4-6

Using Matrices to Solve Systems of Equations

What **YOU'LL LEARN**

- To solve systems of linear equations by using inverse matrices.

Why **IT'S IMPORTANT**

You can use matrices and systems of equations to solve problems involving chemistry and business.

Real World APPLICATION

Business

Katie earns extra money by making stuffed teddy bears and rabbits and then selling them to a local craft store. She has arranged to sell a total of 15 stuffed animals to the owner of the craft store each week. Her profit on each rabbit is $5 and each bear is $9. Her goal is to earn at least $120 each week. Of course, Katie could make 15 bears and easily meet her goals. But it takes her longer to make a bear than a rabbit, and she doesn't have time to make 15 bears. Katie wants to know what combination of stuffed animals she should make each week to guarantee a profit of $120. *This problem will be solved in Example 3.*

The problem above can be solved by using a system of equations. You have already learned several methods for solving a system of equations, including graphing, substitution, addition and subtraction, and Cramer's rule. Matrices can also be used to solve systems of equations. In this lesson, you will be using inverses of matrices.

Remember, two matrices are equal if their corresponding elements are equal.

Consider the system of equations below. You can write this system with matrices by using the left and right sides of the equations.

$$\begin{aligned} 7x + 5y &= 3 \\ 3x - 2y &= 22 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} 7x + 5y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

Write the matrix on the left as the product of the coefficients and variables.

$$\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

coefficient matrix   *variable matrix*   *constant matrix*

LOOK BACK

You can refer to Lessons 3-1, 3-2, and 3-3 for information about solving systems of equations.

The system of equations is now expressed as a **matrix equation**. *This system will be solved in Example 2.*

Example 1 **Write each system of equations as a matrix equation.**

a. $3x - 2y = 7$
$4x + y = 8$

The matrix equation is

$$\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}.$$

b. $3a - 5b + 2c = 9$
$4a + 7b + c = 3$
$2a - c = 12$

The matrix equation is

$$\begin{bmatrix} 3 & -5 & 2 \\ 4 & 7 & 1 \\ 2 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 12 \end{bmatrix}.$$

A linear equation in the form $ax = b$ and a matrix equation in the form $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix, can be solved in a similar manner.

Since matrix multiplication is not commutative, the inverse matrix should be at the left on each side of the matrix equation.

$ax = b$		$AX = B$
$\left(\frac{1}{a}\right)ax = \left(\frac{1}{a}\right)b$	*Multiply by the inverse if it exists.*	$A^{-1}AX = A^{-1}B$
$1x = \left(\frac{1}{a}\right)b$	*1 and I are identities.*	$IX = A^{-1}B$
$x = \left(\frac{1}{a}\right)b$		$X = A^{-1}B$

The solution of the linear equation is the product of the inverse of the coefficient and the constant term. In the matrix equation, the solution is the product of the inverse of the coefficient matrix and the constant matrix.

Example 2 **Use a matrix equation to solve the system of equations.**
$7x + 5y = 3$
$3x - 2y = 22$

The matrix equation is $\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 22 \end{bmatrix}$.

First, find the inverse of the coefficient matrix. The inverse of $\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix}$ is $\frac{1}{-14 - (15)}\begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix}$ or $-\frac{1}{29}\begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix}$.

Next, multiply each side of the matrix equation by the inverse matrix.

The identity matrix on the left verifies that the inverse matrix has been calculated correctly.

$$-\frac{1}{29}\begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{29}\begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

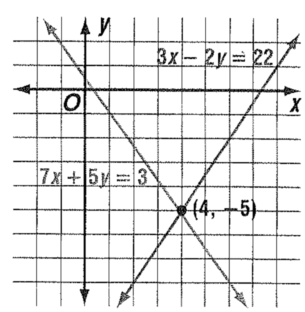
$$-\frac{1}{29}\begin{bmatrix} -29 & 0 \\ 0 & -29 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{29}\begin{bmatrix} -116 \\ 145 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The solution is (4, −5).

The graph at the right confirms the solution.



How might you check the solution?

You will not be asked to find the inverse of a 3 × 3 matrix in this chapter.

To solve a system of equations with three variables, you will use the 3 × 3 identity matrix. However, as you may imagine, finding the inverse of a 3 × 3 matrix can be tedious. Graphing calculators and computer programs offer fast and accurate methods for performing the necessary calculations.

TECHNOLOGY Tips

You may need to use the arrow keys to see the entire matrix on the graphing calculator screen.

EXPLORATION — GRAPHING CALCULATORS

You can use a graphing calculator and a matrix equation to solve systems of equations. Consider the system of equations below.

$$3x - 2y + z = 0$$
$$2x + 3y = 12$$
$$y + 4z = -18$$

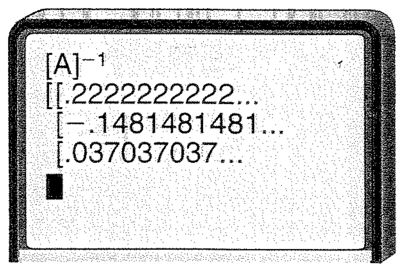
- Write the system so that each equation is in standard form and contains all three variables. Then write the coefficient matrix.

$$\begin{aligned} 3x - 2y + 1z &= 0 \\ 2x + 3y + 0z &= 12 \\ 0x + 1y + 4z &= -18 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$$

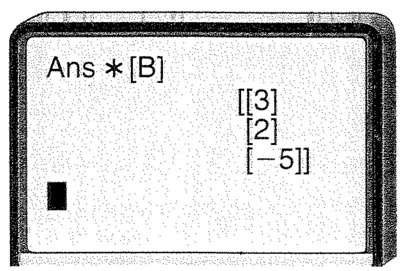
- Write the matrix equation in the form $AX = B$.

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -18 \end{bmatrix}$$

- Use a graphing calculator to find A^{-1}. The inverse is shown below.



- Now, find the product of A^{-1} and B.



Therefore, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, and $x = 3$, $y = 2$, and $z = -5$.

Your Turn

Solve the matrix equation $AX = B$ for each value of A and B.

a. $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 \\ 2 \\ 7 \end{bmatrix}$

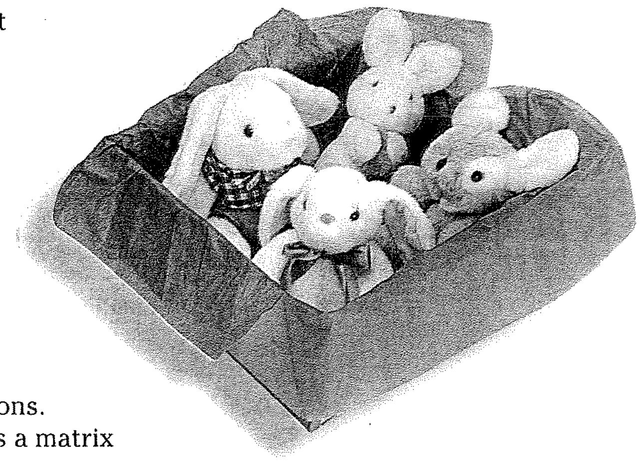
b. $A = \begin{bmatrix} 2 & 6 & 8 \\ -2 & 9 & -12 \\ 4 & 6 & -4 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$

You can use matrices to help solve problems that involve systems of equations. Matrices often simplify the process of solving these systems.

Example 3 Refer to the application at the beginning of the lesson. What combination of stuffed animals should Katie make each week to guarantee her a profit of $120?

Real World APPLICATION

Business



Explore Let b represent the number of bears, and let r represent the amount of rabbits. Therefore, $b + r = 15$.

The total number of animals must be 15.

Now, write an equation that represents Katie's profit.

$$9b + 5r = 120$$

Plan Write a system of equations. Then write the system as a matrix equation.

$$\begin{aligned} b + r &= 15 \\ 9b + 5r &= 120 \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 \\ 9 & 5 \end{bmatrix} \cdot \begin{bmatrix} b \\ r \end{bmatrix} = \begin{bmatrix} 15 \\ 120 \end{bmatrix}$$

Solve To solve the equation, first find the inverse of the coefficient matrix.

$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \rightarrow -\frac{1}{4}\begin{bmatrix} 5 & -1 \\ -9 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -1.25 & 0.25 \\ 2.25 & -0.25 \end{bmatrix}.$$

Now multiply each side of the matrix equation by the inverse and solve.

$$\begin{bmatrix} -1.25 & 0.25 \\ 2.25 & -0.25 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 9 & 5 \end{bmatrix} \cdot \begin{bmatrix} b \\ r \end{bmatrix} = \begin{bmatrix} -1.25 & 0.25 \\ 2.25 & -0.25 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 120 \end{bmatrix}$$

$$\begin{bmatrix} b \\ r \end{bmatrix} = \begin{bmatrix} 11.25 \\ 3.75 \end{bmatrix}$$

Examine Since Katie cannot make a fraction of an animal, the solution (11.25, 3.75) is unreasonable. However, try $b = 11$, $r = 4$ and $b = 12$, $r = 3$ as possible solutions.

Bears	Rabbits	Profit
11	4	9(11) + 5(4) or $119
12	3	9(12) + 5(3) or $123 ✓

Therefore, one solution to the problem is 12 bears and 3 rabbits.

What other solutions are possible?

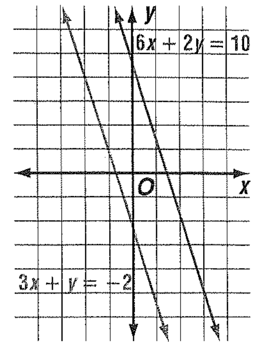
LOOK BACK

You can refer to Lesson 3-1 for information about consistent and inconsistent systems.

When the determinant of the coefficient matrix is 0, the system of equations has no unique solution. You can graph the equations to determine whether the system is consistent and has infinitely many solutions or is inconsistent and has no solutions.

Example 4 Solve the matrix equation $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 10 \end{bmatrix}$.

The determinant of the coefficient matrix is 0, so there is no unique solution. Graph the system of equations.



The matrix equation represents the system of equations below.

$3x + y = -2$

$6x + 2y = 10$

Since the lines are parallel, this system has no solution. Therefore, the system is inconsistent.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Write** a matrix equation for the system.
   $2x - 8y = 3$
   $7x - 2y = 5$
2. **Write** the matrix equation $\begin{bmatrix} 2 & -5 \\ 3 & 8 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ as a system of linear equations.
3. **List** the steps you would use to solve a system of linear equations with inverse matrices.
4. **Write** an example of a system of equations that does not have a unique solution.

Guided Practice

Write a matrix equation for each system.

5. $4x - 7y = 2$
   $3x + 5y = 9$
6. $2a + 3b - 5c = 1$
   $7a + 3c = 7$
   $3a - 6b + c = -5$
7. $y = 3x$
   $x + 2y = -21$
8. Given that the inverse of the coefficient matrix is $-\frac{1}{28}\begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix}$, solve the matrix equation $\begin{bmatrix} 4 & 8 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$.

Use a matrix equation to solve each system of equations.

9. $x + 2y = 8$
   $3x + 2y = 6$
10. $5s + 4t = 12$
    $3s = -4 + 4t$
11. Determine whether the system of equations has *one* solution, *no* solution, or *infinitely many* solutions.
    $x + 2y = 5$
    $3x - 15 = -6y$

EXERCISES

Practice

Write a matrix equation for each system.

12. $5a - 6b = -47$
    $3a + 2b = -17$

13. $3m - 7n = -43$
    $6m + 5n = -10$

14. $2r + 3s = -17$
    $s = r - 4$

15. $y = -x$
    $y = 2x$

16. $y = x + 3$
    $3y + x = 5$

17. $2x = 3y$
    $y = 4x - 5$

Matrix M^{-1} is the inverse of the coefficient matrix. Use M^{-1} to solve each matrix equation.

18. $\begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 24 \end{bmatrix}$ $\quad M^{-1} = -\frac{1}{10}\begin{bmatrix} -2 & -1 \\ -4 & 3 \end{bmatrix}$

19. $\begin{bmatrix} 3 & 1 & 1 \\ -6 & 5 & 3 \\ 9 & -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -9 \\ 5 \end{bmatrix}$ $\quad M^{-1} = -\frac{1}{9}\begin{bmatrix} 1 & -1 & -2 \\ 21 & -12 & -15 \\ -33 & 15 & 21 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 3 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -4 \end{bmatrix}$ $\quad M^{-1} = -\frac{1}{9}\begin{bmatrix} -1 & -10 & 4 \\ -3 & -3 & 3 \\ -1 & 8 & -5 \end{bmatrix}$

Solve each matrix equation or system of equations by using inverse matrices.

21. $\begin{bmatrix} 2 & 6 \\ 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

22. $\begin{bmatrix} 8 & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 16 \\ -9 \end{bmatrix}$

23. $\begin{bmatrix} 5 & -3 \\ 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -30 \\ 1 \end{bmatrix}$

24. $3x = 13 - y$
    $2x - y = 2$

25. $6a + 2b = 11$
    $3a = 8b + 1$

26. $4x = -3y + 5$
    $8x = 9y$

Determine whether each system has *one* unique solution, *no* solution, or *infinitely many* solutions. If it has one unique solution, name it. If it has no solution or many solutions, graph the system.

27. $3x - y = 4$
    $6x + 2y = -8$

28. $x + 3y = 6$
    $3x - 18 = -9y$

29. $3x - 8y = 4$
    $6x - 42 = 16y$

Graphing Calculator

Use a graphing calculator to solve each system of equations.

30. $5x + y = 1$
    $9x + 3y = 1$

31. $1.8x + 5y = 19.5$
    $5.2x - 2.9y = 4.3$

Critical Thinking

32. According to Cramer's rule, the solution of the system $\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$ is (x, y), where $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$. Generalize Cramer's rule to be used with a system of equations in three variables.

Applications and Problem Solving

Real World

33. Chemistry Sonia Ramos is a chemist who is preparing an acid solution to be used as a cleaner for machine parts. The machine shop needs several 200-mL batches of solution at a 48% concentration. Sonia only has 60% and 40% concentration solutions. The two solutions can be combined in some ratio to make the 48% solution. How much of each solution should Sonia use to make 200 mL of solution?

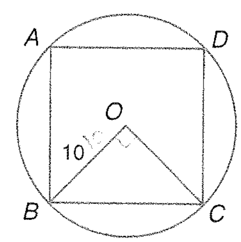
34. Food Service The manager of the Snack Shack is gathering information about the time it takes to make and serve hamburgers and chicken sandwiches. It takes 5 minutes to prepare a hamburger and 2 additional minutes to serve it with cheese, lettuce, and ketchup. It takes 7 minutes to prepare a chicken sandwich and 1 additional minute to serve it with lettuce, tomato, and mayonnaise. How many sandwiches can be prepared and served by one employee if 42 minutes is spent on preparation and 15 minutes is spent on serving?

Mixed Review

35. *True* or *false*: $\begin{bmatrix} \frac{9}{2} & \frac{1}{2} \\ 4 & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} & -\frac{1}{2} \\ -4 & \frac{9}{2} \end{bmatrix} = I.$ (Lesson 4–5)

36. Find $\begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} + 3\begin{bmatrix} -6 \\ -3 \\ -1 \end{bmatrix}$. (Lesson 4–2)

37. ACT Practice In the figure, $ABCD$ is a square inscribed in the circle centered at O. If $\overline{OB}$ is 10 units long, how many units long is minor arc BC?



A $\frac{5}{2}\pi$ **B** 5π **C** 10π

D 20π **E** 100π

38. Solve the system of equations by graphing. (Lesson 3–1)

$3x + 4y = 8$

$6y - 8x = 12$

39. Oceanography The Mariana Trench is the deepest point in any of the oceans. It is located in the western Pacific Ocean north of Australia. The deepest point in the trench is 36,198 feet, or about 6.8 miles below sea level. Water pressure in the ocean is represented by the function $f(x) = 1.15x$, where x is the depth in miles and $f(x)$ is the pressure in tons per square inch. Find the approximate water pressure at the deepest point in the Mariana Trench. (Lesson 2–2)

For Extra Practice, see page 884.

40. Retail Sales Leon bought a 10-speed bicycle on sale for 75% of its original price. The sale price was $41 less than the original price. Find the original price and the sale price. (Lesson 1–4)

4-7 Using Augmented Matrices

What YOU'LL LEARN

- To solve systems of linear equations by using augmented matrices.

Why IT'S IMPORTANT

You can use augmented matrices to solve problems involving finance and business.

Real World APPLICATION

Investing

When Cleveland Jackson inherited \$5000, he went to a financial planner for help in investing the money. The financial planner suggested that he invest the money in a stock fund that earns an average of 6%, a bond fund that earns an average of 7%, and a term fund that earns an average of 4%. The planner told Mr. Jackson that he shouldn't invest all the money in the highest paying fund, because although it would make more money, higher paying funds are more risky. Since the earnings from the stock fund will be available sooner, Mr. Jackson wants to earn three times as much from the stock fund as he will from the term fund. If he wants to earn a total of \$300, how much should he invest in each fund? *This problem will be solved in Example 2.*

In the last lesson, you solved systems of equations using inverse matrices. A system of equations may also be solved using a matrix called an **augmented matrix**. The augmented matrix of a system contains the coefficient matrix with an extra column containing the constant terms. Study how the system below is written as an augmented matrix.

$$\begin{aligned} x + 5y + 6z &= -8 \\ 3x - 2y - 2z &= 17 \\ 2x + 3y + 4z &= 1 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} 1 & 5 & 6 & -8 \\ 3 & -2 & -2 & 17 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

The system of equations can be solved by manipulating the rows of the matrix rather than the equations themselves. For example, you could multiply each side of the first equation by -2. The result would be $-2x - 10y - 12z = 16$. The corresponding change in the matrix is that the first row becomes $[-2 \quad -10 \quad -12 \quad 16]$.

When you use an augmented matrix, you perform the same operations as you would in working with the equations, but you do not have to bother writing the variables or worrying about the order in which the terms are written—the organization of the matrix keeps all of this in its proper place. Here is a summary of the **row operations** that can be performed on an augmented matrix.

Notice that the row operations can only be performed on rows, not columns.

- Any two rows can be interchanged.
- Any row can be replaced with a nonzero multiple of that row.
- Any row can be replaced with the sum of that row and a multiple of another row.

The solution of the system of equations above is $(4, -3, 0.5)$; that is, $x = 4$, $y = -3$, and $z = 0.5$.

Suppose we write these three equations in the form of an augmented matrix.

$$\begin{aligned} x &= 4 \\ y &= -3 \\ z &= 0.5 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0.5 \end{array}\right]$$

Notice that the first three columns are the same as a 3×3 identity matrix. When doing row operations, your goal should be to find an augmented identity matrix.

Just as there is no single order of steps to solve a system of equations, there is also no one single group of row operations that arrives at the correct solution. The order in which you solve a system may be different from the way a classmate solves it, but you may both be correct.

Example 1 **Use an augmented matrix to solve the system of equations.**

$$\begin{aligned} \mathbf{a + 2b + c} &= \mathbf{0} \\ \mathbf{2a + 5b + 4c} &= \mathbf{-1} \\ \mathbf{a - b - 9c} &= \mathbf{-5} \end{aligned}$$

Write the augmented matrix. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 4 & -1 \\ 1 & -1 & -9 & -5 \end{array}\right]$ *The first element in row 1 is already 1.*

Multiply row 1 by -1 and add to row 3. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 4 & -1 \\ \mathbf{0} & \mathbf{-3} & \mathbf{-10} & \mathbf{-5} \end{array}\right]$ *The first element in row 3 is now 0.*

Multiply row 1 by -2 and add to row 2. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ 0 & -3 & -10 & -5 \end{array}\right]$ *The first element in row 2 is now 0, and the second element in row 2 is now 1.*

Multiply row 2 by -2 and add to row 1. $\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{-3} & \mathbf{2} \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -10 & -5 \end{array}\right]$ *The second element in row 1 is now 0.*

Multiply row 2 by 3 and add to row 3. $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \\ \mathbf{0} & \mathbf{0} & \mathbf{-4} & \mathbf{-8} \end{array}\right]$ *The second element in row 3 is now 0.*

Notice that the matrix has all zeros in the "bottom triangle." The system can now be solved by setting $c = 2$ and substituting into the other equations.

Multiply row 3 by $-\frac{1}{4}$. $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ *The third element in row 3 is now 1.*

Multiply row 3 by 3 and add to row 1. $\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{8} \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ *The third element in row 1 is now 0.*

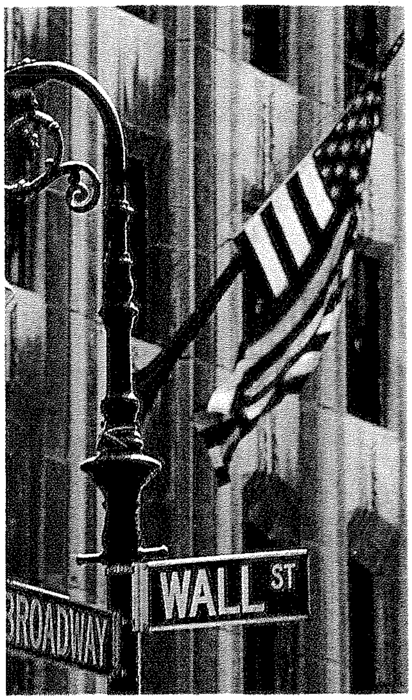
Multiply row 3 by -2 and add to row 2. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-5} \\ 0 & 0 & 1 & 2 \end{array}\right]$ *This matrix contains an augmented identity matrix. Now you can read the solution.*

The solution is $(8, -5, 2)$.

The process of performing row operations to get the desired matrix is called **reducing a matrix**. The resulting matrix is called a **reduced matrix**. Reducing a matrix is the method used by computers to solve systems of equations with many variables and equations.

Example 2 **Real World APPLICATION**

Investing



Refer to the application at the beginning of the lesson. How much money should Mr. Jackson invest in each fund so that the total earnings will be $300?

Explore Let s represent the amount in stocks, let b represent the amount in bonds, and let t represent the amount in the term fund.

Plan Write a system of equations.

$s + b + t = 5000$ *The total amount invested is $5000.*

$0.06s + 0.07b + 0.04t = 300$ *The total amount earned is $300.*

$0.06s = 3(0.04)t$ *The amount earned from the stock fund is three times the amount earned from the term fund.*

Solve Then write an augmented matrix.

$$\begin{aligned} s + b + t &= 5000 \\ 0.06s + 0.07b + 0.04t &= 300 \\ 0.06s &= 3(0.04)t \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5000 \\ 0.06 & 0.07 & 0.04 & 300 \\ 0.06 & 0 & -0.12 & 0 \end{array}\right]$$

After applying row operations on the matrix, we get the following.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2000 \\ 0 & 1 & 0 & 2000 \\ 0 & 0 & 1 & 1000 \end{array}\right]$$

The solution is (2000, 2000, 1000), which means that Mr. Jackson should invest $2000 in the stock fund, $2000 in the bond fund, and $1000 in the term fund.

Examine Examine this solution to see if it makes sense.

Stocks: 6% of $2000 = $120 *Three times the amount earned in the term fund is 3($40) or $120.*
Bonds: 7% of $2000 = $140
Term: 4% of $1000 = $ 40
Total: $300 ✓

As with other methods of solving systems of equations, there is not always a unique solution. In systems where no solution or multiple solutions exist, solving by augmented matrices can identify these solutions. Study the solution of the system shown below.

$$\begin{aligned} x + y + 2z &= -5 \\ 3x + y + 12z &= -19 \\ 2x + y + 7z &= -12 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & -5 \\ 3 & 1 & 12 & -19 \\ 2 & 1 & 7 & -12 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & -7 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The row of zeros indicates that the last equation is some combination of the other two equations, meaning that this is a dependent system and there is no unique solution. However, there is a solution. In fact, there is an infinite number of solutions.

CAREER CHOICES

The most common use of mathematical matrices may be the spreadsheet programs used in the **banking** industry. They are valuable in creating budgets, analyzing financial performance, and tracking loans, mortgages, and stocks.

For more information contact:

American Bankers Association
1120 Connecticut Ave, N.W.
Washington, D.C. 20036

Let's write the equations represented by the matrix and solve each of them in terms of z.

$$x + 5z = -7 \qquad\qquad y - 3z = 2$$
$$x = -5z - 7 \qquad\qquad y = 3z + 2$$

The solution is the ordered triple $(-5z - 7, 3z + 2, z)$. By choosing any value for z, you can find coordinates of points on a line that are solutions to the system.

Let's look at another system of equations.

$$\begin{aligned} 3x - 5y + 2z &= -7 \\ x + 4y - z &= 10 \\ 6x + 7y - z &= -18 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{ccc|c} 3 & -5 & 2 & -7 \\ 1 & 4 & -1 & 10 \\ 6 & 7 & -1 & -18 \end{array}\right] \quad \rightarrow \quad \left[\begin{array}{ccc|c} 1 & 4 & -1 & 10 \\ 0 & -17 & 5 & -37 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

Notice the last row of the final matrix. This represents the equation $0 = 5$. Since this cannot be true, the system is inconsistent, and no solution exists.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Discuss** the advantages and disadvantages of using an augmented matrix to solve a system of equations.
2. **Write** an augmented matrix for the system of equations.

   $x + 3z = 5$

   $2x + y = 5$

   $-2x + 3y - z = 8$
3. **State** the row operations you would use to change $\left[\begin{array}{cc|c} 4 & -1 & -19 \\ 3 & 4 & 19 \end{array}\right]$ to $\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 7 \end{array}\right]$.
4. **Choose** the operation that cannot be used with the augmented matrix below.

   $$\left[\begin{array}{ccc|c} 1 & 5 & 6 & -8 \\ 3 & -2 & -2 & 17 \\ 2 & 3 & 4 & 1 \end{array}\right]$$

   a. Multiply row 1 by -3 and add it to row 2.

   b. Switch row 2 and row 3.

   c. Multiply column 3 by $\frac{1}{2}$.

   d. Multiply row 3 by 6.
5. **Describe** the solution for the system of equations represented by each reduced augmented matrix.

   a. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 8 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & 7 \\ 0 & 2 & -3 & 8 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

MATH JOURNAL

6. **Assess Yourself** List all of the algebraic methods that can be used to solve systems of equations. Which method do you prefer, and why?

Guided Practice

Write a system of equations represented by each augmented matrix.

7. $\left[\begin{array}{cc|c} 3 & -5 & 25 \\ 2 & 4 & 24 \end{array}\right]$
8. $\left[\begin{array}{ccc|c} 3 & -5 & 2 & 9 \\ 1 & -7 & 3 & 11 \\ 4 & 0 & -3 & -1 \end{array}\right]$

Write an augmented matrix for each system. Then solve each system.

9. $4m - 7n = -19$
   $3m + 2n = 22$

10. $3a - b + 5c = -1$
    $a + 3b - c = 25$
    $2a + 4c = 2$

11. Describe the solution for the system of equations represented by

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right].$$

EXERCISES

Practice

Write an augmented matrix for each system of equations. Then solve each system.

12. $3x + 2y = 7$
    $x - 3y = 17$

13. $4x - 3y = 5$
    $2x + 9y = 6$

14. $7m - 3n = 41$
    $2m = -5n$

15. $2a - b + 4c = 6$
    $a + 5b - 2c = -6$
    $3a - 2b + 6c = 8$

16. $3x - 5y + 2z = 22$
    $2x + 3y - z = -9$
    $4x + 3y + 3z = 1$

17. $2q + r + s = 2$
    $-q - r + 2s = 7$
    $-3q + 2r + 3s = 7$

Describe the solution for the system of equations represented by each reduced augmented matrix.

18. $\left[\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 2 & -8 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} 4 & 0 & 1 & 4 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

20. $\left[\begin{array}{ccc|c} 4 & 0 & 0 & -8 \\ 0 & 0 & 1 & 3 \\ 0 & 2 & 0 & 5 \end{array}\right]$

21. $\left[\begin{array}{ccc|c} 1 & 0 & -7 & 2 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right]$

Solve each system of equations by using augmented matrices.

22. $6a + 5b = -12$
    $12a + 10b = -20$

23. $6r + s = 9$
    $3r = -2s$

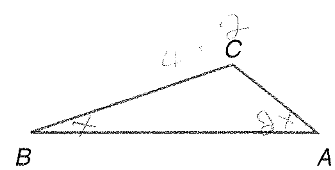
24. $4m + 2n + 5p = 24$
    $3m + 5n - p = -13$
    $m + 7n + 3p = 33$

25. $6x + 2y - 6z = 4$
    $3x - 5y - 3z = -1$
    $2x + 4y + z = 1$

26. $2x - 3y + z = -2$
    $x + y - 2z = 1$
    $4x + 4y - 8z = 4$

27. $2a - b - c = 3$
    $a + b - 3c = 5$
    $4a - 2b - 2c = -2$

28. **Geometry** In triangle ABC, the measure of $\angle A$ is twice the measure of $\angle B$. The measure of $\angle C$ exceeds four times the measure of $\angle B$ by 12 degrees. Find the measure of each angle.



Critical Thinking

29. When one of the three rows in a reduced augmented matrix contains all zeros, the solution of the system of equations is a line. What kind of solution do you have when two rows contain all zeros? Explain your answer.

Applications and Problem Solving

Real World

30. **Geometry** The perimeter of a triangle is 83 inches. The longest side is three times the length of the shortest side and 17 inches more than one-half the sum of the other two sides. Use augmented matrices to find the length of each side.

31. **Business** The Yogurt Shoppe sells cones in three sizes: small, \$0.89; medium, \$1.19; and large, \$1.39. One day, Kyle Miller sold 52 cones. He sold two more than twice as many medium cones as large cones. If he sold \$58.98 in cones, how many of each size did he sell?

Mixed Review

32. Solve the system of equations by using an inverse matrix. (Lesson 4–6)

$3a + 2b = 7$

$-a - 7b = 23$

33. Find the inverse of $\begin{bmatrix} 4 & -5 \\ 2 & -1 \end{bmatrix}$. (Lesson 4–5)

34. Find $\begin{vmatrix} -2 & 0 \\ 7 & -6 \end{vmatrix}$. (Lesson 3–3)

35. **ACT Practice** At what point (x, y) do the two lines with the equations $7x - 3y = 13$ and $y = 2x - 3$ intersect?

A $(-4, -11)$ B $(4, 11)$ C $(4, 5)$ D $(5, 4)$ E $(10, 17)$

For Extra Practice, see page 885.

36. Graph $5 = 5x$. (Lesson 2–2)

37. Evaluate $\frac{3ab^2 - c^3}{a + c}$ if $a = 3$, $b = 7$, and $c = -2$. (Lesson 1–1)

WORKING ON THE

In·ves·ti·ga·tion

Refer to the Investigation on pages 180–181.

3-2-1-Blast-Off!

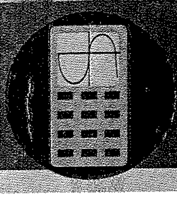
Conduct a shoot-off day. Create a shooting area with masking tape. Label the tape with a scale, marking distances 50 centimeters to 250 centimeters from the launching area.

The teacher selects a distance between 50 centimeters and 250 centimeters and announces that distance to the class. No test shots may be taken after the announcement. Groups are randomly selected to demonstrate their launch system. During these launches, the launcher should be shot from the floor, and the distance should be measured from the launcher's location to the spot where the table tennis ball hits the floor.

Each group should shoot ten shots at the same distance. Then each group must accurately measure and record the shot distances.

1. State the target distance and list the ten distances shot by your launcher.
2. Create a matrix of the class results from the launcher shoot-off. The matrix should consist of all ten distances shot by each of the groups in the class.
3. Find the median, quartiles, greatest and least values, and interquartile range of each set of shots by each of the launchers.
4. How do these three measures compare to the target distance of your launcher? Explain.
5. How do these three measures help to determine which launcher was most accurate? Explain.
6. Make a box-and-whisker plot for each launcher. Which launcher would you consider to be the most accurate? Explain your reasoning.

Add the results of your work to your Investigation Folder.



4–7B Graphing Technology Matrix Row Operations

An Extension of Lesson 4–6

You can solve a system of linear equations by using a graphing calculator and the MATRX function. The row operation functions are located in the MATH menu when you press the MATRX key. Each function is listed below with instructions on the keying procedures. Suppose your augmented matrix has been entered as matrix A.

Row Swap(

MATRX ▶ 8

Interchange two rows.

1. Enter the name of the matrix followed by a comma.
2. Enter one of the rows you want to interchange followed by a comma.
3. Enter the other row you want to interchange followed by).

Example: To interchange rows 1 and 2 in matrix A, enter rowSwap([A], 1, 2).

***Row**

MATRX ▶ 0

Multiply one row by a number.

1. Enter the number you want to multiply by, followed by a comma.
2. Enter the name of the matrix followed by a comma.
3. Enter the row you want multiplied, followed by).

Example: To multiply row 2 by −3 in matrix A, enter *row(−3, [A], 2).

Row+(

MATRX ▶ 9

Add two rows and store the result in the last row you entered.

1. Enter the name of the matrix followed by a comma.
2. Enter the row you want to add followed by a comma.
3. Enter the row you want it added to, followed by).

Example: To add row 2 to row 1 in matrix A, enter row+([A], 2, 1).

***Row+(**

MATRX ▶

ALPHA A

Multiply one row by a number and add the result to another.

1. Enter the number you want to multiply by, followed by a comma.
2. Enter the name of the matrix followed by a comma.
3. Enter the row you want multiplied, followed by a comma.
4. Enter the row you want the result added to, followed by).

Example: To multiply row 1 by $\frac{1}{2}$ and add it to row 2 in matrix A, enter *row+(0.5, [A], 1, 2).

To perform one operation after another in completely reducing a matrix, let ANS be your matrix name so the operations will be done on the matrix you just finished.

Example **Write an augmented matrix for the following system of equations. Then solve the system by reducing the matrix with a graphing calculator.**

$15x + 11y = 36$

$4x - 3y = -26$

The augmented matrix $A = \left[\begin{array}{cc|c} 15 & 11 & 36 \\ 4 & -3 & -26 \end{array}\right]$.

Begin by entering the matrix.

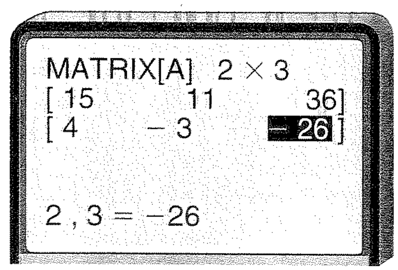
Enter: [MATRX] [▶] [▶] [ENTER]

2 [ENTER] 3 [ENTER] 15 [ENTER]

11 [ENTER] 36 [ENTER] 4

[ENTER] [(–)] 3 [ENTER] [(–)] 26

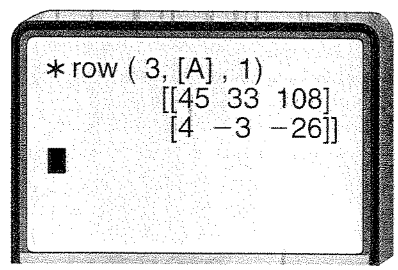
[ENTER] [2nd] [QUIT]



Multiply row 1 by 3.

Enter: [MATRX] [▶] 0 3 [,] [MATRX]

1 [,] 1 [)] [ENTER]

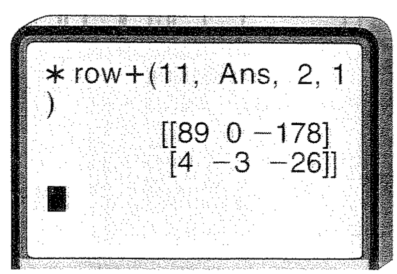


Multiply row 2 by 11 and add it to row 1.

Enter: [MATRX] [▶] [ALPHA] [A] 11

[,] [2nd] [ANS] [,] 2 [,] 1

[)] [ENTER]

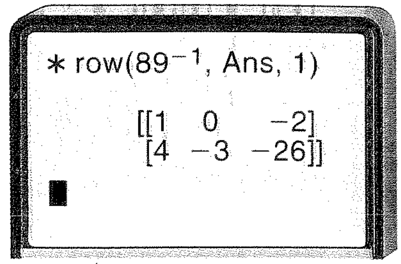


(continued on the next page)

Multiply row 1 by $\frac{1}{89}$.

Enter: [MATRX] [▶] 0 89 [x^{-1}] [,]

[2nd] [ANS] [,] 1 [)] [ENTER]

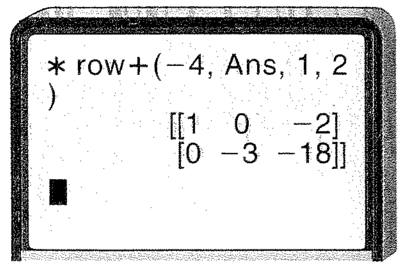


Multiply row 1 by −4 and add it to row 2.

Enter: [MATRX] [▶] [ALPHA] [A] [(−)] 4

[,] [2nd] [ANS] [,] 1 [,] 2

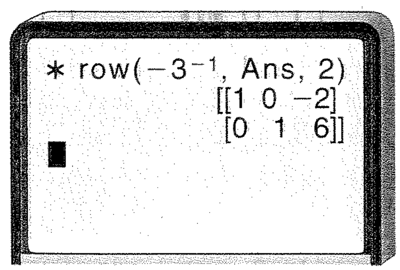
[)] [ENTER]



Multiply row 2 by $-\frac{1}{3}$.

Enter: [MATRX] [▶] 0 [(−)] 3 [x^{-1}] [,]

[2nd] [ANS] [,] 2 [)] [ENTER]



The solution is (−2, 6).

EXERCISES

Write an augmented matrix for each system of equations. Then solve with a graphing calculator.

1. $3x + 2y = -2$
   $2x + 3y = 7$
2. $x - 3y = 5$
   $2x + y = 1$
3. $3x - y = 0$
   $2x - 3y = 1$
4. $2x + y = 5$
   $2x - 3y = 1$
5. $x - y + z = 2$
   $x - z = 1$
   $y + 2z = 0$
6. $3x - 2y + z = -2$
   $x - y + 3z = 5$
   $-x + y + z = -1$
7. $3x + y + 3z = 2$
   $2x + y + 2z = 1$
   $4x + 2y + 5z = 5$
8. $-3x + y - 2z = -7$
   $2x - y - 3z = 1$
   $x + 2y + z = -2$
9. $-2x + y + z = 4$
   $4x - 3y - 2z = -2$
   $-3x + y + z = 5$

4–8 Integration: Statistics Box-and-Whisker Plots

What YOU'LL LEARN

- To find the range, quartiles, and interquartile range for a set of data,
- to determine if any values in a set of data are outliers, and
- to represent data using box-and-whisker plots.

Why IT'S IMPORTANT

Box-and-whisker plots are a useful way to display data. They allow you to see important characteristics of the data at a glance.

Real World APPLICATION

News Media

A recent edition of the *Editor and Publisher International Yearbook* listed the total number of morning and evening newspapers that are published in each state. One way to organize these data is to show them in a 1×50 matrix.

$$N = [26\ \ 7\ \ 22\ \ 32\ \ 104\ \ldots]$$

However, this is not the best way to display the data. A more convenient way is to organize them in the table below.

Number of Morning and Evening Newspapers in the United States

State	Number	State	Number	State	Number	State	Number
AL	26	IN	73	NE	18	SC	16
AK	7	IA	39	NV	8	SD	11
AZ	22	KS	47	NH	11	TN	27
AR	32	KY	23	NJ	21	TX	92
CA	104	LA	25	NM	18	UT	6
CO	28	ME	7	NY	71	VT	8
CT	19	MD	15	NC	49	VA	28
DE	3	MA	39	ND	10	WA	24
FL	40	MI	52	OH	84	WV	23
GA	34	MN	25	OK	46	WI	36
HI	6	MS	22	OR	19	WY	9
ID	12	MO	45	PA	89		
IL	68	MT	11	RI	6		

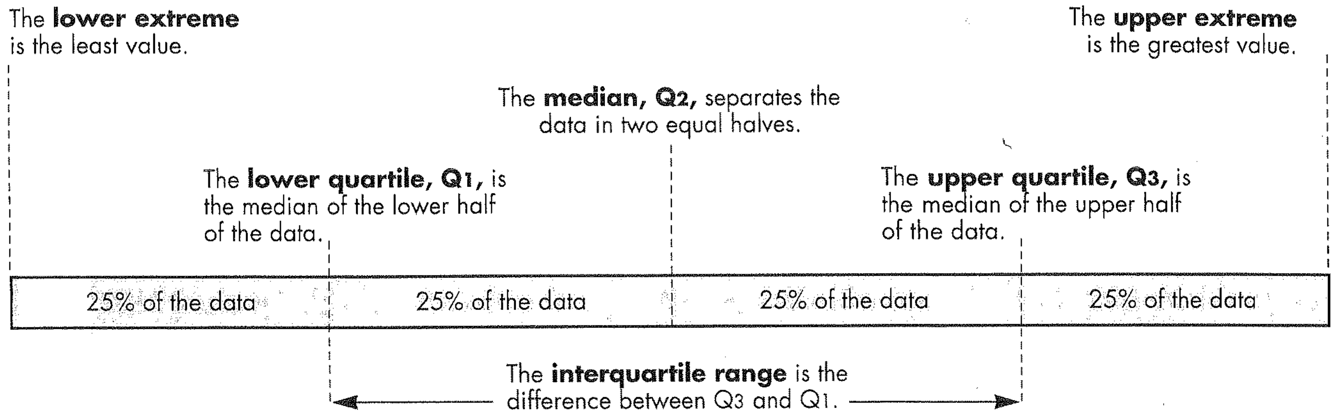
Even when the data are organized in a table, they are difficult to analyze because the values vary and there are no trends apparent in the data. However, there are several ways to measure variation in the data. The simplest is called the **range.**

Definition of Range	**The range of a set of data is the difference between the greatest and least values in the set.**

In this case, the greatest number of newspapers in any state is 104 in California, and the least is 3 in Delaware. So the range of the number of newspapers is $104 - 3$ or 101.

Because the range is the difference between the greatest and least values in a set of data, it is affected by extreme values. In these cases, it is not a good measure of variation.

Another way to analyze a set of data is to determine how the data are distributed between the least and greatest values. The **quartiles** are the values in a set that separate the data into four sections, each containing 25% of the data.



Example **1**

Real World APPLICATION

Sports

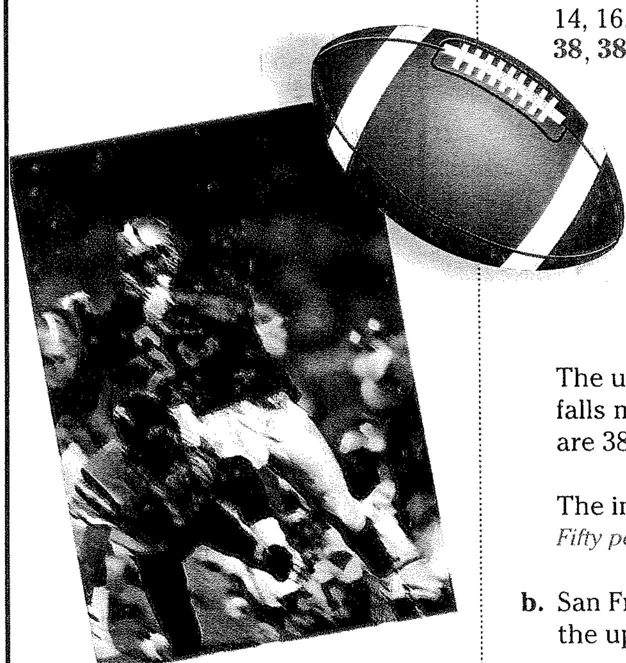
On January 29, 1995, the San Francisco 49ers became the first team to win five Super Bowls as they defeated the San Diego Chargers 49–26. The number of points scored by the winning teams in all Super Bowls through 1995 are as follows.

35, 33, 16, 23, 16, 24, 14, 24, 16, 21, 32, 27, 35, 31, 27, 26, 27, 38, 38, 46, 39, 42, 20, 55, 20, 37, 52, 30, 49

a. Find Q_1, Q_2, Q_3, the range, and the interquartile range.
b. Analyze how San Francisco's score in 1995 compares to the other winning scores.

a. First arrange the data in order.

14, 16, 16, 16, 20, 20, **21**, **23**, 24, 24, 26, 27, 27, 27, **30**, 31, 32, 33, 35, 35, 37, **38**, **38**, 39, 42, 46, 49, 52, 55



The range is 55 − 14 or 41.

There are 29 values in all. The median, Q_2, is the middle, or 15th, value. Therefore, the median score is 30.

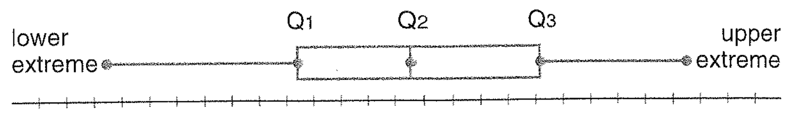
The lower quartile, Q_1, is the median of the lower half of the data. Since there are 14 values in the lower half, the lower quartile falls midway between the 7th and 8th value. The lower quartile is $\frac{21 + 23}{2}$ or 22.

The upper quartile, Q_3, is the median of the upper half of the data. It falls midway between the 22nd and 23rd values. Since both these values are 38, the upper quartile is 38.

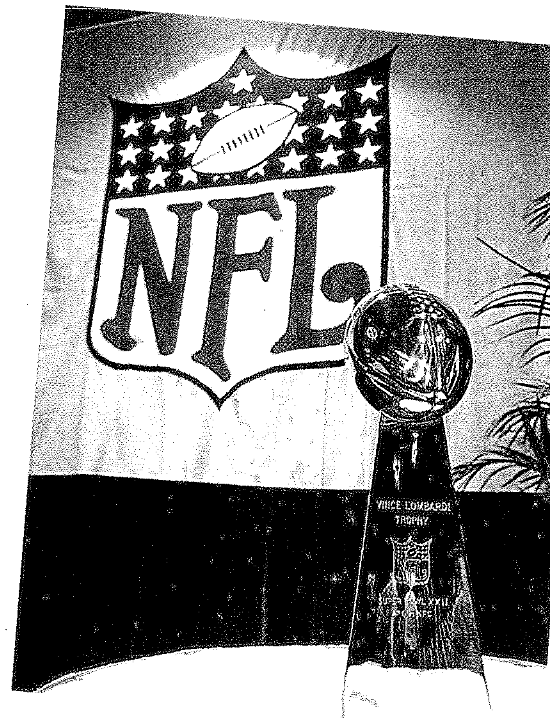
The interquartile range is $Q_3 - Q_1 = 38 - 22$ or 16.
Fifty percent of the time, the number of points scored was between 22 and 38.

b. San Francisco's score of 49 is above the upper quartile. Therefore, it is in the upper 25% of the data.

Numerical data can often be represented using a **box-and-whisker plot**. In a box-and-whisker plot, the quartiles and the extreme values of a set of data are displayed using a number line.



Thus, a box-and-whisker plot is a pictorial representation of the variability of the data and a way to summarize a data set with five points.



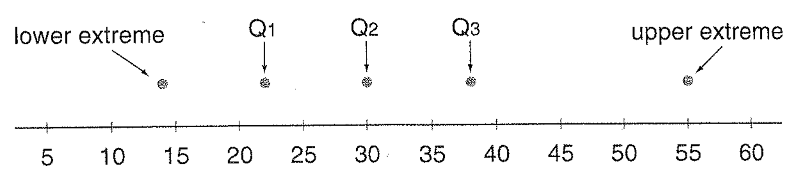
Example 2

Real World APPLICATION

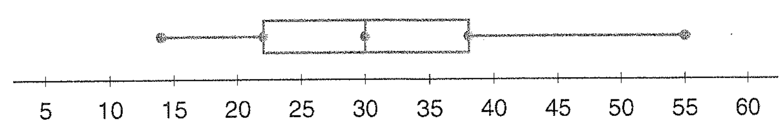
Sports

Refer to the data in Example 1 about the Super Bowl winners. Make a box-and-whisker plot of the data.

To make a box-and-whisker plot, draw a number line and plot the quartiles, the median, and the extreme values. The lower extreme is 14, the lower quartile is 22, the median is 30, the upper quartile is 38, and the upper extreme is 55.



Draw a box to designate the interquartile range and mark the median by drawing a segment containing its point in the box. Draw segments (whiskers) connecting the lower quartile to the least value and the upper quartile to the greatest value.



The dimensions of the box-and-whisker plot can help you characterize the data. Each whisker and each small box contains 25% of the data. If the whisker or box is short, the data are concentrated over a narrower range of values. The longer the whisker or box, the more diverse the data. Extreme values are referred to as **outliers**.

Definition of Outlier	**An outlier is any value in the set of data that is at least 1.5 interquartile ranges beyond the upper or lower quartile.**

Example 3 Refer to the data in the application at the beginning of the lesson about the number of newspapers in circulation.

Real World APPLICATION

News Media

a. Find Q_1, Q_2, Q_3, and the interquartile range.

b. List any outliers.

c. Make a box-and-whisker plot.

d. If any outliers exist, analyze them to determine possible reasons why they exist.

a. First arrange the data in order.

3, 6, 6, 6, 7, 7, 8, 8, 9, 10, 11, 11, 11, 12, 15, 16, 18, 18, 19, 19, 21, 22, 22, 23, 23, 24, 25, 25, 26, 27, 28, 28, 32, 34, 36, 39, 39, 40, 45, 46, 47, 49, 52, 68, 71, 73, 84, 89, 92, 104

There are 50 values in all.

Q_1 is the 13th value, 11.

Q_2 is between the 25th value, 23, and the 26th value, 24. Therefore, Q_2 is 23.5.

Q_3 is the 38th value, 40.

The interquartile range is $40 - 11$ or 29.

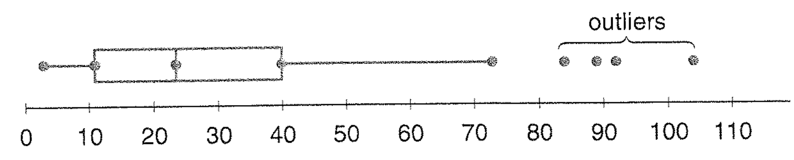
b. Find the outliers.

$Q_1 - 1.5(29) = 11 - 43.5$ or -32.5 *There are no values less than -32.5.*

$Q_3 + 1.5(29) = 40 + 43.5$ or 83.5 *There are four values greater than 83.5.*

Therefore, 84, 89, 92, and 104 are outliers.

c. Draw a number line and plot the quartiles, the lower extreme, the upper extreme, and the outliers. Also, plot 73, since this is the last data value that is not an outlier. Extend the whiskers to the lower extreme, 14, and to 73. The outliers remain as single points.

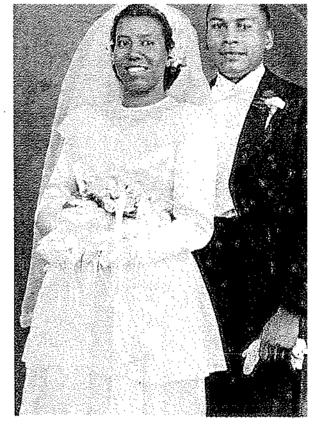


d. The outliers are from the states of Ohio, Pennsylvania, Texas, and California. All of these states have large populations that can support many daily newspapers.

In addition to showing how data within a set vary, box-and-whisker plots can be used to compare two or more sets of data.

Example 4 CONNECTION Sociology

The table below shows the median ages of men and women at the time of their first marriage for the decades of 1890 through 1990.

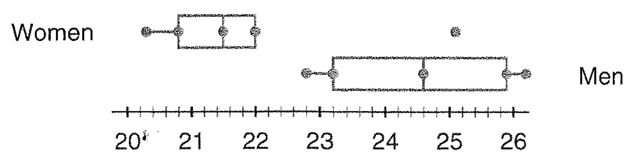


Year	Men	Women	Year	Men	Women
1890s	26.1	22.0	1950s	22.8	20.3
1900s	25.9	21.9	1960s	22.8	20.3
1910s	25.1	21.6	1970s	23.2	20.8
1920s	24.6	21.2	1980s	24.7	22.0
1930s	24.3	21.3	1990s	26.2	25.1
1940s	24.3	21.5			

a. Make a box-and-whisker plot for the men's and women's ages.
b. Analyze the information as it is displayed in the two plots.

a. For the men's data, the lower extreme is 22.8, the upper extreme is 26.2, and the quartiles are 23.2, 24.6, and 25.9. There are no outliers.

For the women's data, the lower extreme is 20.3, the upper extreme is 25.1, and the quartiles are 20.8, 21.5, and 22.0. There is one outlier, 25.1. The last value of the data that is not an outlier is 22.0.



b. It appears from the data that over the years, men marry at a later age than women. It also appears that the interquartile range for the women is 1.2 years, but for the men it is 2.7 years. For 50% of the years, women married between the ages of 20.8 and 22.0. Thus, there is not much difference or spread in the ages at which women married, except in 1990. The ages at which men married varies much more.

EXPLORATION — GRAPHING CALCULATORS

The table below shows the 1990 populations of ten large cities and the predicted populations of those same cities for the year 2000.

City	1990 Population (millions)	2000 Population (millions)
Tokyo	27.0	30.0
Mexico City	20.2	27.9
Sao Paulo	18.1	25.4
Seoul	16.3	22.0
New York	14.6	14.6
Osaka	13.8	14.3
Bombay	11.8	15.4
Calcutta	11.7	14.1
Buenos Aires	11.5	12.9
Rio de Janeiro	11.4	14.2

Source: U.S. Bureau of the Census, International Data Base, 1994

(continued on the next page)

There are many ways in which these data could be displayed. You can make box-and-whisker plots using these data fairly quickly by using a graphing calculator.

Your Turn

a. Press STAT 1. Enter the 1990 population values into L1 and the 2000 population values into L2.

b. Press 2nd STAT PLOT 1. Turn on Plot 1 and define it as a box-and-whisker plot, using L1 and Frequency 1. Press 2nd STAT PLOT 2. Turn on Plot 2 and define it as a box-and-whisker plot, using L2 and Frequency 1.

c. Clear the Y= list and change the window settings to Xscl = 1, Ymin = 0, and Yscl = 0. Then press ZOOM 9.

d. Write a paragraph to describe the difference in the spread of the data between 1990 and 2000.

CHECK FOR UNDERSTANDING

Communicating Mathematics

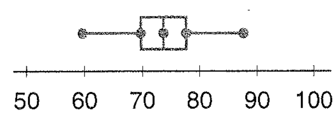
Study the lesson. Then complete the following.

1. **Show** how a set of data can be separated into quartiles.

2. **Describe** what you can tell about a set of data from a box-and-whisker plot.

3. **Write** an example of two sets of data with the same lower and upper extreme but different interquartile ranges.

4. **Explain** how to find the outliers in a set of data if $Q_1 = 52.5$, $Q_2 = 60$, and $Q_3 = 72.5$.

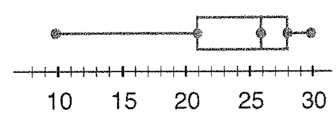


5. **Describe** how the data shown in the box-and-whisker plot at the right are distributed.

6. **You Decide** Michelle thinks that $Q_3 - Q_2 = Q_2 - Q_1$ for any set of data. Mei thinks that this isn't necessarily true. Who is correct? Explain your reasoning.

Guided Practice

7. Use the box-and-whisker plot at the right to answer each question.



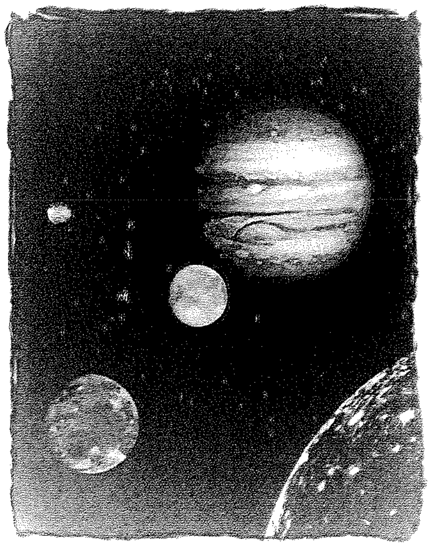
   a. What is the range of the data?
   b. What is the median of the data?
   c. What percent of the data is greater than 28?
   d. Between what two values of the data is the middle 50% of the data?

Find the range, quartiles, interquartile range, and outliers for each set of data. Then make a box-and-whisker plot for each set of data.

8. {12, 19, 20, 1, 15, 14, 19}

9. {24, 32, 38, 38, 26, 33, 37, 39, 23, 31, 40, 21}

10. **Astronomy** The table at the right lists the approximate length of a day, in Earth hours, for each of the planets in our solar system.



Planet	Length of Day (Earth hours)
Mercury	1416
Venus	5832
Earth	24
Mars	24
Jupiter	10
Saturn	11
Uranus	22
Neptune	16
Pluto	153

a. Which planet's day length separates the data into two equal halves?

b. What is the median length of a day for the planets in our solar system?

c. Fifty percent of the data lie between what two values?

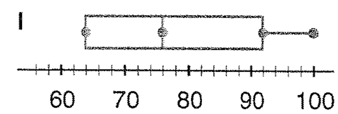
d. Estimate which value(s) may be outliers. Then determine if the data set has any outliers.

e. What problems would you encounter if you tried to make a box-and-whisker plot with these data?

EXERCISES

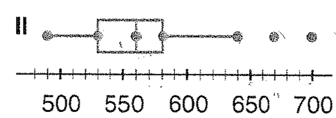
Practice

11. Use box-and-whisker plot I to answer each question.



a. What percent of the data is less than 76?
b. What percent of the data is less than 92?
c. What percent of the data is greater than 64 and less than 92?
d. Under what conditions would a set of data have this type of box-and-whisker plot?

12. Use box-and-whisker plot II to answer each question.



a. What is the range of the data?
b. What values of the data are outliers?
c. What percent of the data is greater than 560?
d. What percent of the data is less than 580?

Find the range, quartiles, interquartile range, and outliers for each set of data. Then make a box-and-whisker plot for each set of data.

13. {25, 46, 31, 53, 39, 59, 48, 43, 68, 64, 29}

14. {25, 51, 29, 43, 32, 17, 21, 29, 36, 47}

15. {51, 69, 46, 27, 60, 53, 55, 39, 81, 54, 46, 23}

16. {110, 110, 330, 200, 88, 110, 88, 110, 165, 390, 150, 440, 536, 200, 110, 165, 88, 147, 110, 165}

17. {13.6, 15.1, 14.9, 15.7, 16.0, 14.1, 16.3, 14.3, 13.8}

18. {15.1, 11.5, 5.8, 6.2, 10.5, 7.6, 9.0, 8.5, 8.8, 8.5}

Graphing Calculator

19. **Meteorology** San Francisco, California, and Springfield, Missouri, are both located at 37° N latitude. However, their average monthly high temperatures are very different.
    a. Use a graphing calculator to make box-and-whisker plots for each set of data.
    b. Write a sentence that compares the average monthly high temperatures in each city.
    c. In which city would you prefer to live? Give reasons to support your answer.

Average High Temperatures (°F)		
Month	**San Francisco**	**Springfield**
January	59	41
February	61	48
March	61	57
April	62	69
May	65	72
June	69	82
July	69	89
August	70	88
September	72	80
October	70	70
November	61	59
December	58	45

Source: *1995 Weather Almanac*

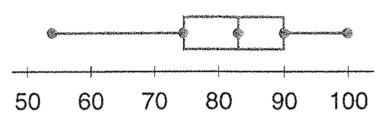
Critical Thinking

20. Suppose a set of data has a small interquartile range and a large range. What does this tell you about how the data are distributed?
21. Give an example of a set of data having ten values and containing at least one outlier at each end of the data. What conditions must be present for this to happen?

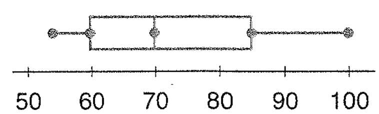
Applications and Problem Solving

22. **School** Two geometry classes took the same exam, and their scores are shown in box-and-whisker plots below.

Period 1



Period 2



   a. Which class has the higher median?
   b. Which class has the greater range?
   c. Which class has the greater interquartile range?
   d. Which class appears to have done better?
   e. Describe the spread of the scores in the two classes.

23. **History** The stem-and-leaf plot below represents the age at death of the presidents of the United States.

Stem	Leaf
4	6 9
5	3 6 6 7 8
6	0 0 3 3 4 4 5 6 7 7 7 8
7	0 1 1 2 3 4 7 8 8 9
8	0 1 3 5 8
9	0 0 *4 | 6 = 46*

   a. Make a box-and-whisker plot of the data.
   b. What are some advantages and disadvantages of representing these data in a stem-and-leaf plot? What are some advantages and disadvantages of representing these data in a box-and-whisker plot?

24. Food The number of Calories in a regular serving of French fries at nine different restaurants are listed below.

250 240 220 200 125 239 211 240 327

a. Make a box-and-whisker plot of the data.

b. Explain why this box-and-whisker plot has such short whiskers.

25. Literature Maya Angelou is a Reynolds Professor of African Studies at Wake Forest University in North Carolina and is well known for her poetry. She was commissioned by then President-elect Bill Clinton to write a poem for his 1993 inauguration. An excerpt from *On the Pulse of Morning* is shown below.

A Rock, A River, A Tree
Hosts to species long since departed,
Marked the mastodon,
The dinosaur, who left dried tokens
Of their sojourn here
On our planet floor,
Any broad alarm of their hastening doom
Is lost in the gloom of dust and ages.

a. Make a box-and-whisker plot that shows the number of letters in each word of this excerpt.

b. Find a newspaper article and make a box-and-whisker plot that shows the number of letters in each word of the article.

c. Compare and contrast the two plots.

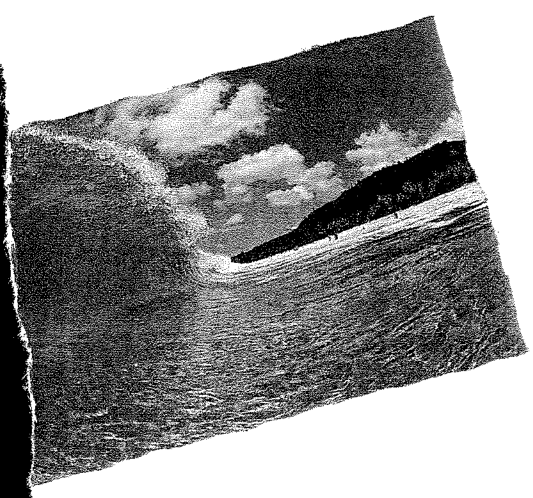
26. Demographics The chart shows the Asian and Pacific Islander population projections for 2000 and 2010 in the western United States.

a. For each year, make a box-and-whisker plot.

b. How will the median population change from 2000 until 2010?

c. How will the interquartile range change from 2000 until 2010?

d. Write a sentence that summarizes how the population of Asian and Pacific Islanders is expected to change from 2000 to 2010.



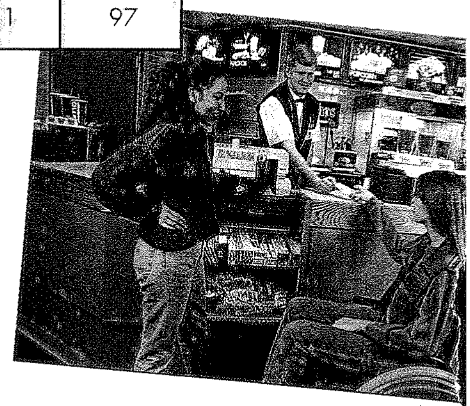
Population Projections (thousands)

State	2000	2010
Montana	8	12
Idaho	21	31
Wyoming	6	9
Colorado	117	166
New Mexico	35	56
Arizona	127	204
Utah	77	122
Nevada	99	148
Washington	424	635
Oregon	153	238
California	4906	7169
Alaska	39	59
Hawaii	681	746

Source: U.S. Bureau of the Census

27. **Snacks** Who can resist the snack counter when you're at the movies? Not many people! But the Center for Science in the Public Interest reports that many snacks contain as many Calories and fat as a fast-food lunch. The chart below lists the Calories and fat content for several popular movie snacks.

Snack	Calories	Fat (grams)
Cookie bar/4 oz	588	33
Red licorice twists/5 oz	500	5
Candy bar/4 oz	492	21
Peanut butter cups/3.2 oz	380	22
Peanut candies/2.6 oz	363	20
Chocolate mints/3 oz	360	9
Plain candies/2.6 oz	350	16
Chocolate-covered peanuts/2.2 oz	320	21
Fruit candies/2.6 oz	286	2
Chocolate-covered raisins/2.3 oz	270	10
Buttered popcorn popped in coconut oil/medium bucket	1221	97



a. Make box-and-whisker plots for the number of Calories and grams of fat.

b. Add the following values to the data.

Unbuttered popcorn popped in coconut oil/medium bucket: 901 Calories, 60 grams fat

Plain air-popped popcorn/medium bucket: 180 Calories, <1 gram fat

How do these values affect the plot in part a?

c. Find a snack that would fall in the bottom 25% of the data for both number of Calories and grams of fat.

Mixed Review

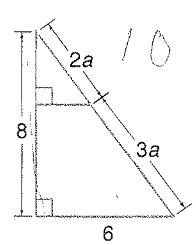
28. Solve the system of equations by using augmented matrices. (Lesson 4–7)

$$2x + y + z = 0$$
$$3x - 2y - 3z = -21$$
$$4x + 5y + 3z = -2$$

29. Find $[2 \ \ -6 \ \ 3] \cdot \begin{bmatrix} 3 & -3 \\ 9 & 0 \\ -2 & 4 \end{bmatrix}$. (Lesson 4–3)

30. **SAT Practice** In the figure, a =

A 1 B 2 C 3 D 4 E 5



31. Find the slope of the line that passes through (5, 4) and (2, 2). (Lesson 2–3)

32. Name the sets of numbers to which 2.121221222 . . . belongs. (Lesson 1–2)

33. **Finance** You are about to buy a new car. Your mother offers you a simple interest loan to finance it. Simple interest is calculated using the formula $I = prt$, where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the amount of interest you would pay for a two-year loan if the principal is \$6000 and the rate is 12%. (Lesson 1–1)

For Extra Practice, see page 885.

CHAPTER 4 HIGHLIGHTS

INTERNET CONNECTION
Chapter Review For additional lesson-by-lesson review, visit: **www.algebra2.glencoe.com**

VOCABULARY

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Discrete Mathematics
addition of matrices (p. 195)
augmented matrix (p. 226)
column matrix (p. 187)
coordinate matrix (p. 189)
determinant (p. 205)
dimensions (p. 187)
discrete mathematics (p. 188)
element (p. 186)
equal matrices (p. 188)
expansion by minors (p. 205)
identity matrix (p. 213)
inverse of a matrix (p. 213)
matrix (p. 186)
matrix equation (p. 219)
minor (p. 205)
multiplying matrices (p. 199)
probability matrix (p. 200)
reduced matrix (p. 227)
reducing a matrix (p. 227)
row operations (p. 226)
row matrix (p. 187)
scalar (p. 188)
scalar multiplication (p. 188)
square matrix (p. 187)
third-order determinant (p. 205)
transition matrix (p. 200)
translation matrix (p. 196)

Problem Solving
matrix logic (p. 187)

Geometry
dilation (p. 190)
rotation (p. 201)
transformation (p. 190)
translation (p. 195)

Statistics
box-and-whisker plot (p. 237)
interquartile range (p. 236)
lower extreme (p. 236)
lower quartile (p. 236)
median (p. 236)
outlier (p. 237)
quartiles (p. 236)
range (p. 235)
upper extreme (p. 236)
upper quartile (p. 236)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term to complete each sentence.

augmented matrix
determinant
dilation
dimensions
equal matrices
identity matrix
reduced matrix
rotation
scalar multiplication
translation

1. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a(n) ________ for multiplication.
2. The matrix $\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -3 \end{array}\right]$ is a(n) ________.
3. ________ is the process of multiplying a matrix by a constant.
4. A(n) ________ is when a figure is moved around a center point.
5. The ________ of $\begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$ is -5.
6. The matrix $\left[\begin{array}{ccc|c} 1 & 5 & 3 & 4 \\ 4 & -3 & 2 & 4 \\ 8 & -6 & 4 & 14 \end{array}\right]$ is a(n) ________.
7. The ________ of a matrix tell how many rows and columns are in the matrix.
8. A ________ occurs when a figure is moved from one location to another on the coordinate plane without changing its size, shape, or orientation.
9. The matrices $\begin{bmatrix} 3x \\ x + 2y \end{bmatrix}$ and $\begin{bmatrix} y \\ 7 \end{bmatrix}$ are ________ if $x = 1$ and $y = 3$.
10. A ________ is when a geometric figure is enlarged or reduced.

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Upon completing this chapter, you should be able to:

- perform scalar multiplication on a matrix (Lesson 4–1)

$$4\begin{bmatrix} 2 & -3 \\ 4 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4(2) & 4(-3) \\ 4(4) & 4(1) \\ 4(0) & 4(3) \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -12 \\ 16 & 4 \\ 0 & 12 \end{bmatrix}$$

- solve matrices for variables (Lesson 4–1)

To find x and y in $\begin{bmatrix} 2x \\ y \end{bmatrix} = \begin{bmatrix} 32 + 6y \\ 7 - x \end{bmatrix}$, solve the system $2x = 32 + 6y$ and $y = 7 - x$.

$x = 9.25, y = -2.25$

- add and subtract matrices (Lesson 4–2)

$$2\begin{bmatrix} 8 & -1 \\ 3 & 4 \end{bmatrix} - 3\begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 16 & -2 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} -3 & -18 \\ 6 & 9 \end{bmatrix} = \begin{bmatrix} 13 & -20 \\ 12 & 17 \end{bmatrix}$$

- multiply matrices (Lesson 4–3)

$$[6\ 4\ 1] \cdot \begin{bmatrix} 2 & 5 \\ -3 & 0 \\ -1 & 3 \end{bmatrix} = [12 - 12 - 1 \quad 30 + 0 + 3] = [-1 \quad 33]$$

REVIEW EXERCISES

Use these exercises to review and prepare for the chapter test.

Find each product.

11. $3\begin{bmatrix} 8 & -3 & 2 \\ 4 & 1 & 7 \end{bmatrix}$

12. $-5\begin{bmatrix} -3 & 2 \\ 6 & 4 \end{bmatrix}$

13. $\frac{2}{3}\begin{bmatrix} 3 & \frac{3}{4} & -6 \end{bmatrix}$

14. $1.2\begin{bmatrix} -2 \\ 0.3 \\ 1 \end{bmatrix}$

15. $4\begin{bmatrix} 1.3 & 5.1 \\ -2 & -3.7 \\ -2.8 & 4.5 \end{bmatrix}$

16. $-\frac{1}{2}\begin{bmatrix} -2 & 4 \\ -8 & 2 \end{bmatrix}$

Solve for the variables.

17. $\begin{bmatrix} 2y - x \\ x \end{bmatrix} = \begin{bmatrix} 3 \\ 4y - 1 \end{bmatrix}$

18. $\begin{bmatrix} 7x \\ x + y \end{bmatrix} = \begin{bmatrix} 5 + 2y \\ 11 \end{bmatrix}$

19. $\begin{bmatrix} 3x + y \\ x - 3y \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

20. $\begin{bmatrix} 2x - y \\ 6x - y \end{bmatrix} = \begin{bmatrix} 2 \\ 22 \end{bmatrix}$

Perform the indicated operations.

21. $\begin{bmatrix} -4 & 3 \\ -5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 3 & -8 \end{bmatrix}$

22. $[0.2 \ \ 1.3 \ \ -0.4] - [2 \ \ 1.7 \ \ 2.6]$

23. $\begin{bmatrix} 1 & -5 \\ -2 & 3 \end{bmatrix} + \frac{3}{4}\begin{bmatrix} 0 & 4 \\ -16 & 8 \end{bmatrix}$

24. $\begin{bmatrix} 1 & 0 & -3 \\ 4 & -5 & 2 \end{bmatrix} - 2\begin{bmatrix} -2 & 3 & 5 \\ -3 & -1 & 2 \end{bmatrix}$

Perform the indicated operations, if possible.

25. $[2 \ \ 7] \cdot \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

26. $\begin{bmatrix} 8 & -3 \\ 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 1 & -5 \end{bmatrix}$

27. $\begin{bmatrix} 3 & 4 \\ 1 & 0 \\ 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} -2 & 4 & 5 \\ 3 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

OBJECTIVES AND EXAMPLES

- evaluate the determinant of a 3×3 matrix (Lesson 4–4)

$$\begin{array}{ccc|cc} 0 & -3 & 2 & & \\ \hline 3 & 1 & 5 & 3 & 1 \\ 1 & -2 & 1 & 1 & -2 \\ 0 & -1 & 2 & 0 & -1 \\ \hline -12 & 0 & -5 & & \end{array}$$

$= -12 + 0 + (-5) - 0 - (-3) - 2$

$= -16$

REVIEW EXERCISES

Determine whether each matrix has a determinant. Write *yes* or *no*. If *yes*, find the value of the determinant.

28. $\begin{bmatrix} 4 & 11 \\ -7 & 8 \end{bmatrix}$

29. $\begin{bmatrix} 7 & -4 & 5 \\ 1 & 3 & -6 \\ 5 & -1 & -2 \end{bmatrix}$

30. $\begin{bmatrix} 5 & -1 & 2 \\ -6 & -7 & 3 \\ 7 & 0 & 4 \end{bmatrix}$

31. $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 7 & 8 \\ 2 & 1 & 3 \end{bmatrix}$

32. $\begin{bmatrix} -2 & 9 \\ 7 & -4 \\ -6 & 1 \end{bmatrix}$

33. $\begin{bmatrix} 6 & 3 & -2 \\ -4 & 2 & 5 \\ -3 & -1 & 0 \end{bmatrix}$

- find the inverse of a matrix (Lesson 4–5)

Any matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, will have an inverse M^{-1} if and only if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

Then $M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Find the inverse of each matrix, if it exists. If it does not exist, explain why not.

34. $\begin{bmatrix} 3 & 2 \\ 4 & -2 \end{bmatrix}$

35. $\begin{bmatrix} 8 & 6 \\ 9 & 7 \end{bmatrix}$

36. $\begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$

37. $\begin{bmatrix} -6 & 2 \\ 3 & 1 \end{bmatrix}$

38. $\begin{bmatrix} 0 & 2 \\ 5 & -4 \end{bmatrix}$

39. $\begin{bmatrix} 6 & -1 & 0 \\ 5 & 8 & -2 \end{bmatrix}$

- solve systems of linear equations by using inverse matrices (Lesson 4–6)

$$\begin{bmatrix} 4 & 8 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$$-\frac{1}{28} \cdot \begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 8 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{28} \cdot \begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$$-\frac{1}{28} \cdot \begin{bmatrix} -28 & 0 \\ 0 & -28 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{28} \cdot \begin{bmatrix} -21 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{2} \end{bmatrix} \text{ or } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{2} \end{bmatrix}$$

The solution is $\left(\frac{3}{4}, \frac{1}{2}\right)$.

Solve each matrix equation or system of equations by using inverse matrices.

40. $\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix}$

41. $\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$

42. $3x + 8 = -y$
$4x - 2y = -14$

43. $3x - 5y = -13$
$4x + 3y = 2$

OBJECTIVES AND EXAMPLES

- solve systems of linear equations by using augmented matrices (Lesson 4–7)

$5a - 3b = 7$

$3a + 9b = -3$

$$\left[\begin{array}{cc|c} 5 & -3 & 7 \\ 3 & 9 & -3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -\frac{2}{3} \end{array}\right]$$

The solution is $\left(1, -\frac{2}{3}\right)$.

REVIEW EXERCISES

Solve each system of equations by using augmented matrices.

44. $9a - b = 1$
$3a + 2b = 12$

45. $x + 5y = 14$
$-2x + 6y = 4$

46. $6x - 7z = 13$
$8y + 2z = 14$
$7x + z = 6$

47. $2a - b - 3c = -20$
$4a + 2b + c = 6$
$2a + b - c = -6$

OBJECTIVES AND EXAMPLES

- find the range, quartiles, and interquartile range for a set of data and represent the data using box-and-whisker plots (Lesson 4–8)

Find the range, interquartile range, and any outliers for the set of data below.

60, 61, 62, 72, 72, 78, 80, 82, 83, 83, 99

greatest value = 99 least value = 60
range = 39 median = 6th score, 78
lower quartile = 62 upper quartile = 83
interquartile range = 83 − 62 or 21

There are no outliers.

REVIEW EXERCISES

Find the range, quartiles, interquartile range, and outliers for each set of data. Then make a box-and-whisker plot for each set of data.

48. {90, 92, 78, 93, 70, 85, 89, 88, 84, 86}

49. {10, 50, 90, 40, 60, 40, 50, 90, 0}

50. {0.4, 0.2, 0.5, 0.9, 0.3 0.4, 0.5, 1.9, 0.5, 0.7, 0.8, 0.6, 0.2, 0.1, 0.4}

51. {1055, 1075, 1095, 1125, 1005, 975, 1125, 1100, 1145, 1025, 1075}

APPLICATIONS AND PROBLEM SOLVING

52. Horticulture A rose garden is being planted as a border around two sides of a triangular shaped lawn in a city park. Two of the vertices of the triangle have coordinates (−2, 4) and (3, −5). The gardener wishes to locate the third vertex so that the lawn's area is 25 square feet. Find the value of *f* if the coordinates of the third vertex are (3, *f*). (Lesson 4–4)

53. Auto Mechanics Ann Braun is inventory manager for a local repair shop. If she orders 6 batteries, 5 cases of spark plugs, and two dozen pairs of wiper blades, she will pay $830. If she orders 3 batteries, 7 cases of spark plugs, and 4 dozen pairs of wiper blades, she will pay $820. If the batteries are $22 less than twice the price of a dozen wiper blades, what is the cost of each item on her order? (Lesson 4–7)

A practice test for Chapter 4 is provided on page 915.

ALTERNATIVE ASSESSMENT

COOPERATIVE LEARNING PROJECT

Who Owns Muffin? In this project, you will manipulate data by using logic. You will not need to use operational algebraic skills, but you will need to write things down, keep track of information, and analyze data. There are 16 pieces of information below. From this information, you are to figure out who owns Muffin and who has no children.

1. There are five houses in a row.
2. The owners of the SPORTS CAR live in the RED house.
3. The MINI-VAN owners have TWO children.
4. The family that lives in the GREEN house has a pet named SPOT.
5. The SEDAN owners' pet is called ROVER.
6. The GREEN house is just to the right of the WHITE house.
7. The family that owns a DUCK has FOUR children.
8. A PIG lives in the YELLOW house.
9. The family in the MIDDLE house has an animal named ROSIE.
10. The vehicle at the FIRST house is a TRUCK.
11. The CAT lives next door to the house with FIVE children.
12. The family that owns the PIG lives next door to the family with SEVEN children.
13. THOMAS, the GOAT, lives next door to SPOT.
14. The STATION WAGON'S owners have a ZEBRA.
15. The TRUCK's owners live next door to the BLUE house.
16. ROSIE's owners have a SPORTS CAR.

Follow these steps to solve your problem.

- Set up a table.
- Determine what information you can be sure of and put it in the table.
- The remaining information will be retrieved by elimination and deduction.
- Continue in this manner until you have filled in the table.
- The two empty spaces point out where Muffin lives and where no children live.
- Write a paragraph describing your plan and how you attacked the problem.

THINKING CRITICALLY

- Find the new coordinates of quadrilateral $ROSE$, with vertices $R(-2, -1)$, $O(3, 0)$, $S(2, 2)$, and $E(-1, 2)$, if it is rotated 90° counterclockwise about the origin *twice*. Compare the new coordinates to the original ones. Make a conjecture about what effect this rotation has on any figure.
- If the determinant of a coefficient matrix is 0, can you use inverse matrices to solve the system of equations? Why or why not? Describe the graph of such a system of equations.

PORTFOLIO

Using the addition, subtraction, and multiplication operations with matrices, determine whether the following properties, when used with matrices, parallel these same properties under whole number operations.

- associative property
- commutative property
- distributive property

Write a convincing proof or argument for each property. Place this in your portfolio.

CHAPTERS 1–4

Section One: MULTIPLE CHOICE

There are eight multiple-choice questions in this section. After working each problem, write the letter of the correct answer on your paper.

1. Write the domain of $g = \{(9, 0), (3, 1), (12, 7), (1, -4), (12, 8), (-11, -3), (0, -6)\}$ and determine if g is a function.

   A. $\{9, 3, 12, 1, -11, 0\}$; g is a function.

   B. $\{0, 1, 7, -4, 8, -3, -6\}$; g is not a function.

   C. $\{9, 3, 12, 1, -11, 0\}$; g is not a function.

   D. $\{0, 1, 7, -4, 8, -3, -6\}$; g is a function.

2. The augmented matrix for a system is $\begin{bmatrix} 1 & 5 & 1 \\ 2 & -3 & 15 \end{bmatrix}$. What is the solution?

   A. $(6, -1)$ B. $(1, 15)$

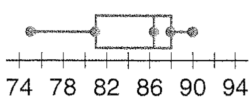
   C. $(5, -13)$ D. $(13, 1)$

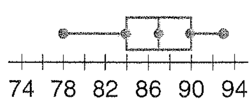
3. Write an algebraic expression for the verbal expression, "twice the sum of a number and 5 is at most 18."

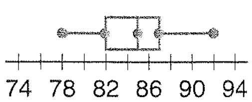
   A. $2x + 5 \geq 18$ B. $2(x + 5) > 18$

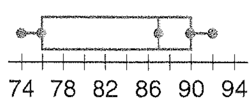
   C. $2(x + 5) \leq 18$ D. $2x + 2(5) = 18$

4. A set of data has a mean of 86.4, a median of 87, a range of 15, an interquartile range of 6, and an upper quartile of 90. Which box-and-whisker plot represents this information?

   A.


   B.


   C.


   D.


5. What is the slope-intercept form of a line that passes through $(1, -4)$ and is perpendicular to a line whose equation is $3x - 2y = 8$?

   A. $y = -\frac{2}{3}x - \frac{10}{3}$

   B. $2x + 3y = -10$

   C. $y = \frac{3}{2}x - 4$

   D. $y = \frac{2}{3}x - 4$

6. A feasible region has vertices $(0, 0)$, $(4, 0)$, $(5, 5)$, and $(0, 8)$. Find the maximum and minimum of the function $f(x,y) = x + 3y$ over this region.

   A. max: $f(0, 8) = 24$
   min: $f(0, 0) = 0$

   B. min: $f(0, 0) = 0$
   max: $f(5, 5) = 20$

   C. max: $f(5, 5) = 20$
   min: $f(0, 8) = 8$

   D. min: $f(4, 0) = 4$
   max: $f(0, 0) = 0$

7. Name all the sets of numbers to which -12 belongs.

   A. integers

   B. integers, rationals

   C. rationals, reals

   D. integers, rationals, reals

8. Find the value for a for which the graph of $y = ax - 3$ is perpendicular to the graph of $6x + y = 4$.

   A. $\frac{11}{4}$

   B. $\frac{1}{6}$

   C. $\frac{2}{3}$

   D. 6