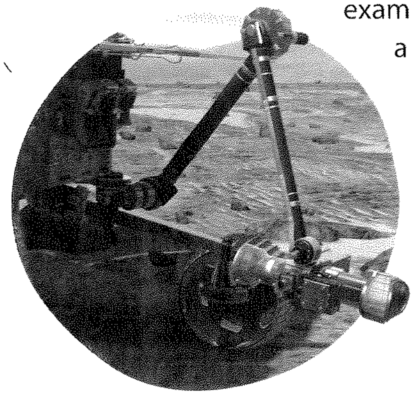
3.1 Measurements and Their Uncertainty

Connecting to Your World

On January 4, 2004, the Mars Exploration Rover Spirit landed on Mars. Equipped with five scientific instruments and a rock abrasion tool (shown at left), Spirit was sent to examine the Martian surface around Gusev Crater, a wide basin that may have once held a lake. Each day of its mission, Spirit recorded measurements for analysis. This data helped scientists learn about the geology and climate on Mars. All measurements have some uncertainty. In the chemistry laboratory, you must strive for accuracy and precision in your measurements.



Guide for Reading

Key Concepts

- How do measurements relate to science?
- How do you evaluate accuracy and precision?
- Why must measurements be reported to the correct number of significant figures?
- How does the precision of a calculated answer compare to the precision of the measurements used to obtain it?

Vocabulary

measurement
scientific notation
accuracy
precision
accepted value
experimental value
error
percent error
significant figures

Reading Strategy

Building Vocabulary As you read, write a definition of each vocabulary term in your own words.

Using and Expressing Measurements

Your height (67 inches), your weight (134 pounds), and the speed you drive on the highway (65 miles/hour) are some familiar examples of measurements. A **measurement** is a quantity that has both a number and a unit. Everyone makes and uses measurements. For instance, you decide how to dress in the morning based on the temperature outside. If you were baking cookies, you would measure the volumes of the ingredients as indicated in the recipe.

Such everyday situations are similar to those faced by scientists. **Measurements are fundamental to the experimental sciences. For that reason, it is important to be able to make measurements and to decide whether a measurement is correct.** The units typically used in the sciences are those of the International System of Measurements (SI).

In chemistry, you will often encounter very large or very small numbers. A single gram of hydrogen, for example, contains approximately 602,000,000,000,000,000,000,000 hydrogen atoms. The mass of an atom of gold is 0.000 000 000 000 000 000 000 327 gram. Writing and using such large and small numbers is very cumbersome. You can work more easily with these numbers by writing them in scientific, or exponential, notation.

In **scientific notation,** a given number is written as the product of two numbers: a coefficient and 10 raised to a power. For example, the number 602,000,000,000,000,000,000,000 written in scientific notation is 6.02×10^{23}. The coefficient in this number is 6.02. In scientific notation, the coefficient is always a number equal to or greater than one and less than ten. The power of 10, or exponent, in this example is 23. Figure 3.1 illustrates how to express the number of stars in a galaxy by using scientific notation. For more practice on writing numbers in scientific notation, refer to page R56 of Appendix C.

$$\underbrace{200{,}000{,}000{,}000.}_{\text{Decimal moves 11 places to the left.}} = 2 \times 10^{\overset{\text{Exponent is 11}}{11}}$$

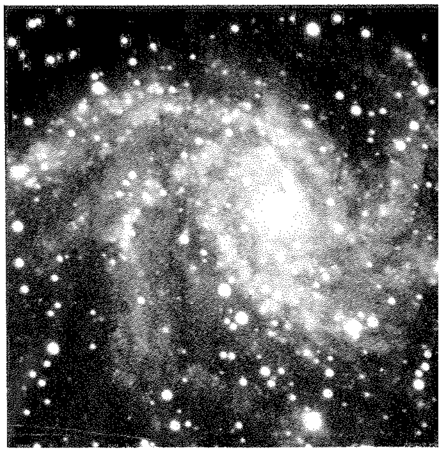
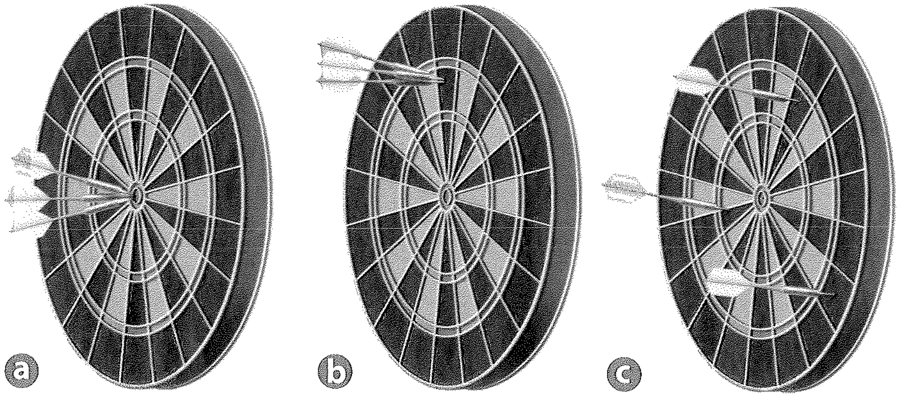


Figure 3.1 Expressing very large numbers, such as the estimated number of stars in a galaxy, is easier if scientific notation is used.

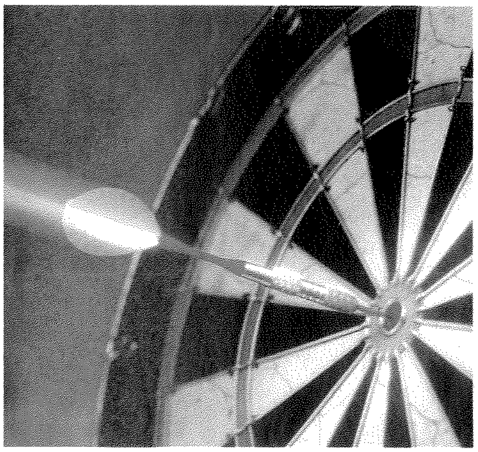


Good accuracy
Good precision

Poor accuracy
Good precision

Poor accuracy
Poor precision

Figure 3.2 The distribution of darts illustrates the difference between accuracy and precision. ⓐ Good accuracy and good precision: The darts are close to the bull's-eye and to one another. ⓑ Poor accuracy and good precision: The darts are far from the bull's-eye but close to one another. ⓒ Poor accuracy and poor precision: The darts are far from the bull's-eye and from one another.



Accuracy, Precision, and Error

Your success in the chemistry lab and in many of your daily activities depends on your ability to make reliable measurements. Ideally, measurements should be both correct and reproducible.

Accuracy and Precision Correctness and reproducibility relate to the concepts of accuracy and precision, two words that mean the same thing to many people. In chemistry, however, their meanings are quite different. **Accuracy** is a measure of how close a measurement comes to the actual or true value of whatever is measured. **Precision** is a measure of how close a series of measurements are to one another. **To evaluate the accuracy of a measurement, the measured value must be compared to the correct value. To evaluate the precision of a measurement, you must compare the values of two or more repeated measurements.**

Darts on a dartboard illustrate accuracy and precision in measurement. Let the bull's-eye of the dartboard represent the true, or correct, value of what you are measuring. The closeness of a dart to the bull's-eye corresponds to the degree of accuracy. The closer it comes to the bull's-eye, the more accurately the dart was thrown. The closeness of several darts to one another corresponds to the degree of precision. The closer together the darts are, the greater the precision and the reproducibility.

Look at Figure 3.2 as you consider the following outcomes.

a. All of the darts land close to the bull's-eye and to one another. Closeness to the bull's-eye means that the degree of accuracy is great. Each dart in the bull's-eye corresponds to an accurate measurement of a value. Closeness of the darts to one another indicates high precision.

b. All of the darts land close to one another but far from the bull's-eye. The precision is high because of the closeness of grouping and thus the high level of reproducibility. The results are inaccurate, however, because of the distance of the darts from the bull's-eye.

c. The darts land far from one another and from the bull's-eye. The results are both inaccurate and imprecise.

Checkpoint ***How does accuracy differ from precision?***

Determining Error Note that an individual measurement may be accurate or inaccurate. Suppose you use a thermometer to measure the boiling point of pure water at standard pressure. The thermometer reads 99.1°C. You probably know that the true or accepted value of the boiling point of pure water under these conditions is actually 100.0°C. There is a difference between the **accepted value,** which is the correct value based on reliable references, and the **experimental value,** the value measured in the lab. The difference between the experimental value and the accepted value is called the **error.**

$$\text{Error} = \text{experimental value} - \text{accepted value}$$

Error can be positive or negative depending on whether the experimental value is greater than or less than the accepted value.

For the boiling-point measurement, the error is 99.1°C − 100.0°C, or −0.9° C. The magnitude of the error shows the amount by which the experimental value differs from the accepted value. Often, it is useful to calculate the relative error, or percent error. The **percent error** is the absolute value of the error divided by the accepted value, multiplied by 100%.

$$\text{Percent error} = \frac{|\text{error}|}{\text{accepted value}} \times 100\%$$

Using the absolute value of the error means that the percent error will always be a positive value. For the boiling-point measurement, the percent error is calculated as follows.

$$\begin{aligned}\text{Percent error} &= \frac{|99.1^\circ\text{C} - 100.0^\circ\text{C}|}{100.0^\circ\text{C}} \times 100\% \\ &= \frac{0.9\cancel{^\circ\text{C}}}{100.0\cancel{^\circ\text{C}}} \times 100\% \\ &= 0.009 \times 100\% \\ &= 0.9\%\end{aligned}$$

Just because a measuring device works doesn't necessarily mean that it is accurate. As Figure 3.3 shows, a weighing scale that does not read zero when nothing is on it is bound to yield error. In order to weigh yourself accurately, you must first make sure that the scale is zeroed.

Word *Origins*

Percent comes from the Latin words *per,* meaning "by" or "through," and *centum,* meaning "100." **What do you think the phrase *per annum* means?**

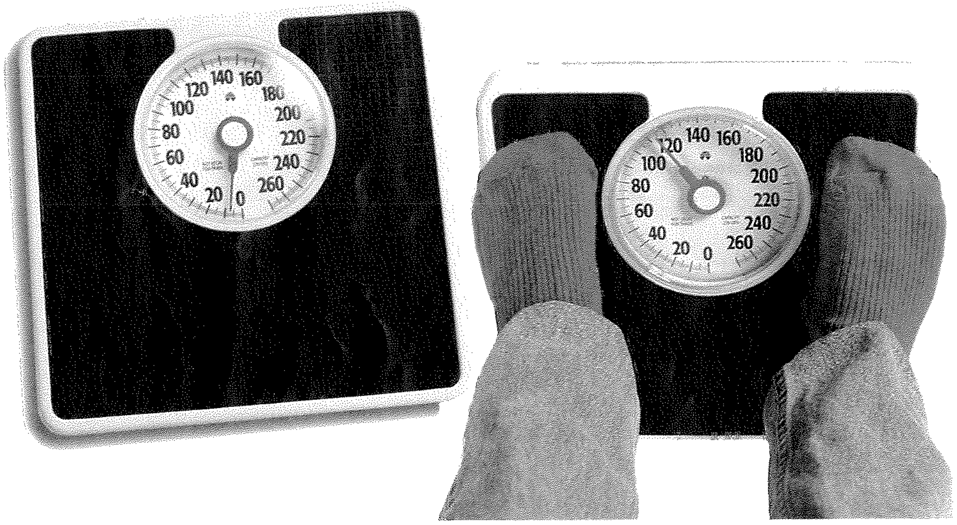


Figure 3.3 The scale below has not been properly zeroed, so the reading obtained for the person's weight is inaccurate. There is a difference between the person's correct weight and the measured value. **Calculating** ***What is the percent error of a measured value of 114 lb if the person's actual weight is 107 lb?***

Figure 3.4 The precision of a weighing scale depends on how finely it is calibrated.

Significant Figures in Measurements

Supermarkets often provide scales like the one in Figure 3.4. Customers use these scales to measure the weight of produce that is priced per pound. If you use a scale that is calibrated in 0.1-lb intervals, you can easily read the scale to the nearest tenth of a pound. With such a scale, however, you can also estimate the weight to the nearest hundredth of a pound by noting the position of the pointer between calibration marks.

Suppose you estimate a weight that lies between 2.4 lb and 2.5 lb to be 2.46 lb. The number in this estimated measurement has three digits. The first two digits in the measurement (2 and 4) are known with certainty. But the rightmost digit (6) has been estimated and involves some uncertainty. These three reported digits all convey useful information, however, and are called significant figures. The **significant figures** in a measurement include all of the digits that are known, plus a last digit that is estimated. **Measurements must always be reported to the correct number of significant figures because calculated answers often depend on the number of significant figures in the values used in the calculation.**

Instruments differ in the number of significant figures that can be obtained from their use and thus in the precision of measurements. The three meter sticks in Figure 3.5 can be used to make successively more precise measurements of the board.

Rules for determining whether a digit in a measured value is significant:

1 Every nonzero digit in a reported measurement is assumed to be significant. The measurements 24.7 meters, 0.743 meter, and 714 meters each express a measure of length to three significant figures.

2 Zeros appearing between nonzero digits are significant. The measurements 7003 meters, 40.79 meters, and 1.503 meters each have four significant figures.

3 Leftmost zeros appearing in front of nonzero digits are not significant. They act as placeholders. The measurements 0.0071 meter, 0.42 meter, and 0.000 099 meter each have only two significant figures. The zeros to the left are not significant. By writing the measurements in scientific notation, you can eliminate such placeholding zeros: in this case, 7.1×10^{-3} meter, 4.2×10^{-1} meter, and 9.9×10^{-5} meter.

4 Zeros at the end of a number and to the right of a decimal point are always significant. The measurements 43.00 meters, 1.010 meters, and 9.000 meters each have four significant figures.

Interactive Textbook

Animation 2 See how the precision of a calculated result depends on the sensitivity of the measuring instruments.

with ChemASAP

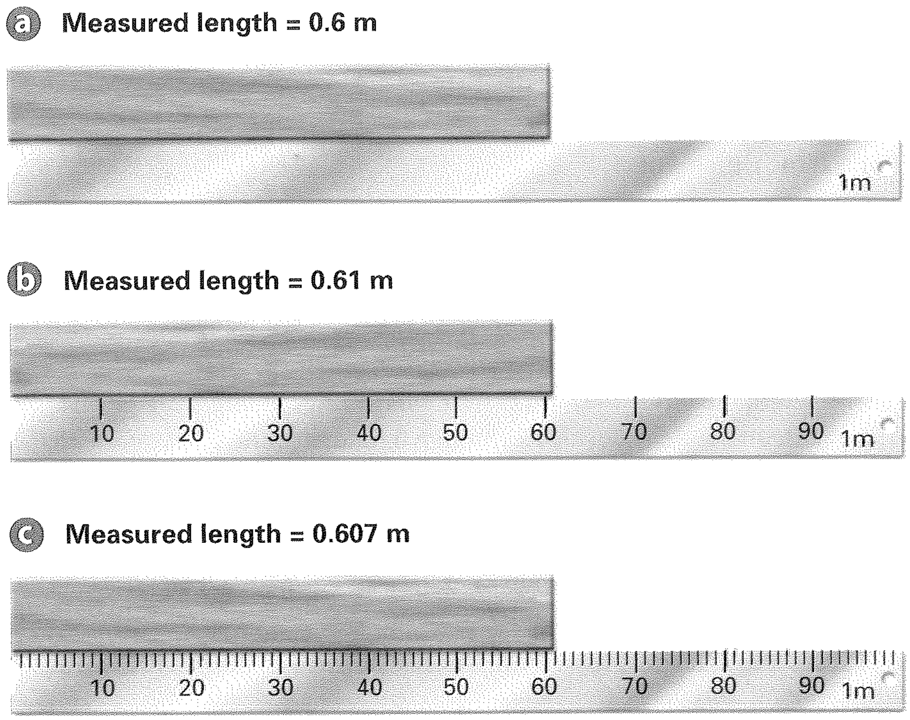
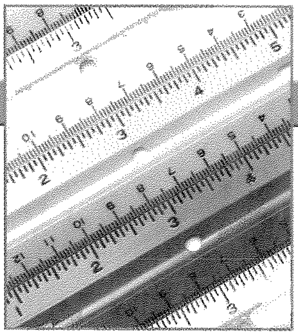


Figure 3.5 Three differently calibrated meter sticks are used to measure the length of a board. ⓐ A meter stick calibrated in a 1-m interval. ⓑ A meter stick calibrated in 0.1-m intervals. ⓒ A meter stick calibrated in 0.01-m intervals. **Measuring** ***How many significant figures are reported in each measurement?***

5 Zeros at the rightmost end of a measurement that lie to the left of an understood decimal point are not significant if they serve as placeholders to show the magnitude of the number. The zeros in the measurements 300 meters, 7000 meters, and 27,210 meters are not significant. The numbers of significant figures in these values are one, one, and four, respectively. If such zeros were known measured values, however, then they would be significant. For example, if all of the zeros in the measurement 300 meters were significant, writing the value in scientific notation as 3.00×10^2 meters makes it clear that these zeros are significant.

6 There are two situations in which numbers have an unlimited number of significant figures. The first involves counting. If you count 23 people in your classroom, then there are exactly 23 people, and this value has an unlimited number of significant figures. The second situation involves exactly defined quantities such as those found within a system of measurement. When, for example, you write 60 min = 1 hr, or 100 cm = 1 m, each of these numbers has an unlimited number of significant figures. As you shall soon see, exact quantities do not affect the process of rounding an answer to the correct number of significant figures.



CONCEPTUAL PROBLEM 3.1

Counting Significant Figures in Measurements

How many significant figures are in each measurement?

a. 123 m
b. 40,506 mm
c. 9.8000×10^4 m
d. 22 meter sticks
e. 0.070 80 m
f. 98,000 m

① **Analyze** ***Identify the relevant concepts.***

The location of each zero in the measurement and the location of the decimal point determine which of the rules apply for determining significant figures.

② **Solve** ***Apply the concepts to this problem.***

All nonzero digits are significant (rule 1). Use rules 2 through 6 to determine if the zeros are significant.

a. three (rule 1)
b. five (rule 2)
c. five (rule 4)
d. unlimited (rule 6)
e. four (rules 2, 3, 4)
f. two (rule 5)

Practice Problems

1. Count the significant figures in each length.
- **a.** 0.057 30 meters
- **b.** 8765 meters
- **c.** 0.000 73 meters
- **d.** 40.007 meters

2. How many significant figures are in each measurement?
- **a.** 143 grams
- **b.** 0.074 meter
- **c.** 8.750×10^{-2} gram
- **d.** 1.072 meter

Interactive Textbook

Problem-Solving 3.2 Solve Problem 2 with the help of an interactive guided tutorial.

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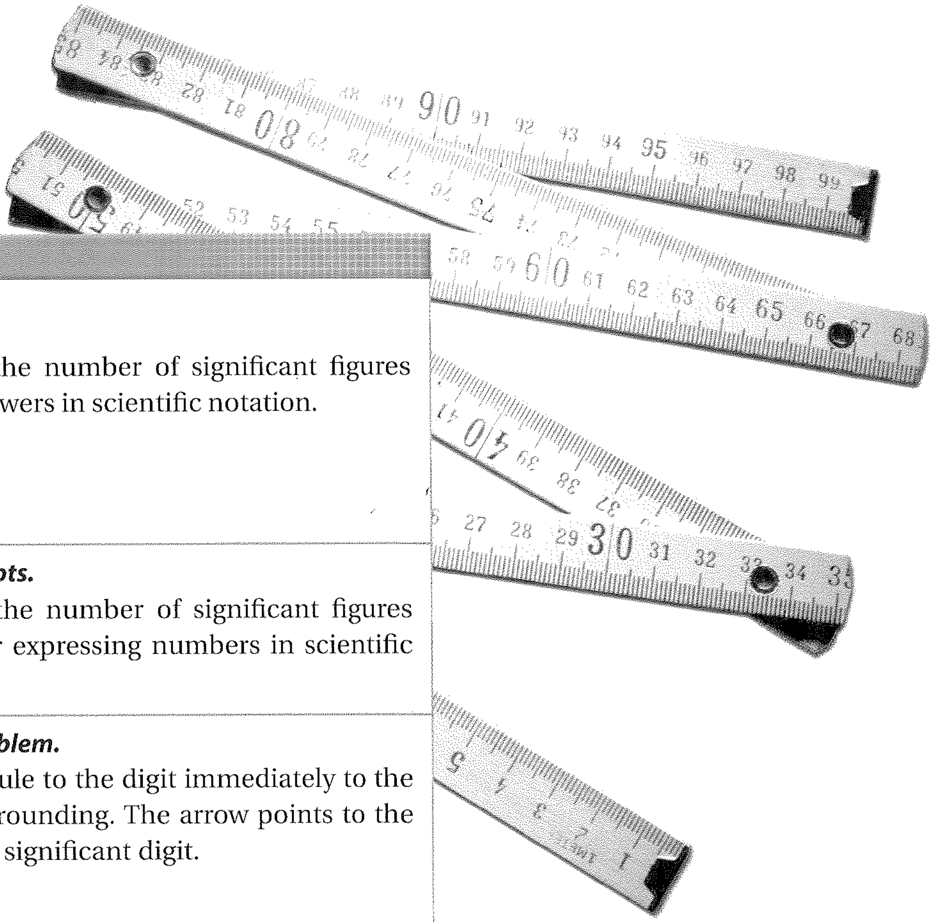
Significant Figures in Calculations

Suppose you use a calculator to find the area of a floor that measures 7.7 meters by 5.4 meters. The calculator would give an answer of 41.58 square meters. The calculated area is expressed to four significant figures. However, each of the measurements used in the calculation is expressed to only two significant figures. So the answer must also be reported to two significant figures (42 m^2). **In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated.** The calculated value must be rounded to make it consistent with the measurements from which it was calculated.

Rounding To round a number, you must first decide how many significant figures the answer should have. This decision depends on the given measurements and on the mathematical process used to arrive at the answer. Once you know the number of significant figures your answer should have, round to that many digits, counting from the left. If the digit immediately to the right of the last significant digit is less than 5, it is simply dropped and the value of the last significant digit stays the same. If the digit in question is 5 or greater, the value of the digit in the last significant place is increased by 1.

✓ **Checkpoint** ***Why must a calculated answer generally be rounded?***



SAMPLE PROBLEM 3.1

Rounding Measurements

Round off each measurement to the number of significant figures shown in parentheses. Write the answers in scientific notation.

a. 314.721 meters (four)
b. 0.001 775 meter (two)
c. 8792 meters (two)

① **Analyze** ***Identify the relevant concepts.***

Round off each measurement to the number of significant figures indicated. Then apply the rules for expressing numbers in scientific notation.

② **Solve** ***Apply the concepts to this problem.***

Count from the left and apply the rule to the digit immediately to the right of the digit to which you are rounding. The arrow points to the digit immediately following the last significant digit.

a. 314.721 meters
↑
2 is less than 5, so you do not round up.
314.7 meters = 3.147×10^2 meters

b. 0.001 775 meter
↑
7 is greater than 5, so round up.
0.0018 meter = 1.8×10^{-3} meter

c. 8792 meters
↑
9 is greater than 5, so round up
8800 meters = 8.8×10^3 meters

③ **Evaluate** ***Do the results make sense?***

The rules for rounding and for writing numbers in scientific notation have been correctly applied.

Practice Problems

3. Round each measurement to three significant figures. Write your answers in scientific notation.
 a. 87.073 meters
 b. 4.3621×10^8 meters
 c. 0.01552 meter
 d. 9009 meters
 e. 1.7777×10^{-3} meter
 f. 629.55 meters

4. Round each measurement in Practice Problem 3 to one significant figure. Write each of your answers in scientific notation.

Math Handbook

For help with scientific notation, go to page R56.

Interactive Textbook

Problem-Solving 3.3 Solve Problem 3 with the help of an interactive guided tutorial.

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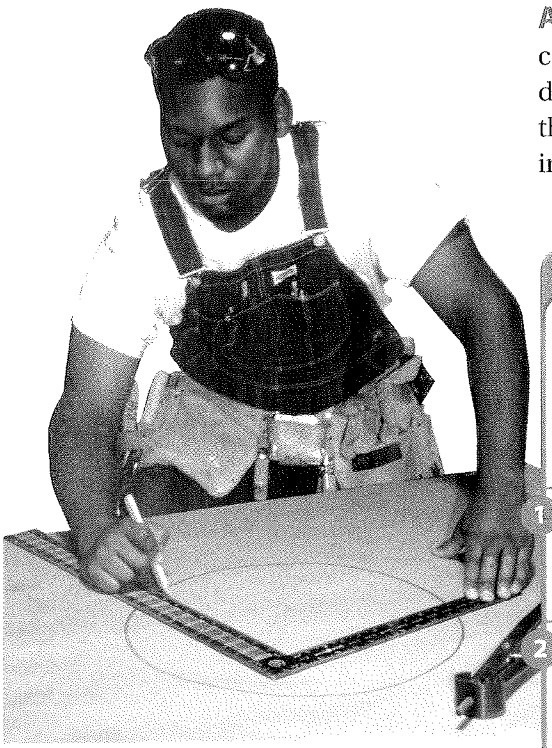

Addition and Subtraction The answer to an addition or subtraction calculation should be rounded to the same number of decimal places (not digits) as the measurement with the least number of decimal places. Work through Sample Problem 3.2 below which provides an example of rounding in an addition calculation.

SAMPLE PROBLEM 3.2

Significant Figures in Addition

Calculate the sum of the three measurements. Give the answer to the correct number of significant figures.

12.52 meters + 349.0 meters + 8.24 meters

1 **Analyze** ***Identify the relevant concepts.***
Calculate the sum and then analyze each measurement to determine the number of decimal places required in the answer.

2 **Solve** ***Apply the concepts to this problem.***
Align the decimal points and add the numbers. Round the answer to match the measurement with the least number of decimal places.

$$\begin{array}{rl} 12.52 & \text{meters} \\ 349.0 & \text{meters} \\ +\quad 8.24 & \text{meters} \\ \hline 369.76 & \text{meters} \end{array}$$

The second measurement (349.0 meters) has the least number of digits (one) to the right of the decimal point. Thus the answer must be rounded to one digit after the decimal point. The answer is rounded to 369.8 meters, or 3.698×10^2 meters.

3 **Evaluate** ***Does the result make sense?***
The mathematical operation has been correctly carried out and the resulting answer is reported to the correct number of decimal places.

Practice Problems

5. Perform each operation. Express your answers to the correct number of significant figures.
  a. 61.2 meters + 9.35 meters + 8.6 meters
  b. 9.44 meters − 2.11 meters
  c. 1.36 meters + 10.17 meters
  d. 34.61 meters − 17.3 meters

6. Find the total mass of three diamonds that have masses of 14.2 grams, 8.73 grams, and 0.912 gram.

Math Handbook

For help with significant figures, go to page R59.

Interactive Textbook

Problem-Solving 3.6 Solve Problem 6 with the help of an interactive guided tutorial.

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Multiplication and Division In calculations involving multiplication and division, you need to round the answer to the same number of significant figures as the measurement with the least number of significant figures. The position of the decimal point has nothing to do with the rounding process when multiplying and dividing measurements. The position of the decimal point is important only in rounding the answers of addition or subtraction problems.

✓ **Checkpoint** ***How many significant figures must you round an answer to when performing multiplication or division?***

SAMPLE PROBLEM 3.3

Significant Figures in Multiplication and Division

Perform the following operations. Give the answers to the correct number of significant figures.

a. 7.55 meters × 0.34 meter
b. 2.10 meters × 0.70 meter
c. 2.4526 meters ÷ 8.4

① **Analyze** ***Identify the relevant concepts.***

Perform the required math operation and then analyze each of the original numbers to determine the correct number of significant figures required in the answer.

② **Solve** ***Apply the concepts to this problem.***

Round the answers to match the measurement with the least number of significant figures.

a. 7.55 meters × 0.34 meter = 2.567 $(\text{meter})^2$ = 2.6 meters^2
(0.34 meter has two significant figures)
b. 2.10 meters × 0.70 meter = 1.47 $(\text{meter})^2$ = 1.5 meters^2
(0.70 meter has two significant figures)
c. 2.4526 meters ÷ 8.4 = 0.291 976 meter = 0.29 meter
(8.4 has two significant figures)

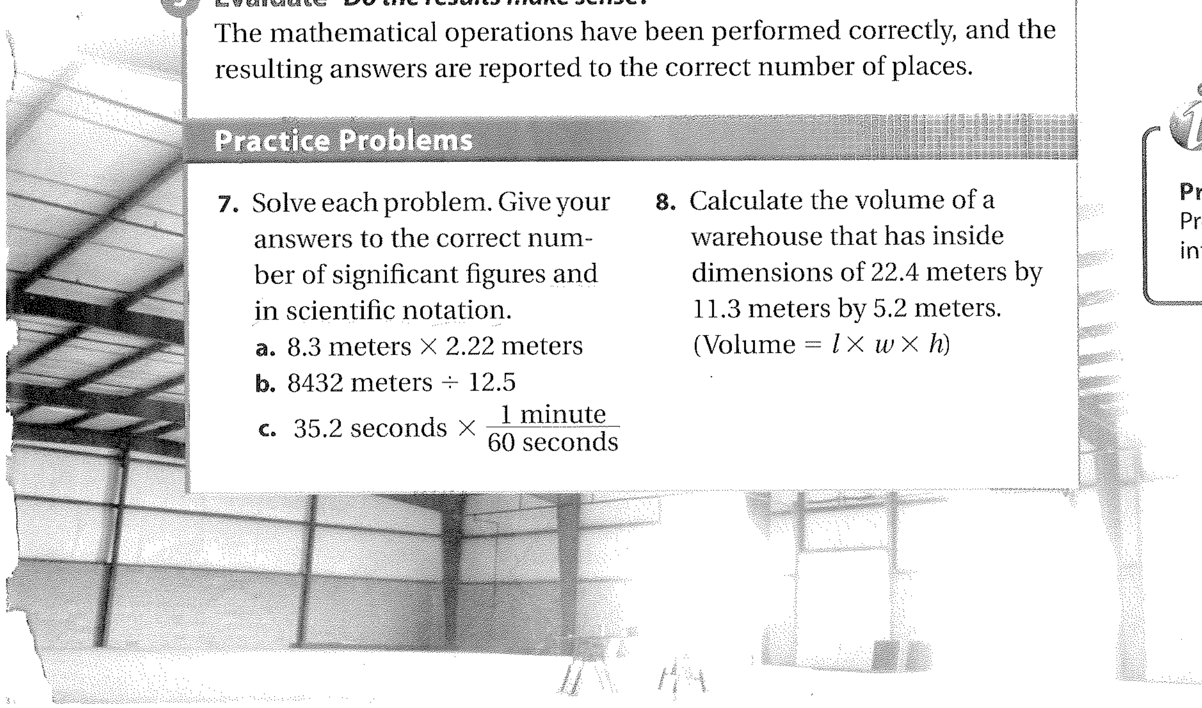
③ **Evaluate** ***Do the results make sense?***

The mathematical operations have been performed correctly, and the resulting answers are reported to the correct number of places.

Practice Problems

7. Solve each problem. Give your answers to the correct number of significant figures and in scientific notation.
  a. 8.3 meters × 2.22 meters
  b. 8432 meters ÷ 12.5
  c. 35.2 seconds × $\frac{\text{1 minute}}{\text{60 seconds}}$

8. Calculate the volume of a warehouse that has inside dimensions of 22.4 meters by 11.3 meters by 5.2 meters. (Volume = $l \times w \times h$)



Math Handbook

For help with using a calculator, go to page R62.

Interactive Textbook

Problem-Solving 3.8 Solve Problem 8 with the help of an interactive guided tutorial.

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Quick LAB

Accuracy and Precision

Purpose

To measure the dimensions of an object as accurately and precisely as possible and to apply rules for rounding answers calculated from the measurements.

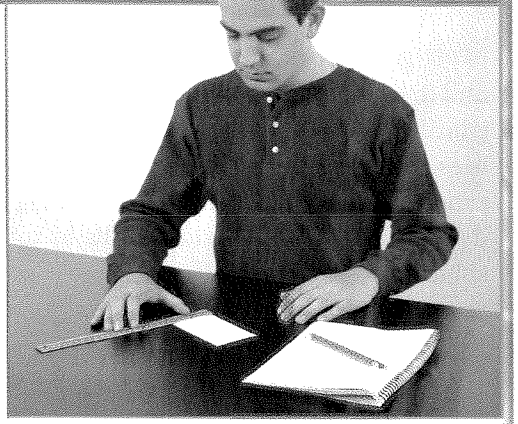
Materials

- 3 inch × 5 inch index card
- metric ruler

Procedure

1. Use a metric ruler to measure in centimeters the length and width of an index card as accurately and precisely as you can. The hundredths place in your measurement should be estimated.
2. Calculate the perimeter [2 × (length + width)] and the area (length × width) of the index card. Write both your unrounded answers and your correctly rounded answers on the chalkboard.

Analyze and Conclude



1. How many significant figures are in your measurements of length and of width?
2. How do your measurements compare with those of your classmates?
3. How many significant figures are in your calculated value for the area? In your calculated value for the perimeter? Do your rounded answers have as many significant figures as your classmates' measurements?
4. Assume that the correct (accurate) length and width of the card are 12.70 cm and 7.62 cm, respectively. Calculate the percent error for each of your two measurements.

3.1 Section Assessment

9. **Key Concept** How do measurements relate to experimental science?
10. **Key Concept** How are accuracy and precision evaluated?
11. **Key Concept** Why must a given measurement always be reported to the correct number of significant figures?
12. **Key Concept** How does the precision of a calculated answer compare to the precision of the measurements used to obtain it?
13. A technician experimentally determined the boiling point of octane to be 124.1°C. The actual boiling point of octane is 125.7°C. Calculate the error and the percent error.
14. Determine the number of significant figures in each of the following.
    a. 11 soccer players **b.** 0.070 020 meter
    c. 10,800 meters **d.** 5.00 cubic meters
15. Solve the following and express each answer in scientific notation and to the correct number of significant figures.
    a. $(5.3 \times 10^{4}) + (1.3 \times 10^{4})$
    b. $(7.2 \times 10^{-4}) \div (1.8 \times 10^{3})$
    c. $10^{4} \times 10^{-3} \times 10^{6}$
    d. $(9.12 \times 10^{-1}) - (4.7 \times 10^{-2})$
    e. $(5.4 \times 10^{4}) \times (3.5 \times 10^{9})$

Writing Activity

Explanatory Paragraph Explain the differences between the accuracy, precision, and error of a measurement.

Interactive Textbook

Assessment 3.1 Test yourself on the concepts in Section 3.1.

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