

1-5

Roots and Real Numbers

Objectives

Simplify expressions containing roots.

Classify numbers within the real number system.

Vocabulary

square root

principal square root

perfect square

cube root

natural numbers

whole numbers

integers

rational numbers

terminating decimal

repeating decimal

irrational numbers

Why learn this?

Square roots can be used to find the side length of a square garden when you know its area. (See Example 3.)

A number that is multiplied by itself to form a product is a **square root** of that product. The radical symbol $\sqrt{\quad}$ is used to represent square roots. For nonnegative numbers, the operations of squaring and finding a square root are inverse operations. In other words, for $x \geq 0$, $\sqrt{x} \cdot \sqrt{x} = x$.

Positive real numbers have two square roots. The **principal square root** of a number is the positive square root and is represented by $\sqrt{\quad}$. A negative square root is represented by $-\sqrt{\quad}$. The symbol $\pm\sqrt{\quad}$ is used to represent both square roots.

$$4 \cdot 4 = 4^2 = 16 \longrightarrow \sqrt{16} = 4 \longleftarrow \text{Positive square root of 16}$$

$$(-4)(-4) = (-4)^2 = 16 \longrightarrow -\sqrt{16} = -4 \longleftarrow \text{Negative square root of 16}$$

A **perfect square** is a number whose positive square root is a whole number. Some examples of perfect squares are shown in the table.

0	1	4	9	16	25	36	49	64	81	100
0^2	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2

A number that is raised to the third power to form a product is a **cube root** of that product. The symbol $\sqrt[3]{\quad}$ indicates a cube root. Since $2^3 = 8$, $\sqrt[3]{8} = 2$. Similarly, the symbol $\sqrt[4]{\quad}$ indicates a fourth root: $2^4 = 16$, so $\sqrt[4]{16} = 2$.

EXAMPLE 1 Finding Roots

Find each root.

A $\sqrt{49}$
 $\sqrt{49} = \sqrt{7^2}$
 $= 7$

Think: What number squared equals 49?

B $-\sqrt{36}$
 $-\sqrt{36} = -\sqrt{6^2}$
 $= -6$

Think: What number squared equals 36?

C $\sqrt[3]{-125}$
 $\sqrt[3]{-125} = \sqrt[3]{(-5)^3}$
 $= -5$

Think: What number cubed equals -125 ?
 $(-5)(-5)(-5) = 25(-5) = -125$

Writing Math

The small number to the left of the root is the **index**. In a square root, the index is understood to be 2. In other words, $\sqrt{\quad}$ is the same as $\sqrt[2]{\quad}$.



Find each root.

1a. $\sqrt{4}$

1b. $-\sqrt{25}$

1c. $\sqrt[4]{81}$

EXAMPLE 2 Finding Roots of Fractions

Find $\sqrt{\frac{1}{4}}$.

$$\sqrt{\frac{1}{4}} = \sqrt{\left(\frac{1}{2}\right)^2}$$

Think: What number squared equals $\frac{1}{4}$?

$$\sqrt{\frac{1}{4}} = \frac{1}{2}$$



CHECK IT OUT!

Find each root.

2a. $\sqrt{\frac{4}{9}}$

2b. $\sqrt[3]{\frac{1}{8}}$

2c. $-\sqrt{\frac{4}{49}}$

Square roots of numbers that are not perfect squares, such as 15, are not whole numbers. A calculator can approximate the value of $\sqrt{15}$ as 3.872983346... Without a calculator, you can use the square roots of perfect squares to help estimate the square roots of other numbers.

EXAMPLE 3 Gardening Application

Nancy wants to plant a square garden of wildflowers. She has enough wildflower seeds to cover 19 ft^2 . Estimate to the nearest tenth the side length of a square with an area of 19 ft^2 .

Since the area of the square is 19 ft^2 , then each side of the square is $\sqrt{19}$ ft. 19 is not a perfect square, so find the two consecutive perfect squares that 19 is between: 16 and 25. $\sqrt{19}$ is between $\sqrt{16}$ and $\sqrt{25}$, or 4 and 5. Refine the estimate.

4.3:	$4.3^2 = 18.49$	too low	$\sqrt{19}$ is greater than 4.3.
4.4:	$4.4^2 = 19.36$	too high	$\sqrt{19}$ is less than 4.4.
4.35:	$4.35^2 = 18.9225$	too low	$\sqrt{19}$ is greater than 4.35.

Since 4.35 is too low and 4.4 is too high, $\sqrt{19}$ is between 4.35 and 4.4. Rounded to the nearest tenth, $\sqrt{19} \approx 4.4$.

The side length of the plot is $\sqrt{19} \approx 4.4$ ft.



CHECK IT OUT!

3. Estimate to the nearest tenth the side length of a cube with a volume of 26 ft^3 .

Real numbers can be classified according to their characteristics.

Natural numbers are the counting numbers: 1, 2, 3, ...

Whole numbers are the natural numbers and zero: 0, 1, 2, 3, ...

Integers are the whole numbers and their opposites: ..., -3, -2, -1, 0, 1, 2, 3, ...

Writing Math

The symbol \approx means "is approximately equal to."

Writing Math

To show that one or more digits repeat continuously, write a bar over those digits.

$$1.333333333... = 1.\overline{3}$$

$$2.14141414... = 2.\overline{14}$$

Rational numbers are numbers that can be expressed in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$. When expressed as a decimal, a rational number is either a *terminating decimal* or a *repeating decimal*.

- A **terminating decimal** has a finite number of digits after the decimal point (for example, 1.25, 2.75, and 4.0).
- A **repeating decimal** has a block of one or more digits after the decimal point that repeat continuously (where all digits are not zeros).

Irrational numbers are all real numbers that are not rational. They cannot be expressed in the form $\frac{a}{b}$ where a and b are both integers and $b \neq 0$. They are neither terminating decimals nor repeating decimals. For example:

0.10100100010000100000... After the decimal point, this number contains 1 followed by one 0, and then 1 followed by two 0's, and then 1 followed by three 0's, and so on.

This decimal neither terminates nor repeats, so it is an irrational number.

If a whole number is not a perfect square, then its square root is irrational. For example, 2 is not a perfect square, and $\sqrt{2}$ is irrational.

The real numbers are made up of all rational and irrational numbers.

Know It!

Note

Reading Math

Note the symbols for the sets of numbers.

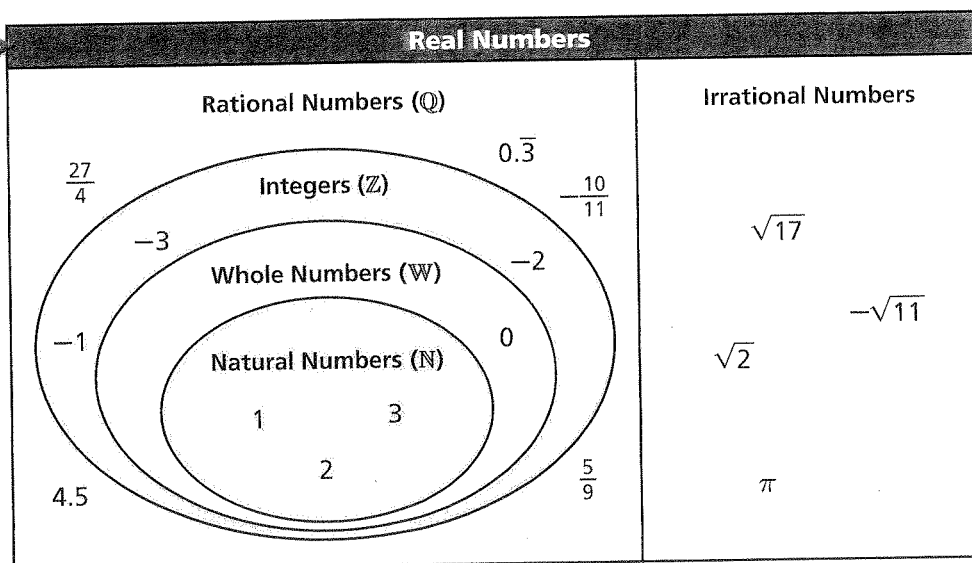
\mathbb{R} : real numbers

\mathbb{Q} : rational numbers

\mathbb{Z} : integers

\mathbb{W} : whole numbers

\mathbb{N} : natural numbers



EXAMPLE 4 Classifying Real Numbers

Write all classifications that apply to each real number.

A $\frac{8}{9}$

$\frac{8}{9}$ is in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

$8 \div 9 = 0.8888\ldots$

$= 0.\overline{8}$

rational, repeating decimal

$\frac{8}{9}$ can be written as a repeating decimal.

B 18

$18 = \frac{18}{1}$ 18 can be written in the form $\frac{a}{b}$.

$18 = 18.0$ 18 can be written as a terminating decimal.

rational, terminating decimal, integer, whole, natural

C $\sqrt{20}$

irrational 20 is not a perfect square, so $\sqrt{20}$ is irrational.



Write all classifications that apply to each real number.

4a. $7\frac{4}{9}$

4b. -12

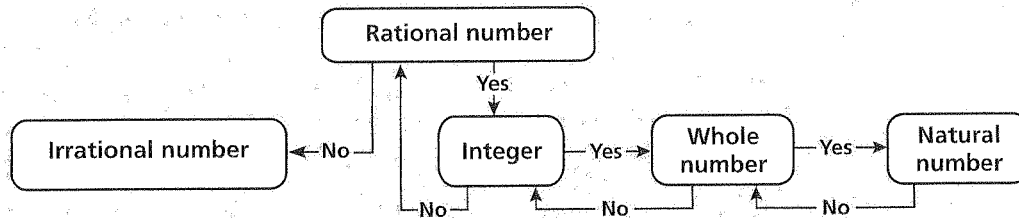
4c. $\sqrt{10}$

4d. $\sqrt{100}$

THINK AND DISCUSS

1. Write $\frac{2}{3}$ and $\frac{3}{5}$ as decimals. Identify what number classifications the two numbers share and how their classifications are different.

2. **GET ORGANIZED** Copy the graphic organizer and use the flowchart to classify each of the given numbers. Write each number in the box with the most specific classification that applies. 4, $\sqrt{25}$, 0, $\frac{1}{3}$, -15, -2.25, $\frac{1}{4}$, $\sqrt{21}$, 2^4 , $(-1)^2$



1-5

Exercises



go.hrw.com

Homework Help Online

KEYWORD: MA11 1-5

Parent Resources Online

KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Give an example of a *square root* that is not a *rational number*.

Find each root.

EXAMPLE 1
p. 32

2. $\sqrt{64}$

3. $-\sqrt{225}$

4. $\sqrt[3]{-64}$

5. $\sqrt[4]{625}$

6. $\sqrt{81}$

7. $-\sqrt[3]{27}$

8. $-\sqrt[3]{-27}$

9. $-\sqrt{16}$

EXAMPLE 2
p. 33

10. $\sqrt{\frac{1}{16}}$

11. $\sqrt[3]{\frac{8}{27}}$

12. $-\sqrt{\frac{1}{9}}$

13. $\sqrt{\frac{9}{64}}$

14. $\sqrt{\frac{1}{36}}$

15. $\sqrt[3]{\frac{1}{64}}$

16. $-\sqrt{\frac{4}{81}}$

17. $\sqrt[3]{-\frac{1}{125}}$

EXAMPLE 3
p. 33

18. A contractor is told that a potential client's kitchen floor is in the shape of a square. The area of the floor is 45 ft². Estimate to the nearest tenth the side length of the floor.

EXAMPLE 4
p. 34

- Write all classifications that apply to each real number.

19. -27

20. $\frac{1}{6}$

21. $\sqrt{33}$

22. -6.8

PRACTICE AND PROBLEM SOLVING

Find each root.

23. $\sqrt{121}$

24. $\sqrt[3]{-1000}$

25. $-\sqrt{100}$

26. $\sqrt[4]{256}$

27. $\sqrt{\frac{1}{25}}$

28. $\sqrt[4]{\frac{1}{16}}$

29. $\sqrt[3]{-\frac{1}{8}}$

30. $-\sqrt{\frac{25}{36}}$

31. An artist makes glass paperweights in the shape of a cube. He uses 68 cm³ of glass to make each paperweight. Estimate to the nearest tenth the side length of a paperweight.

Independent Practice

For Exercises	See Example
23–26	1
27–30	2
31	3
32–35	4

Extra Practice

Skills Practice p. S5
Application Practice p. S28

Write all classifications that apply to each real number.

32. $\frac{5}{12}$

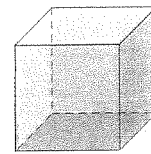
33. $\sqrt{49}$

34. -3

35. $\sqrt{18}$

36. **Geometry** The cube root of the volume of a cube gives the length of one side of the cube.

- Find the side length of the cube shown.
- Find the area of each face of the cube.



Volume = 343 cm^3

Compare. Write $<$, $>$, or $=$.

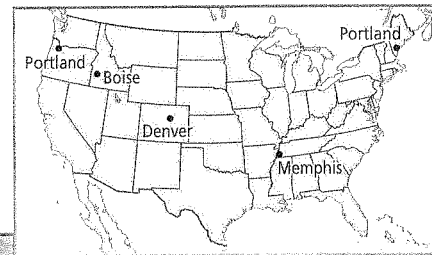
37. $8 \sqrt{63}$

38. $\sqrt{88}$ 9

39. $6 \sqrt{40}$

40. $\sqrt{\frac{9}{25}}$ 0.61

Travel During a cross-country road trip, Madeline recorded the distance between several major cities and the time it took her to travel between those cities. Find Madeline's average speed for each leg of the trip and classify that number.



Madeline's Cross-Country Road Trip					
	Distance (mi)	Time (h)	Speed (mi/h)	Classification	
41. Portland, ME, to Memphis, TN	1485	33			
42. Memphis, TN, to Denver, CO	1046	27			
43. Denver, CO, to Boise, ID	831	24			
44. Boise, ID, to Portland, OR	424	9			

Determine whether each statement is sometimes, always, or never true. If it is sometimes true, give one example that makes the statement true and one example that makes it false. If it is always true, explain. If it is never true, rewrite the statement so that it is always true.

- Mixed numbers are rational numbers.
- The decimal form of an irrational number is a repeating decimal.
- A terminating decimal is a rational number.
- A negative number is irrational.
- Critical Thinking** A positive number has two square roots, one that is positive and one that is negative. Is the same thing true for the cube root of a positive number? What about the fourth root of a positive number? Explain.

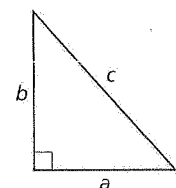
MULTI-STEP TEST PREP



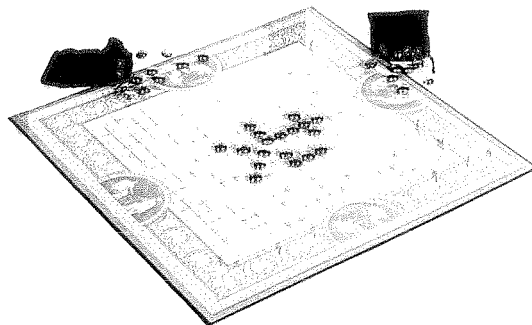
50. This problem will prepare you for the Multi-Step Test Prep on page 38.

The equation $a^2 + b^2 = c^2$ relates the lengths of the sides of a right triangle. Sides a and b make the right angle of the triangle.

- What is the value of c^2 when $a = 5$ and $b = 12$? Determine the square root of c^2 to find the value of c .
- A diver is a horizontal distance of 50 feet from a boat and 120 feet beneath the surface of the water. What distance will the diver swim if he swims diagonally to the boat?

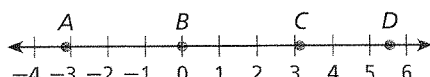


51. **Entertainment** In a board game, players place different-colored stones on a grid. Each player tries to make rows of 5 or more stones in their color while preventing their opponent(s) from doing the same. The square game board has 324 squares on it. How many squares are on each side of the board?



52. **Write About It** Explain why you cannot take the square root of a negative number but you can take the cube root of a negative number.

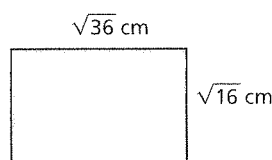
53. Which point on the number line is closest to $-\sqrt[3]{36}$?



- (A) A (B) B (C) C (D) D

54. What is the area of the figure at right?

- (F) 24 cm^2 (H) 104 cm^2
(G) 52 cm^2 (J) 576 cm^2



55. Which number is irrational?

- (A) $-\sqrt{9}$ (B) 4.0005
(C) $2.\overline{17}$ (D) $\sqrt{40}$

56. The square root of 175 is between which two whole numbers?

- (F) 11 and 12 (G) 12 and 13 (H) 13 and 14 (J) 14 and 15

CHALLENGE AND EXTEND

Find each root.

57. $\sqrt{0.81}$ 58. $\sqrt{0.25}$ 59. $\sqrt[3]{-0.001}$ 60. $\sqrt{2.25}$

Evaluate each expression for $a = 9$ and $b = 7$.

61. $\sqrt{a+b}$ 62. $b\sqrt{a} - a$ 63. $\sqrt[4]{b+a} + ab$ 64. $\sqrt{ab+1}$

65. The *Density Property of Real Numbers* states that between any two real numbers, there is another real number.

- a. Does the set of integers have this property? Explain.
b. Use the Density Property to write a convincing argument that there are infinitely many real numbers between 0 and 1.

SPIRAL REVIEW

Add or subtract. (Lesson 1-2)

66. $-14 + (-16)$ 67. $-\frac{1}{4} - \left(-\frac{3}{4}\right)$ 68. $25 - 17.6$

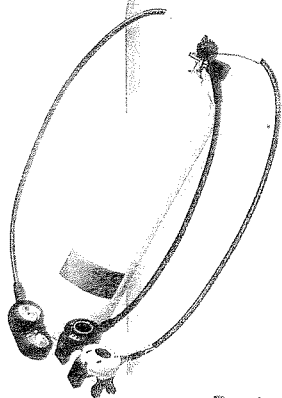
Multiply or divide. (Lesson 1-3)

69. $\frac{1}{8} \div \left(-\frac{2}{3}\right)$ 70. $(-2.5)(-8)$ 71. $-\frac{21}{6}$

Simplify each expression. (Lesson 1-4)

72. -3^4 73. $\left(-\frac{2}{5}\right)^3$ 74. 14^2 75. 4^3

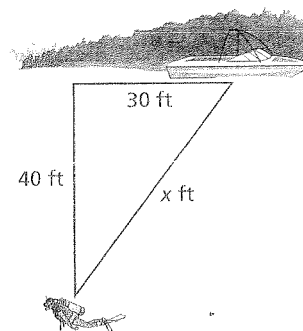
MULTI-STEP TEST PREP



The Language of Algebra

Under Pressure Atmospheric pressure is 14.7 pounds per square inch (psi). Underwater, the water exerts additional pressure. The total pressure on a diver underwater is the atmospheric pressure plus the water pressure.

1. As a diver moves downward in the water, the water pressure increases by 14.7 psi for approximately every 33 ft of water. Make a table to show the total pressure on a diver at 0, 33, 66, and 99 ft below the surface of the water. At what depth would the total pressure equal 73.5 psi? Explain your method.
2. A diver is 40 ft below the surface of the water when a hot-air balloon flies over her. The hot-air balloon is 849 ft above the surface of the water. Draw a diagram and write an expression to find the distance between the diver and the balloon when the balloon is directly above her.
3. The diver swam 62.5 ft in 5 minutes. How fast was she swimming? What total distance will she have traveled after an additional 4 minutes if she maintains this same speed?
4. The total pressure on each square foot of the diver's body is given by the expression $2116.8 + 64.145d$, where d is the depth in feet. At a depth of 66 ft, what is the total pressure on each square foot of her body? What is the total pressure on each square inch of her body at this depth? How does your answer compare to your results for part a?
5. The diver realizes that she has drifted horizontally about 30 ft from the boat she left. She is at a depth of 40 ft from the surface. What is the diver's diagonal distance from the boat?



Quiz for Lessons 1-1 Through 1-5

✓ 1-1 Variables and Expressions

Give two ways to write each algebraic expression in words.

1. $4 + n$
2. $m - 9$
3. $\frac{g}{2}$
4. $4z$
5. Bob earns \$15 per hour. Write an expression for the amount of money he earns in h hours.
6. A soccer practice is 90 minutes long. Write an expression for the number of minutes left after m minutes have elapsed.

Evaluate each expression for $x = 3$, $y = 6$, and $z = 2$.

7. $y \div z$
8. xy
9. $x + y$
10. $x - z$

✓ 1-2 Adding and Subtracting Real Numbers

Add or subtract.

11. $81 + (-15)$
12. $27 - 32$
13. $2 - \left(-1\frac{1}{4}\right)$
14. $-7 + (-14)$
15. Brandon's bank statement shows a balance of $-\$45.00$. What will the balance be after Brandon deposits \$70.00?

✓ 1-3 Multiplying and Dividing Real Numbers

Find the value of each expression if possible.

16. $9(-9)$
17. $6 \div \frac{3}{5}$
18. $9.6 \div 0$
19. $\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$
20. Simon drove for $2\frac{1}{2}$ hours to get from his house to the beach. Simon averaged 55 miles per hour on the trip. What is the distance from Simon's house to the beach?

✓ 1-4 Powers and Exponents

Simplify each expression.

21. $(-3)^2$
22. -3^2
23. $\left(-\frac{2}{3}\right)^3$
24. $\left(-\frac{1}{2}\right)^5$
25. The number of bytes in a kilobyte is 2 to the 10th power. Express this number in two ways.

✓ 1-5 Roots and Real Numbers

Find each root.

26. $\sqrt{225}$
27. $-\sqrt{49}$
28. $\sqrt[3]{8}$
29. $\sqrt{\frac{16}{25}}$
30. Mindy is building a patio that is in the shape of a square. The patio will cover 56 square yards. Find the length of a side of the patio to the nearest tenth of a yard.

Write all classifications that apply to each real number.

31. $\frac{1}{11}$
32. $\sqrt{12}$
33. $\sqrt{400}$
34. -6

1-6 Order of Operations

Objective

Use the order of operations to simplify expressions.

Vocabulary

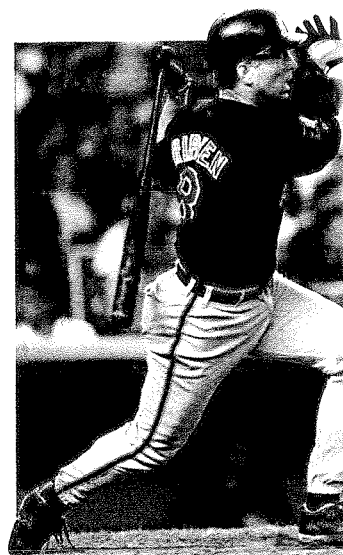
order of operations

Who uses this?

Sports statisticians use the order of operations to calculate data. (See Example 5.)

A baseball player must run to first, second, and third bases before running back to home plate. In math, some tasks must be done in a certain order.

When a numerical or algebraic expression contains more than one operation symbol, the **order of operations** tells you which operation to perform first.



Know It!

Note

Order of Operations

First:	Perform operations inside grouping symbols.
Second:	Simplify powers and roots.
Third:	Perform multiplication and division from left to right.
Fourth:	Perform addition and subtraction from left to right.

Grouping symbols include parentheses $()$, brackets $[]$, and braces $\{\}$. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

EXAMPLE 1 Simplifying Numerical Expressions

Simplify each expression.

A $-4^2 + 24 \div 3 \cdot 2$

$$-4^2 + 24 \div 3 \cdot 2$$

$$-16 + 24 \div 3 \cdot 2$$

$$-16 + 8 \cdot 2$$

$$-16 + 16$$

$$0$$

There are no grouping symbols.

Simplify powers. The exponent applies only to the 4.

Divide.

Multiply.

Add.

B $4[25 - (5 - 2)^2]$

$$4[25 - (5 - 2)^2]$$

$$4[25 - 3^2]$$

$$4[25 - 9]$$

$$4 \cdot 16$$

$$64$$

There are two sets of grouping symbols.

Perform the operation in the innermost set.

Simplify powers within the brackets.

Subtract within the brackets.

Multiply.

Helpful Hint

The first letters of these words can help you remember the order of operations.

Please	Parentheses
Excuse	Exponents
My	Multiply/
Dear	Divide
Aunt	Add/
Sally	Subtract



Simplify each expression.

1a. $8 \div \frac{1}{2} \cdot 3$

1b. $5.4 - 3^2 + 6.2$

1c. $-20 \div [-2(4+1)]$

EXAMPLE 2 Evaluating Algebraic ExpressionsEvaluate each expression for the given value of x .

A $21 - x + 2 \cdot 5$ for $x = 7$

$21 - x + 2 \cdot 5$

$21 - 7 + 2 \cdot 5$ First substitute 7 for x .

$21 - 7 + 10$ Multiply.

$14 + 10$ Subtract.

24 Add.

B $5^2(30 - x)$ for $x = 24$

$5^2(30 - x)$

$5^2(30 - 24)$ First substitute 24 for x .

$5^2(6)$ Perform the operation inside the parentheses.

$25(6)$ Simplify powers.

150 Multiply.

**CHECK IT OUT!**Evaluate each expression for the given value of x .

2a. $14 + x^2 \div 4$ for $x = 2$

2b. $(x \cdot 2^2) \div (2 + 6)$ for $x = 6$

Fraction bars, radical symbols, and absolute-value symbols can also be used as grouping symbols. Remember that a fraction bar indicates division.

EXAMPLE 3 Simplifying Expressions with Other Grouping Symbols

Simplify each expression.

A $\frac{-22 - 2^2}{5 - 3}$

$\frac{(-22 - 2^2)}{(5 - 3)}$

The fraction bar acts as a grouping symbol. Simplify the numerator and the denominator before dividing.

$\frac{-22 - 4}{5 - 3}$

Simplify the power in the numerator.

$\frac{-26}{5 - 3}$

Subtract to simplify the numerator.

$\frac{-26}{2}$

Subtract to simplify the denominator.

-13

Divide.

B $|10 - 5^2| \div 5$

$|10 - 5^2| \div 5$

The absolute-value symbols act as grouping symbols.

$|10 - 25| \div 5$

Simplify the power.

$|-15| \div 5$

Subtract within the absolute-value symbols.

$15 \div 5$

Write the absolute value of -15 .

3

Divide.

**CHECK IT OUT!**

Simplify each expression.

3a. $\frac{5 + 2(-8)}{(-2)^3 - 3}$

3b. $|4 - 7|^2 \div (-3)$

3c. $3\sqrt{50 - 1}$

Helpful Hint

You may need to add grouping symbols to simplify expressions when using a scientific or graphing calculator.

To simplify $\frac{2+3}{5-4}$ with a calculator, enter $(2 + 3) \div (5 - 4)$.

You may need to use grouping symbols when translating from words to numerical or algebraic expressions. Remember that operations inside grouping symbols are performed first.

EXAMPLE 4 Translating from Words to Math

Remember!

Look for words that imply mathematical operations.

difference → subtract

sum → add

product → multiply

quotient → divide

Translate each word phrase into a numerical or algebraic expression.

- A** one half times the difference of -5 and 3

$$\frac{1}{2}(-5 - 3) \quad \text{Use parentheses so that the difference is evaluated first.}$$

- B** the square root of the quotient of -12 and n

$$\sqrt{\frac{-12}{n}} \quad \text{Show the square root of a quotient.}$$



4. Translate the word phrase into a numerical or algebraic expression: the product of 6.2 and the sum of 9.4 and 8 .

EXAMPLE 5 Sports Application

Hank Aaron's last season in the Major Leagues was in 1976. A player's total number of bases can be found using the expression $S + 2D + 3T + 4H$. Use the table to find Hank Aaron's total bases for 1976.

$$S + 2D + 3T + 4H$$

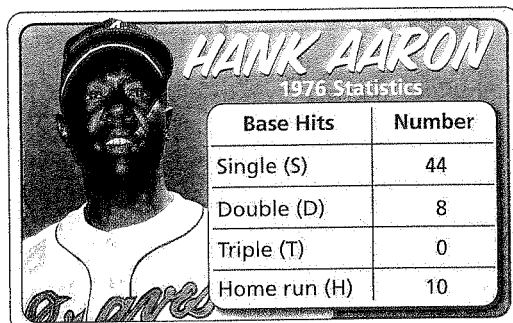
$$44 + 2(8) + 3(0) + 4(10)$$

$$44 + 16 + 0 + 40$$

$$60 + 0 + 40$$

$$100$$

Hank Aaron's total number of bases for 1976 was 100.



HANK AARON 1976 Statistics	
Base Hits	Number
Single (S)	44
Double (D)	8
Triple (T)	0
Home run (H)	10

First substitute values for each variable.

Multiply.

Add from left to right.

Add.



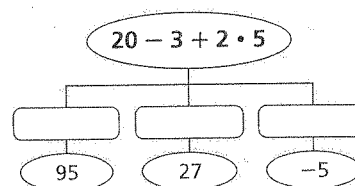
5. Another formula for a player's total number of bases is $\text{Hits} + D + 2T + 3H$. Use this expression to find Hank Aaron's total bases for 1959, when he had 223 hits, 46 doubles, 7 triples, and 39 home runs.

THINK AND DISCUSS

1. Explain whether you always perform addition before subtraction when simplifying a numerical or algebraic expression.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, show how grouping symbols can be placed so that the expression is equal to the number shown.

Know It!

Note





GUIDED PRACTICE

1. **Vocabulary** Explain why the *order of operations* is necessary for simplifying numerical expressions.

EXAMPLE 1 Simplify each expression.

p. 40

2. $5 - 12 \div (-2)$

3. $30 - 5 \cdot 3$

4. $50 - 6 + 8$

5. $12 \div (-4)(3)$

6. $(5 - 8)(3 - 9)$

7. $16 + [5 - (3 + 2^2)]$

EXAMPLE 2 Evaluate each expression for the given value of the variable.

p. 41

8. $5 + 2x - 9$ for $x = 4$

9. $30 \div 2 - d$ for $d = 14$

10. $51 - 91 + g$ for $g = 20$

11. $2(3 + n)$ for $n = 4$

12. $4(b - 4)^2$ for $b = 5$

13. $12 + [20(5 - k)]$ for $k = 1$

EXAMPLE 3 Simplify each expression.

p. 41

14. $24 \div |4 - 10|$

15. $4.5 - \sqrt{2(4.5)}$

16. $5(2) + 16 \div |-4|$

17. $\frac{0 - 24}{6 + 2}$

18. $\frac{2 + 3(6)}{2^2}$

19. $-44 \div \sqrt{12 \div 3}$

EXAMPLE 4 Translate each word phrase into a numerical or algebraic expression.

p. 42

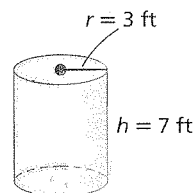
20. 5 times the absolute value of the sum of s and -2

21. the product of 12 and the sum of -2 and 6

22. 14 divided by the sum of 52 and -3

EXAMPLE 5 **Geometry** The surface area of a cylinder can be found using the expression $2\pi r(h + r)$. Find the surface area of the cylinder shown. (Use 3.14 for π and give your final answer rounded to the nearest tenth.)

p. 42



PRACTICE AND PROBLEM SOLVING

Simplify each expression.

24. $3 + 4(-5)$

25. $20 - 4 + 5 - 2$

26. $41 + 12 \div 2$

27. $3(-9) + (-2)(-6)$

28. $10^2 \div (10 - 20)$

29. $(6 + 2 \cdot 3) \div (9 - 7)^2$

30. $-9 - (-18) + 6$

31. $15 \div (2 - 5)$

32. $5(1 - 2) - (3 - 2)$

Evaluate each expression for the given value of the variable.

33. $-6(3 - p)$ for $p = 7$

34. $5 + (r + 2)^2$ for $r = 4$

35. $13 - [3 + (j - 12)]$ for $j = 5$

36. $(-4 - a)^2$ for $a = -3$

37. $7 - (21 - h)^2$ for $h = 25$

38. $10 + [8 \div (q - 3)]$ for $q = 2$

39. $(4r - 2) + 7$ for $r = 3$

40. $-2(11b - 3)$ for $b = 5$

41. $7x(3 + 2x)$ for $x = -1$

Simplify each expression.

42. $-4|2.5 - 6|$

43. $\frac{8 - 8}{2 - 1}$

44. $\frac{3 + |8 - 10|}{2}$

45. $\sqrt{3^2 - 5} \div 8$

46. $\frac{-18 - 36}{-9}$

47. $\frac{6|5 - 7|}{14 - 2}$

48. $\sqrt{5^2 - 4^2}$

49. $(-6 + 24) \div |-3|$

Independent Practice

For Exercises See Example

4-32 1

3-41 2

2-49 3

1-53 4

54 5

Extra Practice

Practice p. S5

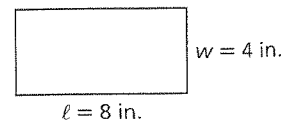
Integration Practice p. S28

Translate each word phrase into a numerical or an algebraic expression.

50. the product of 7 and the sum of 2 and d
51. the difference of 3 and the quotient of 2 and 5
52. the square root of the sum of 5 and -4
53. the difference of 8 and the absolute value of the product of 3 and 5



54. **Geometry** The perimeter of a rectangle can be found using the expression $2(\ell + w)$. Find the perimeter of the rectangle shown.



55. Simplify each expression.

- | | | |
|-------------------------|----------------------|-------------------------|
| a. $50 + 10 \div 2$ | b. $50 \cdot 10 - 2$ | c. $50 \cdot 10 \div 2$ |
| d. $50 \div 10 \cdot 2$ | e. $50 - 10 \cdot 2$ | f. $50 + 10 \cdot 2$ |

Translate each word phrase into a numerical or algebraic expression.

56. the difference of 8 and the product of 4 and n
57. 2 times the sum of 9 and the opposite of x
58. two-thirds of the difference of -2 and 8
59. the square root of 7 divided by the product of 3 and 10

60. **Sports** At the 2004 Summer Olympics, U.S. gymnast Paul Hamm received the scores shown in the table during the individual all-around competition.

2004 Summer Olympics Individual Scores for Paul Hamm						
Event	Floor	Pommel horse	Rings	Vault	Parallel bars	Horizontal bar
Score	9.725	9.700	9.587	9.137	9.837	9.837

- a. Write a numerical expression to show the average of Hamm's scores. (*Hint: The average of a set of values is the sum of the values divided by the number of values.*)
- b. Simplify the expression to find Hamm's average score to the nearest thousandth.

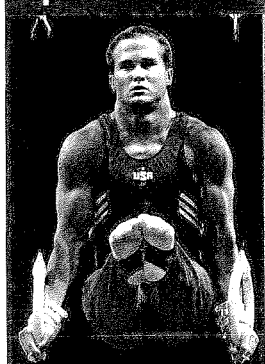
61. **Critical Thinking** Are parentheses required when translating the word phrase "the sum of 8 and the product of 3 and 2" into a numerical phrase? Explain.

Translate each word phrase into a numerical expression. Then simplify.

62. the sum of 8 and the product of -3 and 5
63. the difference of the product of 3 and 5 and the product of 6 and 2
64. the product of $\frac{2}{3}$ and the absolute value of the difference of 3 and -12

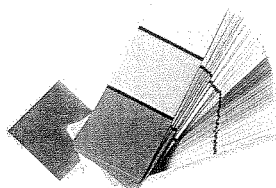


Sports



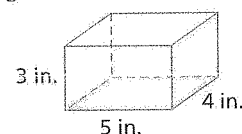
In 2004, Paul Hamm became the first American to win a gold medal in the men's all-around gymnastics competition at the Olympics. He won by a margin of 0.012 point.

MULTI-STEP TEST PREP

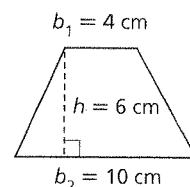


65. This problem will prepare you for the Multi-Step Test Prep on page 60.

- a. Find the area of each face of the prism. Find the sum of these areas to find the total surface area of the prism.
- b. The total surface area of a prism is described by the expression $2(\ell w) + 2(\ell h) + 2(wh)$. Explain how this expression relates to the sum you found in part a.
- c. Use the expression above to find the total surface area of the prism. Explain why your answers to parts a and c should be equal.



- 66. Geometry** The area of a trapezoid is equal to the average of its bases times its height. Use the expression $\left(\frac{b_1 + b_2}{2}\right)h$ to determine the area of the trapezoid.



- 67. Write About It** Many everyday processes must be done in a certain order to be completed successfully. Describe a process that requires several steps, and tell why the steps must be followed in a certain order.

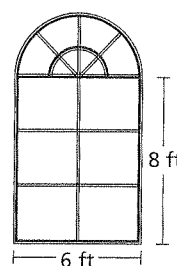


- 68.** Cara's family rented a car for their 3-day vacation to the Grand Canyon. They paid \$29.00 per day and \$0.12 for each mile driven. Which expression represents Cara's family's cost to rent the car for 3 days and drive 318 miles?

(A) $29 + 0.12(318)$ (C) $29(3) + 0.12(318)$
(B) $29 + 3 + 0.12 + 318$ (D) $3[9 + 0.12(318)]$

- 69.** The perimeter of the Norman window shown is approximated by the expression $2(3 + 8) + 3.14(3)$. Which is the closest approximation of the perimeter of the window?

(F) 23.4 feet (H) 31.4 feet
(G) 28.4 feet (J) 51.4 feet



- 70. Gridded Response** Simplify $\sqrt{\frac{54 - (-2)(5)}{20 - 4^2}}$.

CHALLENGE AND EXTEND

Simplify each expression.

71. $\frac{3 + 9 \cdot 2}{2 - 3^2}$

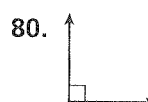
72. $[(-6 \cdot 4) \div (-6) \cdot 4]^2$

73. $\sqrt{\frac{8 + 10^2}{13 + (-10)}}$

- 74.** Use the numbers 2, 4, 5, and 8 to write an expression that has a value of 5. You may use any operations, and you must use each of the numbers at least once.
- 75.** Use the numbers 2, 5, 6, and 9 to write an expression that has a value of 1. You may use any operations, and you must use each of the numbers at least once.
- 76.** If the value of $(\otimes + 5)^2$ is 81, what is the value of $(\otimes + 5)^2 + 1$?
- 77.** If the value of $(\otimes + 1)^2 - 3$ is 22, what is the value of $(\otimes + 1)^2 - 5$?

SPIRAL REVIEW

Identify each angle as acute, right, obtuse, or straight. (Previous course)



Add or subtract. (Lesson 1-2)

81. $51 - (-49)$

82. $-5 + \left(-1\frac{1}{3}\right)$

83. $-3 + (-8)$

84. $2.9 - 5.3$

Find each square root. (Lesson 1-5)

85. $\sqrt{64}$

86. $\sqrt{324}$

87. $\sqrt{\frac{36}{49}}$

88. $-\sqrt{121}$

1-7

Simplifying Expressions

Objectives

Use the Commutative, Associative, and Distributive Properties to simplify expressions.

Combine like terms.

Vocabulary

term
like terms
coefficient

Who uses this?

Triathletes can use the Commutative, Associative, and Distributive Properties to calculate overall times mentally.

A triathlon is an endurance race that includes swimming, biking, and running. The winner is determined by adding the times for each of the three events.

The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange an expression to simplify it.



Know it!

Note

Properties of Addition and Multiplication

WORDS	NUMBERS	ALGEBRA
Commutative Property You can add numbers in any order and multiply numbers in any order.	$2 + 7 = 7 + 2$ $3 \cdot 9 = 9 \cdot 3$	$a + b = b + a$ $ab = ba$
Associative Property When you are only adding or only multiplying, you can group any of the numbers together.	$(6 + 8) + 2 = 6 + (8 + 2)$ $(7 \cdot 4) \cdot 5 = 7 \cdot (4 \cdot 5)$	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$

EXAMPLE 1 Using the Commutative and Associative Properties

Simplify each expression.

A $4 \cdot 9 \cdot 25$

$9 \cdot 4 \cdot 25$

$9 \cdot (4 \cdot 25)$

$9 \cdot 100$

900

Use the Commutative Property of Multiplication.

Use the Associative Property of Multiplication to make groups of compatible numbers.

B $25 + 48 + 75$

$25 + 75 + 48$

$(25 + 75) + 48$

$100 + 48$

148

Use the Commutative Property of Addition.

Use the Associative Property of Addition to make groups of compatible numbers.

Helpful Hint

Compatible numbers help you do math mentally. Try to make multiples of 5 or 10. They are simpler to use when multiplying.



Simplify each expression.

1a. $15\frac{1}{3} + 4 + 1\frac{2}{3}$ 1b. $410 + 58 + 90 + 2$ 1c. $\frac{1}{2} \cdot 7 \cdot 8$

Student to Student

Commutative and Associative Properties



Lorna Anderson
Pearson High School

I used to get the Commutative and Associative Properties mixed up.

To remember the Commutative Property, I think of people commuting back and forth from work. When people commute, they move. I can move the numbers around without changing the value of the expression.

For the Associative Property, I think of associating with my friends. They're the group I hang out with. In math, it's about how numbers are grouped.

The Distributive Property is used with addition to simplify expressions.

Know It!

Note

Distributive Property

WORDS	NUMBERS	ALGEBRA
You can multiply a number by a sum or multiply each addend by the number and then add. The result is the same.	 $3(4 + 8) = 3(4) + 3(8)$	 $a(b + c) = ab + ac$

The Distributive Property also works with subtraction because subtraction is the same as adding the opposite.

EXAMPLE 2 Using the Distributive Property with Mental Math

Write each product using the Distributive Property. Then simplify.

A $15(103)$

$15(100 + 3)$

Rewrite 103 as $100 + 3$.

$15(100) + 15(3)$

Use the Distributive Property.

$1500 + 45$

Multiply.

1545

Add.

B $6(19)$

$6(20 - 1)$

Rewrite 19 as $20 - 1$.

$6(20) - 6(1)$

Use the Distributive Property.

$120 - 6$

Multiply.

114

Subtract.



CHECK IT OUT!

Write each product using the Distributive Property. Then simplify.

2a. $9(52)$

2b. $12(98)$

2c. $7(34)$

The **terms** of an expression are the parts to be added or subtracted. **Like terms** are terms that contain the same variables raised to the same powers. Constants are also like terms.

Like terms Constant

$4x - 3x + 2$

A **coefficient** is a number that is multiplied by a variable. Like terms can have different coefficients. A variable written without a coefficient has a coefficient of 1.

$$1x^2 + 3x$$

Coefficients

Using the Distributive Property can help you combine like terms. You can factor out the common factor to simplify the expression.

$$\begin{aligned} 7x^2 - 4x^2 &= (7 - 4)x^2 && \text{Factor out } x^2 \text{ from both terms.} \\ &= (3)x^2 && \text{Perform operations in parentheses.} \\ &= 3x^2 \end{aligned}$$

Notice that you can combine like terms by adding or subtracting the coefficients and keeping the variables and exponents the same.

EXAMPLE 3 Combining Like Terms

Simplify each expression by combining like terms.

A $12x + 30x$

$$12x + 30x$$

$$42x$$

12x and 30x are like terms.

Add the coefficients.

B $6.8y^2 - y^2$

$$6.8y^2 - y^2$$

$$6.8y^2 - 1y^2$$

$$5.8y^2$$

A variable without a coefficient has a coefficient of 1.

$6.8y^2$ and $1y^2$ are like terms.

Subtract the coefficients.

C $4n + 11n^2$

$$4n + 11n^2$$

$4n$ and $11n^2$ are not like terms. Do not combine.

Caution!

Add or subtract only the coefficients.

$$6.8y^2 - y^2 \neq 6.8$$



Simplify each expression by combining like terms.

3a. $16p + 84p$

3b. $-20t - 8.5t$

3c. $3m^2 + m^3$

EXAMPLE 4 Simplifying Algebraic Expressions

Simplify $2(x + 6) + 3x$. Justify each step with an operation or property.

	Procedure	Justification
1.	$2(x + 6) + 3x$	
2.	$2(x) + 2(6) + 3x$	Distributive Property
3.	$2x + 12 + 3x$	Multiply.
4.	$2x + 3x + 12$	Commutative Property of Addition
5.	$(2x + 3x) + 12$	Associative Property of Addition
6.	$5x + 12$	Combine like terms.



Simplify each expression. Justify each step with an operation or property.

4a. $6(x - 4) + 9$

4b. $-12x - 5x + 3a + x$

THINK AND DISCUSS

1. Tell which property is described by this sentence: When adding three numbers, you can add the first number to the sum of the second and third numbers, or you can add the third number to the sum of the first and second numbers.

Know It!

Note

2. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, give an example to illustrate the given property.

Associative	Commutative	Distributive

1-7

Exercises



go.hrw.com

Homework Help Online

KEYWORD: MA7 1-7

Parent Resources Online

KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** The ____?____ Property states the following:
 $(a + b) + c = a + (b + c)$. (*Associative, Commutative, or Distributive*)

- E EXAMPLE 1** Simplify each expression.

p. 46

2. $-12 + 67 + 12 + 23$

3. $16 + 2\frac{1}{2} + 4 + 1\frac{1}{2}$

4. $27 + 98 + 73$

5. $\frac{1}{3} \cdot 8 \cdot 21$

6. $2 \cdot 38 \cdot 50$

7. $50 \cdot 118 \cdot 20$

- E EXAMPLE 2** Write each product using the Distributive Property. Then simplify.

p. 47

8. $14(1002)$

9. $16(19)$

10. $9(38)$

11. $8(57)$

12. $12(112)$

13. $7(109)$

- E EXAMPLE 3** Simplify each expression by combining like terms.

p. 48

14. $6x + 10x$

15. $35x - 15x$

16. $-3a + 9a$

17. $-8r - r$

18. $17x^2 + x$

19. $3.2x + 4.7x$

- E EXAMPLE 4** Simplify each expression. Justify each step with an operation or property.

p. 48

20. $5(x + 3) - 7x$

21. $9(a - 3) - 4$

22. $5x^2 - 2(x - 3x^2)$

23. $6x - x - 3x^2 + 2x$

24. $12x + 8x + t - 7x$

25. $4a - 2(a - 1)$

PRACTICE AND PROBLEM SOLVING

Simplify each expression.

26. $53 + 28 + 17 + 12$

27. $5 \cdot 14 \cdot 20$

28. $6 \cdot 3 \cdot 5$

29. $4.5 + 7.1 + 8.5 + 3.9$

Write each product using the Distributive Property. Then simplify.

30. $9(62)$

31. $8(29)$

32. $11(25)$

33. $6(53)$

Independent Practice

For Exercises	See Example
26–29	1
30–33	2
34–37	3
38–43	4

Extra Practice

Skills Practice p. S5
Application Practice p. S28

Simplify each expression by combining like terms.

34. $3x + 9x$ 35. $14x^2 - 5x^2$ 36. $-7x + 8x$ 37. $3x^2 - 4$

Simplify each expression. Justify each step with an operation or property.

38. $4(y + 6) + 9$ 39. $-7(x + 2) + 4x$ 40. $3x + 2 - 2x - 1$
41. $5x - 3x + 3x^2 + 9x$ 42. $8x + 2x - 3y - 9x$ 43. $7y - 3 + 6y - 7$

44. **Estimation** Tavon bought a binder, 3 spiral notebooks, and a pen. The binder cost \$4.89, the notebooks cost \$1.99 each, and the pen cost \$2.11. About how much did Tavon spend on school supplies?

45. **Sports** In a triathlon, athletes race in swimming, biking, and running events. The athlete with the shortest total time to complete the events is the winner.

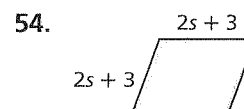
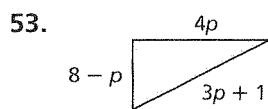
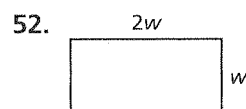
Times from Triathlon			
Athlete	Swim (min:s)	Bike (min:s)	Run (min:s)
Amy	18:51	45:17	34:13
Julie	17:13	40:27	23:32
Mardi	19:09	38:58	25:32
Sabine	13:09	31:37	19:01

- Find the total time for each athlete. (*Hint*: 1 minute = 60 seconds)
- Use the total times for the athletes to determine the order in which they finished the triathlon.


Name the property that is illustrated in each equation.

46. $5 + x = x + 5$ 47. $x - 2 = -2 + x$ 48. $2 + (3 + y) = (2 + 3) + y$
49. $3(2r - 7) = 3(2r) - 3(7)$ 50. $(2 + g) + 3 = 2 + (g + 3)$ 51. $45x - 35 = 5(9x) - 5(7)$

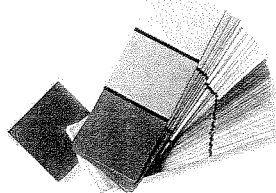
 **Geometry** Give an expression in simplified form for the perimeter of each figure.



55. **Critical Thinking** Evaluate $a - (b - c)$ and $(a - b) - c$ for $a = 10$, $b = 7$, and $c = 3$. Based on your answers, explain whether there is an Associative Property of Subtraction.

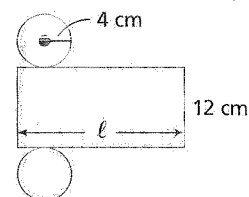
 56. **Write About It** Describe a real-world situation that can be represented by the Distributive Property. Translate your situation into an algebraic expression. Define each variable you use.

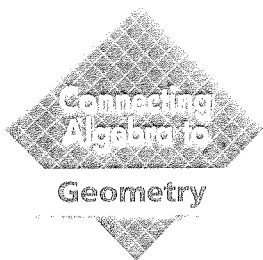
MULTI-STEP TEST PREP



57. This problem will prepare you for the Multi-Step Test Prep on page 60.

- The diagram shows a pattern of shapes that can be folded to make a cylinder. How is the length ℓ of the rectangle related to the circumference of (distance around) each circle?
- An expression for the circumference of each circle is $2\pi r$. Write an expression for the area of the rectangle.
- Write an expression for the total area of the figures. Leave the symbol π in your expression.





Perimeter

The distance around a geometric figure is called the *perimeter*. You can use what you have learned about combining like terms to simplify expressions for perimeter.

A closed figure with straight sides is called a *polygon*. To find the perimeter of a polygon, add the lengths of the sides.

Example 1

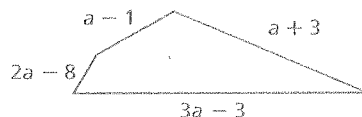
- A** Write an expression for the perimeter of the quadrilateral.

Add the lengths of the four sides.

$$P = (a + 3) + (2a - 8) + (3a - 3) + (a - 1)$$

Combine like terms to simplify.

$$\begin{aligned} P &= (a + 2a + 3a + a) + (3 - 8 - 3 - 1) \\ &= 7a - 9 \end{aligned}$$



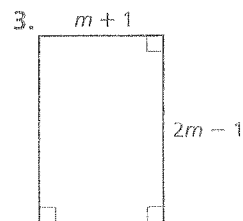
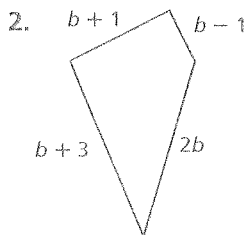
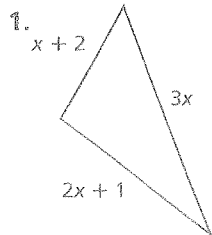
- B** Find the perimeter of this quadrilateral for $a = 5$.

Substitute 5 for a .

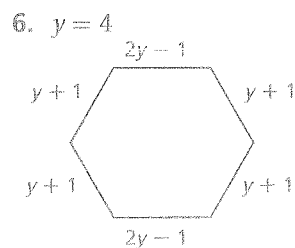
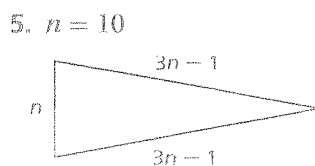
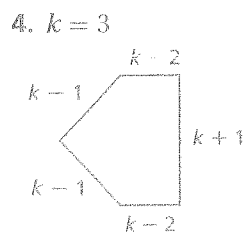
$$\begin{aligned} P &= 7(5) - 9 \\ &= 35 - 9 \\ &= 26 \end{aligned}$$

Try This

Write and simplify an expression for the perimeter of each figure.



Find the perimeter of each figure for the given value of the variable.



Combining like terms is one way to explore what happens to the perimeter when you double the sides of a triangle or other polygon.

Example 2

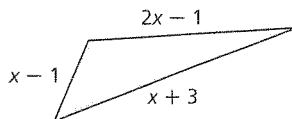
What happens to the perimeter of this triangle when you double the length of each side?

Write an expression for the perimeter of the smaller triangle.
Combine like terms to simplify the expression.

$$(x - 1) + (2x - 1) + (x + 3)$$

$$(x + 2x + x) + (-1 - 1 + 3)$$

$$4x + 1 \quad \text{Perimeter of small triangle}$$

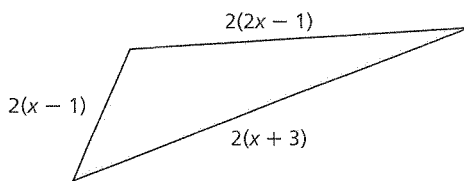


Double the length of each side of the triangle.

$$2(x - 1) = 2x - 2$$

$$2(2x - 1) = 4x - 2$$

$$2(x + 3) = 2x + 6$$



Find the perimeter of the larger triangle.
Combine like terms to simplify.

$$(2x - 2) + (4x - 2) + (2x + 6)$$

Add the lengths of the sides.

$$(2x + 4x + 2x) + (-2 - 2 + 6)$$

Use the Associative and Commutative Properties of Addition and combine like terms.

$$8x + 2$$

Perimeter of large triangle

Use the Distributive Property to show that the new perimeter is twice the original perimeter.

$$8x + 2 = 2(4x + 1)$$

Try This

Each set of expressions represents the side lengths of a triangle. Use the Distributive Property to show that doubling the side lengths doubles the perimeter.

7. $2p + 1$

8. $c - 1$

9. $w + 5$

10. $h - 2$

$3p + 2$

$2c + 1$

$w + 5$

$3h$

$5p$

$3c - 1$

$3w - 1$

$2h + 3$

Solve each problem.

11. Use the triangles in Example 2. Find the side lengths and perimeters for $x = 5$.

12. The sides of a quadrilateral are $2x - 1$, $x + 3$, $3x + 1$, and $x - 1$. Double the length of each side. Then find an expression for the perimeter of the new figure.

13. What happens to the perimeter of this trapezoid when you triple the length of each side? Use the variables a , b , b , and c for the lengths of the sides. Explain your answer using the Distributive Property.

