shows the past and present postage rates.

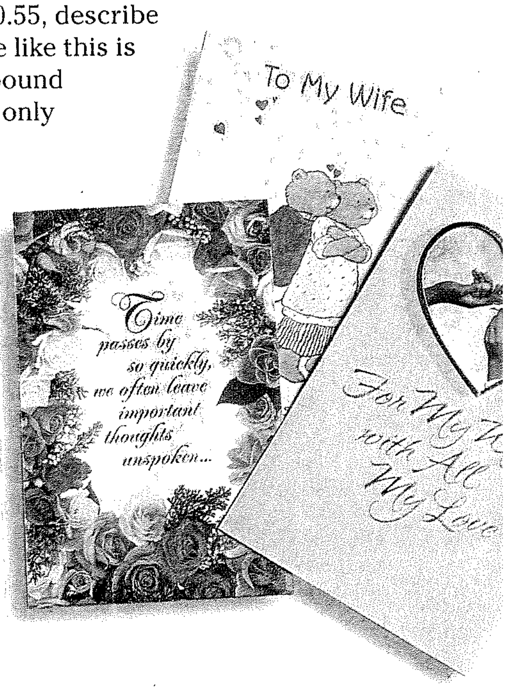


Postal Rates

Item	Before January 1999	After January 1999
Postcard	\$0.20	\$0.20
Birthday card (1 oz)	\$0.32	\$0.33
Heavy letter (2 oz)	\$0.55	\$0.55
Bank statement (3 oz)	\$0.78	\$0.77
Insured mail (\$50)	\$0.75	\$0.85
Priority mail (1 lb)	\$3.10	\$3.20
Registered mail (\$500)	\$5.40	\$6.75
Express mail (8 oz)	\$10.75	\$11.75

Source: U.S. Postal Service

If you were to mail an oversized birthday card, you would expect to pay at least \$0.33 but no more than \$0.55. Let c stand for the cost of mailing the card. The two inequalities, $c \geq 0.33$ and $c \leq 0.55$, describe the cost of mailing the card. A sentence like this is called a **compound inequality.** A compound inequality containing *and* is true if and only if both parts of it are true.



Another way of writing $c \geq 0.33$ and $c \leq 0.55$ is $0.33 \leq c \leq 0.55$. This inequality is read *c is greater than or equal to 0.33 and is less than or equal to 0.55.*

To solve a compound inequality, you must solve each part of the inequality. Thus, the graph of a compound inequality containing *and* is the **intersection** of the graphs of the two inequalities. The intersection can be found by graphing the two inequalities and then determining where these graphs overlap or intersect.

Example 1 **Solve $9 < 3x + 6 < 15$. Then graph the solution set.**

Method 1

Write the compound inequality using the word *and.* Then solve each part.

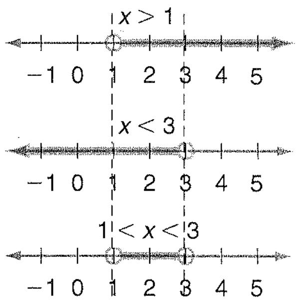
$$9 < 3x + 6 \quad \text{and} \quad 3x + 6 < 15$$
$$3 < 3x \qquad\qquad 3x < 9$$
$$1 < x \qquad\qquad x < 3$$

Method 2

Solve both parts at the same time by adding −6 to each part of the inequality. Then divide each part by 3.

$$9 < 3x + 6 < 15$$
$$9 + (-6) < 3x + 6 + (-6) < 15 + (-6)$$
$$3 \div 3 < 3x \div 3 < 9 \div 3$$
$$1 < x < 3$$

Graph each inequality and find the intersection.



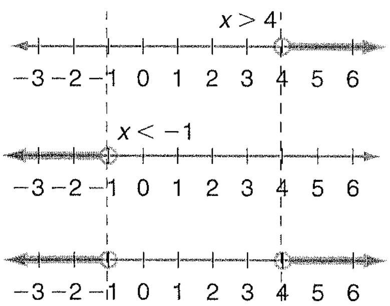
The solution set is $\{x \mid 1 < x < 3\}$.

Another type of compound inequality contains the word *or* instead of *and.* A compound inequality containing *or* is true if one or more of the inequalities is true. The graph of a compound inequality containing *or* is the **union** of the graphs of the two inequalities.

Example 2 **Solve $x - 3 > 1$ or $x + 2 < 1$. Then graph the solution set.**

Solve each part separately.

$$x - 3 > 1 \quad \text{or} \quad x + 2 < 1$$
$$x > 4 \qquad\qquad x < -1$$



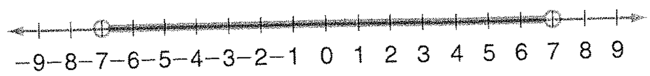
The last graph shows the solution set, $\{x \mid x > 4 \text{ or } x < -1\}$.

There is no short way to write an inequality containing "or."

Recall that the absolute value of a number is its distance from 0 on the number line. You can use this idea to solve inequalities involving absolute value.

Example 3 **Solve $|y| < 7$.**

$|y| < 7$ means that the distance between y and 0 is less than 7 units. To make $|y| < 7$ true, you must substitute values for y that are less than 7 units from 0. *Note that $|y| < 7$ is the same as $y < 7$ and $y > -7$.*

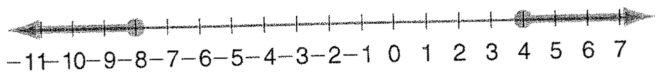


All of the numbers between -7 and 7 are less than 7 units from 0. The solution set is $\{y \mid -7 < y < 7\}$.

Example 4 **Solve $|2x + 4| \geq 12$. Graph the solution set.**

This inequality says that $2x + 4$ is greater than or equal to 12 units from 0.

$$2x + 4 \geq 12 \quad \text{or} \quad 2x + 4 \leq -12$$
$$2x \geq 8 \qquad\qquad 2x \leq -16$$
$$x \geq 4 \qquad\qquad x \leq -8$$



The solution set is $\{x \mid x \geq 4 \text{ or } x \leq -8\}$.

Example 5

Real World APPLICATION

Internet

The Kennedy Space Center website provides up-to-date information on NASA projects. The table shows the number of user sessions on the website from the top ten visiting states for November, 1999.

Kennnedy Space Center Website Top States

State	User Sessions
Virginia	88,888
California	39,250
Connecticut	5697
Texas	5448
Florida	4607
New York	3972
Illinois	3513
New Jersey	3486
Massachusettes	3246
Georgia	3097

Source: WebTrends Corporation

a. **Write a compound inequality that expresses the range of the user sessions from these states.**

b. **Suppose the number of user sessions from people in Virginia is almost always within 8000 of an average of 78, 711 sessions. Write and solve an absolute value inequality for the difference between the average number of user sessions and the number of user sessions in any given month.**

(continued on the next page)

a. Let n represent the number of user sessions. The range can be written as:

$$3097 \leq n \leq 88{,}888$$

b. Let v represent the number of user sessions from Virginia in any given month.

$$|v - 78{,}711| \leq 8000$$

$$v - 78{,}711 \leq 8000 \quad \text{and} \quad v - 78{,}711 \geq -8000$$

$$v \leq 86{,}711 \qquad\qquad v \geq -70{,}711$$

The solution set is $\{v \mid v \geq 70{,}711 \text{ and } v \leq 86{,}711\}$, which may be written as $\{v \mid 70{,}711 \leq v \leq 86{,}711\}$. The usual range of user sessions from Virginia is 70,711 to 86,711.

Some absolute value inequalities have no solution. For example, $|4x - 3| < -6$ is never true. Since the absolute value of a number is never negative, there is no replacement for x that will make this sentence true. So, the solution set to this inequality is the empty set.

Other absolute value inequalities are always true. One such inequality is $|x + 5| > -10$. The solution set of this inequality is all real numbers. Can you see why? *Think of the definition of absolute value.*

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Give** an example of an absolute value inequality whose solution set is the empty set.
2. **State** an absolute value inequality for all numbers less than 5 and greater than -5.
3. **Explain** why $|x + 2| \geq -4$ has all real numbers as its solution set.

MATH JOURNAL

4. Explain the difference between a compound inequality containing the word *or* and the word *and.*

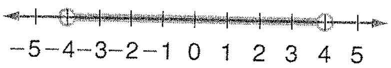
Guided Practice

State an absolute value inequality for each of the following. Then graph each solution set.

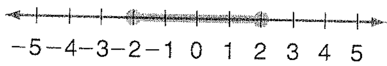
5. all numbers less than 18 and greater than -18
6. all numbers between -3 and 3

State an absolute value inequality for each graph.

7.



8.



Solve each inequality. Graph the solution set.

9. $3x + 1 < 7$ or $7 < 2x - 9$
10. $|x| \geq 4$
11. $|x + 2| > 3$
12. $1 \leq x - 2 \leq 7$
13. $|3x + 12| > 42$
14. $|x| \geq x$

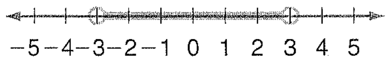
EXERCISES

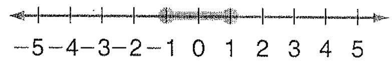
Practice

State an absolute value inequality for each of the following. Then graph each solution set.

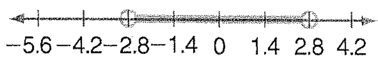
15. all numbers less than 7 and greater than -7
16. all numbers less than or equal to 15, and greater than or equal to -15
17. all numbers greater than 11 or less than -11
18. all numbers less than or equal to 5, and greater than or equal to -5
19. all numbers between -8 and 8
20. all numbers greater than or equal to -10, and less than or equal to 10

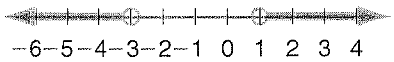
State an absolute value inequality for each graph.

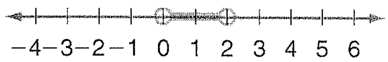
21. 

22. 

23. 

24. 

25. 

26. 

Solve each inequality. Graph the solution set.

27. $|8x| \le 10$
28. $|2x| < 6$
29. $|x| > 5$
30. $|2x - 9| \le 27$
31. $|3x| \ge 7$
32. $|5x| < -25$
33. $|2x| > 1$
34. $x - 4 < 1$ or $x + 2 > 1$
35. $|x - 6| \le -12$
36. $-1 < 3x + 2 < 14$
37. $|3x + 11| > 1$
38. $|x| \le x$
39. $x + 6 \ge -1$ or $x - 2 \le 4$
40. $-4 \le 4x + 24 \le 4$
41. $|3x| + 3 \le 0$
42. $|5x - 7| < 81$
43. $2x - 1 < -5$ or $3x + 2 \ge 5$
44. $|x + 2| - x \ge 0$

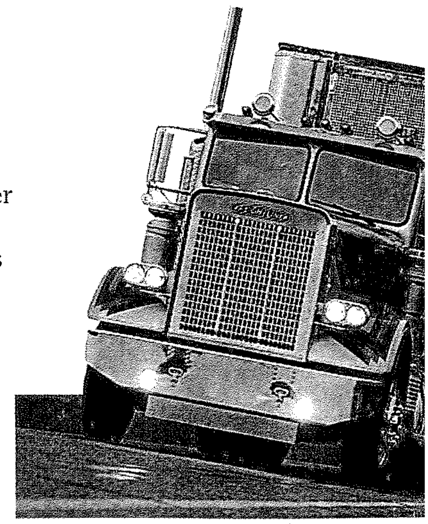
Critical Thinking

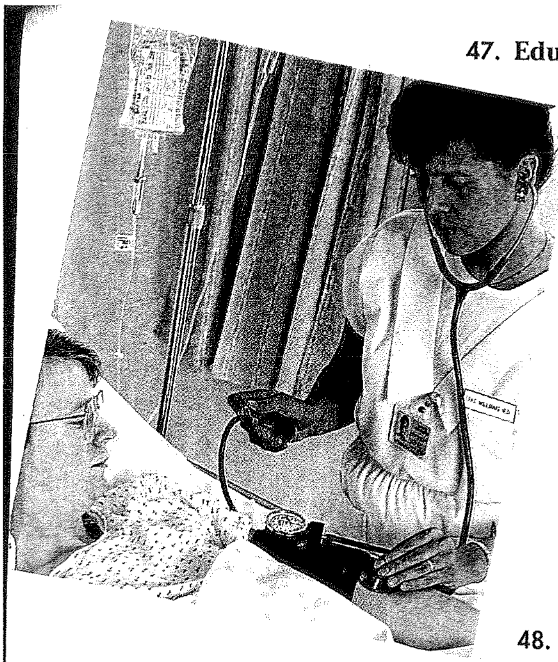
45. Solve $|x + 1| + |x - 1| \le 2$.

Applications and Problem Solving

Real World

46. **Transportation** On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.
    a. Write an inequality to represent the allowable speed for a car on an interstate highway.
    b. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.





47. **Education** Use the chart below to answer the following questions.

Average Nursing Salaries	
Type	**Salary**
Nurse anesthetist	$76,000
Nurse midwife	57,000
Corporate nurse	47,000
Nurse practitioner	43,600
Nurse employed by federal government	43,200
Legal nurse consultant at law firm	40,000

Source: Bureau of Labor Statistics

a. Write an absolute value inequality that describes the range in overall salaries.

b. Write two questions that can be answered using the mean and mode of the data.

48. **Consumerism** Marcus is buying his first tank of gasoline since he got his driver's license. Where he lives, gasoline is selling for between $1.20 and $1.40 per gallon. If he has $10.50 to spend on gas, how many gallons can he buy?

Mixed Review

Solve each inequality. Then graph the solution set. (Lesson 1–6)

49. $9(x + 2) < 72$

50. $3(3x + 2) > 7x - 2$

51. Solve $8x + 5 < 7x - 3$. (Lesson 1–6)

52. Solve $-4(3m - 7) - (3 - m) < 13$. (Lesson 1–6)

53. Solve $|3x - 4| = -1$. (Lesson 1–5)

54. **SAT Practice** Sara has a pitcher containing a ounces of lemonade. She pours b ounces of lemonade into each of c glasses. Which expression represents the amount of lemonade remaining in the pitcher? (Lesson 1–5)

A $\frac{a}{b} + c$ B $ab - c$ C $\frac{a}{bc}$ D $a - bc$ E $\frac{a}{b} - c$

55. **Consumerism** Beto has gone to a doughnut shop with $10 his father gave him. He needs to buy 6 glazed doughnuts at $0.50 each for his father. He can then buy some frosted cake doughnuts for himself at $0.35 each. (Lesson 1–4)

a. Let x represent the number of frosted cake doughnut Beto buys. Translate "Beto bought 6 glazed doughnuts and some frosted cake doughnuts for $10.00" into an equation.

b. Solve the equation to find out how many frosted cake doughnuts Beto can buy.

56. Solve $y = 8(0.3) + 1.2$. (Lesson 1–4)

57. **Olympics** The time in seconds of 15 Olympic Games Champions' scores for the 200-meter run are listed below. Find the median, mode, and mean for these scores. 22.2, 21.6, 22.6, 21.7, 22, 21.6, 21.8, 20.7, 20.7, 20.5, 20.3, 20.01, 19.80, 20.19, 19.75 (Lesson 1–3)

For Extra Practice, see page 878.

CHAPTER 1 HIGHLIGHTS

NET CONNECTION

Chapter Review For additional lesson-by-lesson review, visit: **www.algebra2.glencoe.com**

VOCABULARY

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Algebra

absolute value (p. 37)
addition property of equality (p. 28)
addition property of inequality (p. 43)
algebraic expression (p. 8)
associative properties (p. 14)
commutative properties (p. 14)
compound inequality (p. 49)
distributive property (p. 14)
division property of equality (p. 28)
division property of inequality (p. 44)
empty set (p. 40)
equation (p. 27)
formula (p. 9)
identity properties (p. 14)
intersection (p. 49)
inverse properties (p. 14)
irrational number (p. 13)
multiplication property of equality (p. 28)
multiplication property of inequality (p. 44)
open sentence (p. 27)
order of operations (p. 7)
perfect number (p. 12)
rational number (p. 13)
real numbers (p. 13)
reflexive property of equality (p. 28)
solution (p. 28)
substitution property of equality (p. 28)
subtraction property of equality (p. 28)
subtraction property of inequality (p. 43)
symmetric property of equality (p. 28)
transitive property of equality (p. 28)
trichotomy property (p. 43)
union (p. 50)
variables (p. 27)

Geometry

hypotenuse (p. 9)
legs (p. 9)
Pythagorean theorem (p. 9)

Problem Solving

list possibilities (p. 38)
problem-solving plan (p. 30)
problem-solving strategies (p. 38)

Statistics

back-to-back stem-and-leaf plot (p. 20)
extreme values (p. 22)
line plot (p. 19)
mean (p. 21)
measure of central tendency (p. 21)
median (p. 21)
mode (p. 21)
stem-and-leaf plot (p. 20)

UNDERSTANDING AND USING THE VOCABULARY

Choose the letter that best matches each description.

1. $-b = -b$
2. $(4x) \cdot 1 = 4x$
3. $8(4x - 1) = 32x - 8$
4. $x = 4$, then $4 = x$
5. $4x + 6(y - 5x) - 3z$
6. $10 > t > 4.5$
7. $x = 4y$, $4y = 12$, then $x = 12$
8. $5a - 7(a - 6) = 12$
9. $9 + (4 + 7) = (9 + 4) + 7$
10. $|-3m|$
11. $-5 + 5 = 0$
12. $xy = yx$

a. absolute value
b. algebraic expression
c. associative property
d. commutative property
e. compound inequality
f. distributive property
g. equation
h. identity property
i. inverse property
j. reflexive property
k. symmetric property
l. transitive property

CHAPTER 1 STUDY GUIDE AND ASSESSMENT

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Upon completing this chapter, you should be able to:

- use the order of operations to evaluate expressions (Lesson 1–1)

$$5 - 8(3 - 6) \div 2^2 + 10 = 5 - 8(-3) \div 4 + 10$$
$$= 5 - (-24) \div 4 + 10$$
$$= 5 - (-6) + 10$$
$$= 5 + 6 + 10$$
$$= 21$$

REVIEW EXERCISES

Use these exercises to review and prepare for the chapter test.

Find the value of each expression.

13. $6(5 - 8) \div 9 + 4$
14. $(3 + 7)^2 - 16 \div 2$
15. $(6 + 5)4 - 3$
16. $-7 + [28 \div (18 - 7(2))]$

Evaluate each expression if $a = 4$, $b = 5$, $c = -0.5$, and $d = -3$.

17. $\frac{8c + ab}{a}$
18. $(a - d + b) \div c$

- determine the sets of numbers to which a given number belongs (Lesson 1–2)

Find the value of $3(-4.5)$. Then name the sets of numbers to which this value belongs.

$$3(-4.5) = -13.5$$

rationals, reals

Find the value of each expression. Then name the sets of numbers to which each value belongs.

19. $4 - 12$
20. $42 \div 8$
21. $\sqrt{2 + 3}$
22. $2^3 + 10$
23. $2\pi(8.75)$
24. $-20 \div 2^2$

- use the properties of real numbers to simplify expressions (Lesson 1–2)

$2a(5.4 - 4b) - 4a(3.1 + 8b)$

$= 10.8a - 8ab - 12.4a - 32ab$

$= -1.6a - 40ab$

Name the property illustrated by each equation.

25. $7 \cdot \frac{1}{7} = 1$
26. $4(3 \cdot 5.5) = (4 \cdot 3)\, 5.5$

Simplify each expression.

27. $7a + 2b - 5a - 6b$
28. $3(a + 4b) - 2(4a + 2b)$

- find and use the median, mode, and mean to interpret data (Lesson 1–3)

Find the median, mode, and mean of 78, 67, 73, 69, 84, 68, 74, 76, 66, 78, 70.

66, 67, 68, 69, 70, 73, 74, 76, 78, 78, 84

median: 6th value = 73

mode: 78

mean: $\frac{66 + 67 + 68 + \ldots + 78 + 84}{11} = 73$

Find the median, mode, and mean for each set of data.

29. 5, 92, 64, 18, 25
30. 66, 48, 35, 52, 48, 41, 59, 61
31. 4.6, 6.1, 8.9, 3.6, 6.1, 10, 2.9
32. 3, 6, 7, 10, 7, 14, 19, 21, 10, 10, 31, 17, 16, 9, 7, 17, 20, 14, 7, 10, 13, 10

OBJECTIVES AND EXAMPLES

- solve equations by using the properties of equality (Lesson 1–4)

Solve $2(a - 1) = 8a - 6$.

$2(a - 1) = 8a - 6$	
$2a - 2 = 8a - 6$	*Distributive property*
$-2 = 6a - 6$	*Subtract 2a from each side.*
$4 = 6a$	*Add 6 to each side.*
$\frac{2}{3} = a$	*Divide each side by 6.*

REVIEW EXERCISES

Solve each equation.

33. $12z + 36 = 8z - 48$
34. $4.2x + 6.4 = 40$
35. $14y - 3 = 25$
36. $7w + 2 = 3w + 94$
37. $4 - 2(1 - w) = -38$
38. $4y - \frac{1}{10} = 3y + \frac{4}{5}$
39. $48 + 5y = 96 - 3y$
40. $\frac{x}{3} + \frac{x}{2} = \frac{3}{4}$

- solve equations for a specific variable (Lesson 1–4)

Solve $3(x - 2) = y$ for x.

$3(x - 2) = y$	
$3x - 6 = y$	*Distributive property*
$3x = y + 6$	*Add 6 to each side.*
$x = \frac{y + 6}{3}$	*Divide each side by 3.*

Solve each equation or formula for the variable specified.

41. $A = p + prt$ for t
42. $df - 3g = 4h$ for f
43. $\frac{3a^2 - 1}{2b} = c$ for b
44. $s = \frac{1}{2}gt^2$ for g

- solve equations containing absolute value (Lesson 1–5)

Solve $|r + 14| = 23$.

$|r + 14| = 23$

$r + 14 = 23$ or $r + 14 = -23$

$r = 9$ $\qquad$ $r = -37$

Solve each equation.

45. $|y - 5| - 2 = 10$
46. $|5y - 8| = 12$
47. $|2x - 36| = 14$
48. $|x + 4| + 3 = 17$
49. $|q - 3| + 7 = 2$
50. $4|3x + 4| = 4x + 8$
51. $2|w + 6| = 10$
52. $5|6 - 5x| = 15x - 35$

- solve inequalities and graph the solution sets. (Lesson 1–6)

Solve $3 - 4x \le 6x - 2$.

$3 - 4x \le 6x - 2$	
$3 \le 10x - 2$	*Add 4x to each side.*
$5 \le 10x$	*Add 2 to each side.*
$\frac{1}{2} \le x$	*Divide each side by 10.*

Solve each inequality. Graph the solution set.

53. $5z - 6 > 14$
54. $5(x - 2) < 75$
55. $57 - 4t \ge 13$
56. $-3(2x + 5) > 13x - 4$
57. $18 - 2(y + 6) < 76$
58. $3(6 - 5x) \le 12x - 36$
59. $2 - 3z \ge 7(8 - 2z) + 12$
60. $8(2x - 1) > 11x - 17$

OBJECTIVES AND EXAMPLES

- solve compound inequalities using *and* and *or* (Lesson 1–7)

$$-2 \leq x - 4 < 3$$
$$-2 \leq x - 4 \text{ and } x - 4 < 3$$
$$2 \leq x \qquad x < 7$$
$$2 \leq x < 7$$

REVIEW EXERCISES

Solve each inequality. Graph the solution set.

61. $11 < 3x + 2 < 20$
62. $4x - 10 < -10$ or $6x + 4 \geq 10$
63. $-1 < 3(y - 2) \leq 9$
64. $5y - 4 > 16$ or $3y + 2 < 1$

- solve inequalities involving absolute value and graph the solutions (Lesson 1–7)

$$|3x + 7| \geq 26$$
$$3x + 7 \geq 26 \text{ or } 3x + 7 \leq -26$$
$$3x \geq 19 \qquad 3x \leq -33$$
$$x \geq \frac{19}{3} \qquad x \leq -11$$
$$x \geq \frac{19}{3} \text{ or } x \leq -11$$

Solve each inequality. Graph the solution set.

65. $|2x + 6| \leq 4$
66. $7 + |9 - 5x| > 1$
67. $|4x| + 3 \leq 0$
68. $|x| + 1 < 12$
69. $|3x| < 27$
70. $|2x + 3| - 6 \geq 7$

APPLICATIONS AND PROBLEM SOLVING

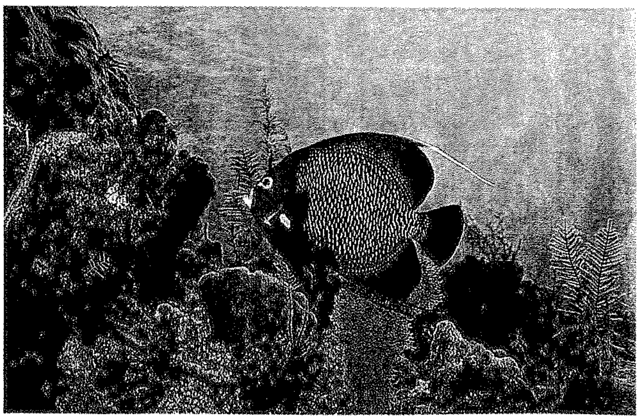
71. **Geometry** The perimeter of a rectangle is 150 centimeters. The length is 15 centimeters greater than the width. Find the dimensions of the rectangle. (Lesson 1–1)

72. **Car Expenses** Rafael spent \$2011 to operate his car last year. He drove 7400 miles. He also paid \$972 for insurance and \$114 for the registration fee. Rafael's only other expense was for gasoline. How much did the gasoline cost per mile? (Lesson 1–4)

73. **Test Scores** Your quiz scores are 73, 75, 89, and 91. What is the lowest score you can obtain on the last quiz and still achieve an average of at least 85? (Lesson 1–6)

74. **Bowling** Bowling at Sunset Lanes cost Danny and Zorina \$9. This included shoe rental of \$0.75 a pair. How much did each game cost if Danny bowled 3 games and Zorina bowled 2 games? (Lesson 1–4)

75. **Oceans** The depths, in meters, of certain points of the oceans and seas of the world are: 10918, 9219, 7455, 5625, 4632, 5016, 4773, 3787, 3658, 2782, 3742, 3777, 660, 22, 421, 6946, and 183. Find the median, mode, and mean for this set. (Lesson 1–3)



A practice test for Chapter 1 is provided on page 912.