3.3 Conversion Problems

Guide for Reading

Key Concepts

- What happens when a measurement is multiplied by a conversion factor?
- Why is dimensional analysis useful?
- What types of problems are easily solved by using dimensional analysis?

Vocabulary

conversion factor

dimensional analysis

Reading Strategy

Monitoring Your Understanding Preview the Key Concepts, the section heads, and boldfaced terms. List three things you expect to learn. After reading, state what you learned about each item listed.

Connecting to Your World

Perhaps you have traveled abroad or are planning to do so. If so, you know—or will soon discover—that different countries have different currencies. As a tourist, exchanging money is essential to the enjoyment of your trip. After all, you must pay for your meals, hotel, transportation, gift purchases, and tickets to exhibits and events. Because each country's currency compares differently with the U.S. dollar, knowing how to convert currency units correctly is very important. Conversion problems are readily solved by a problem-solving approach called dimensional analysis.

Conversion Factors

If you think about any number of everyday situations, you will realize that a quantity can usually be expressed in several different ways. For example, consider the monetary amount \$1.

1 dollar = 4 quarters = 10 dimes = 20 nickels = 100 pennies

These are all expressions, or measurements, of the same amount of money. The same thing is true of scientific quantities. For example, consider a distance that measures exactly 1 meter.

1 meter = 10 decimeters = 100 centimeters = 1000 millimeters

These are different ways to express the same length.

Whenever two measurements are equivalent, a ratio of the two measurements will equal 1, or unity. For example, you can divide both sides of the equation 1 m = 100 cm by 1 m or by 100 cm.

$$\frac{1\text{ m}}{1\text{ m}} = \frac{100\text{ cm}}{1\text{ m}} = 1 \qquad \text{or} \qquad \frac{1\text{ m}}{100\text{ cm}} = \frac{100\text{ cm}}{100\text{ cm}} = 1$$

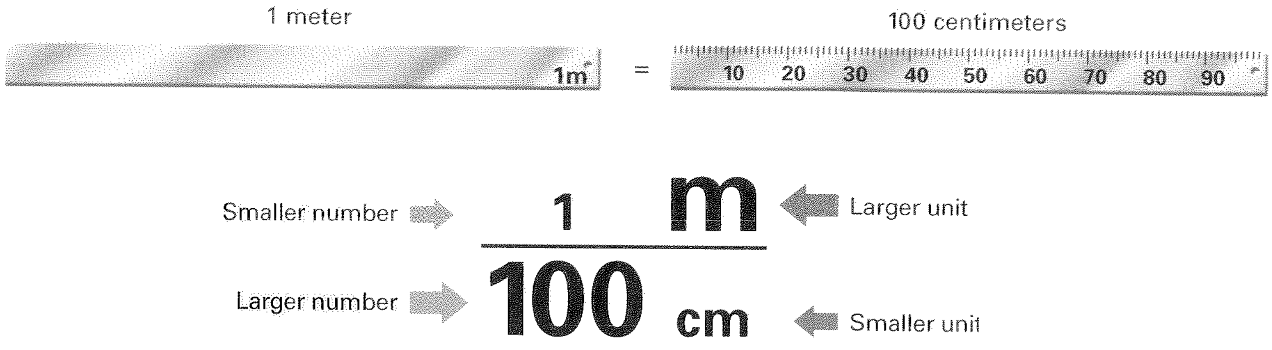
↑ conversion factors ↑

A **conversion factor** is a ratio of equivalent measurements. The ratios 100 cm/1 m and 1 m/100 cm are examples of conversion factors. In a conversion factor, the measurement in the numerator (on the top) is equivalent to the measurement in the denominator (on the bottom). The conversion factors above are read "one hundred centimeters per meter" and "one meter per hundred centimeters." Figure 3.11 illustrates another way to look at the relationships in a conversion factor. Notice that the smaller number is part of the measurement with the larger unit. That is, a meter is physically larger than a centimeter. The larger number is part of the measurement with the smaller unit.

Interactive Textbook

Animation 3 Learn how to select the proper conversion factor and how to use it.

with ChemASAP



A Conversion Factor

Figure 3.11 The two parts of a conversion factor, the numerator and the denominator, are equal.

Conversion factors are useful in solving problems in which a given measurement must be expressed in some other unit of measure. **When a measurement is multiplied by a conversion factor, the numerical value is generally changed, but the actual size of the quantity measured remains the same.** For example, even though the numbers in the measurements 1 g and 10 dg (decigrams) differ, both measurements represent the same mass. In addition, conversion factors within a system of measurement are defined quantities or exact quantities. Therefore, they have an unlimited number of significant figures, and do not affect the rounding of a calculated answer.

Here are some additional examples of pairs of conversion factors written from equivalent measurements. The relationship between grams and kilograms is 1000 g = 1 kg. The conversion factors are:

$$\frac{1000\ \text{g}}{1\ \text{kg}} \quad \text{and} \quad \frac{1\ \text{kg}}{1000\ \text{g}}$$

The scale of the micrograph in Figure 3.12 is in nanometers. Using the relationship 10^9 nm = 1 m, you can write the following conversion factors.

$$\frac{10^9\ \text{nm}}{1\ \text{m}} \quad \text{and} \quad \frac{1\ \text{m}}{10^9\ \text{nm}}$$

Common volumetric units used in chemistry include the liter and the microliter. The relationship 1 L = 10^6 μL yields the following conversion factors.

$$\frac{1\ \text{L}}{10^6\ \mu\text{L}} \quad \text{and} \quad \frac{10^6\ \mu\text{L}}{1\ \text{L}}$$

Based on what you know about metric prefixes, you should be able to easily write conversion factors that relate equivalent metric quantities.

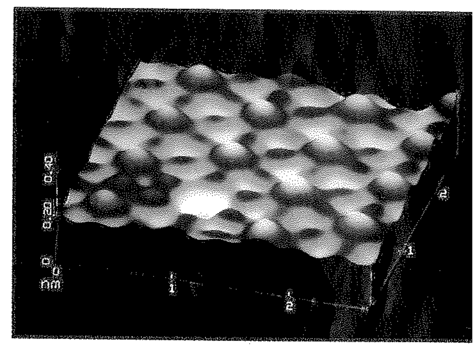
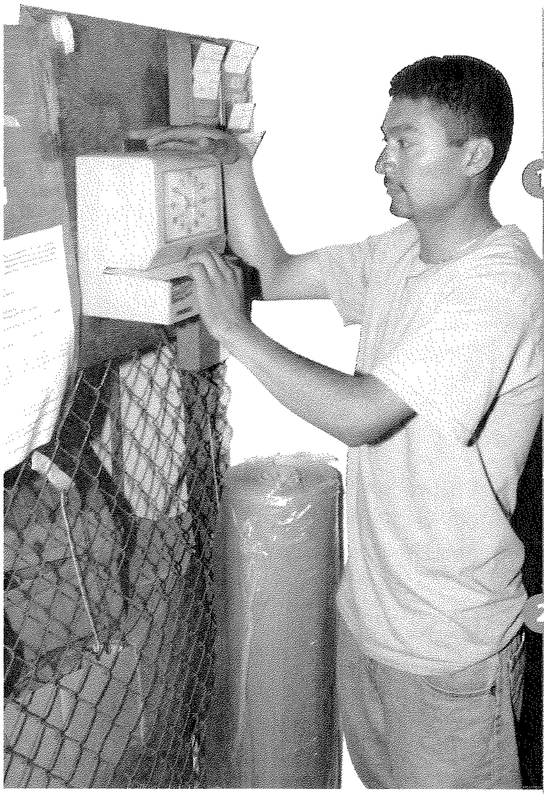


Figure 3.12 In this computer image of atoms, distance is marked off in nanometers (nm). **Inferring** ***What conversion factor would you use to convert nanometers to meters?***

Checkpoint ***How many significant figures does a conversion factor within a system of measurement have?***

Dimensional Analysis

No single method is best for solving every type of problem. Several good approaches are available, and generally one of the best is dimensional analysis. **Dimensional analysis** is a way to analyze and solve problems using the units, or dimensions, of the measurements. The best way to explain this problem-solving technique is to use it to solve an everyday situation.



Math Handbook

For help with dimensional analysis, go to page R66.

Interactive Textbook

Problem-Solving 3.29 Solve Problem 29 with the help of an interactive guided tutorial.

with **ChemASAP**

SAMPLE PROBLEM 3.5

Using Dimensional Analysis

How many seconds are in a workday that lasts exactly eight hours?

1 Analyze ***List the knowns and the unknown.***

Knowns

- time worked = 8 h
- 1 hour = 60 min
- 1 minute = 60 s

Unknown

- seconds worked = ? s

The first conversion factor must be written with the unit hours in the denominator. The second conversion factor must be written with the unit minutes in the denominator. This will provide the desired unit (seconds) in the answer.

2 Calculate ***Solve for the unknown.***

Start with the known, 8 hours. Use the first relationship (1 hour = 60 minutes) to write a conversion factor that expresses 8 hours as minutes. The unit hours must be in the denominator so that the known unit will cancel. Then use the second conversion factor to change the unit minutes into the unit seconds. This conversion factor must have the unit minutes in the denominator. The two conversion factors can be used together in a simple overall calculation.

$$8\,\cancel{h} \times \frac{60\,\cancel{min}}{1\,\cancel{h}} \times \frac{60\,s}{1\,\cancel{min}} = 28{,}800\,s$$
$$= 2.8800 \times 10^4\,s$$

3 Evaluate ***Does the result make sense?***

The answer has the desired unit (seconds). Since the second is a small unit of time, you should expect a large number of seconds in 8 hours. Before you do the actual arithmetic, it is a good idea to make sure that the units cancel and that the numerator and denominator of each conversion factor are equal to each other. The answer is exact since the given measurement and each of the conversion factors is exact.

Practice Problems

28. How many minutes are there in exactly one week?

29. How many seconds are in exactly a 40-hour work week?

There is usually more than one way to solve a problem. When you first read Sample Problem 3.5, you may have thought about different and equally correct ways to approach and solve the problem. Some problems are easily worked with simple algebra. **Dimensional analysis provides you with an alternative approach to problem solving.** In either case, you should choose the problem-solving method that works best.

SAMPLE PROBLEM 3.6

Using Dimensional Analysis

The directions for an experiment ask each student to measure 1.84 g of copper (Cu) wire. The only copper wire available is a spool with a mass of 50.0 g. How many students can do the experiment before the copper runs out?

1 Analyze ***List the knowns and the unknown.***

Knowns

- mass of copper available = 50.0 g Cu
- each student needs 1.84 grams of copper, or $\frac{1.84 \text{ g Cu}}{\text{student}}$.

Unknown

- number of students = ?

From the known mass of copper, calculate the number of students that can do the experiment by using the appropriate conversion factor. The desired conversion is mass of copper $\longrightarrow$ number of students.

2 Calculate ***Solve for the unknown.***

Because students is the desired unit for the answer, the conversion factor should be written with students in the numerator. Multiply the mass of copper by the conversion factor.

$$50.0 \text{ g } \cancel{\text{Cu}} \times \frac{1 \text{ student}}{1.84 \text{ g } \cancel{\text{Cu}}} = 27.174 \text{ students} = 27 \text{ students}$$

Note that because students cannot be fractional, the result is shown rounded down to a whole number.

3 Evaluate ***Does the result make sense?***

The unit of the answer (students) is the one desired. The number of students (27) seems to be a reasonable answer. You can make an approximate calculation using the following conversion factor.

$$\frac{1 \text{ student}}{2 \text{ g Cu}}$$

Multiplying the above conversion factor by 50 g Cu gives the approximate answer of 25 students, which is close to the calculated answer.

Practice Problems

30. An experiment requires that each student use an 8.5-cm length of magnesium ribbon. How many students can do the experiment if there is a 570-cm length of magnesium ribbon available?

31. A 1.00-degree increase on the Celsius scale is equivalent to a 1.80-degree increase on the Fahrenheit scale. If a temperature increases by 48.0°C, what is the corresponding temperature increase on the Fahrenheit scale?

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Conversion Problems

A conversion factor is a ratio of two quantities that are equal to one another. When doing conversions, write the conversion factors so that the unit of a given measurement cancels, leaving the correct unit for your answer. Note that the equalities needed to write a particular conversion may be given in the problem. In other cases, you will need to know or look up the necessary equalities.

Math Handbook

For help with conversion problems, go to page R66.

Interactive Textbook

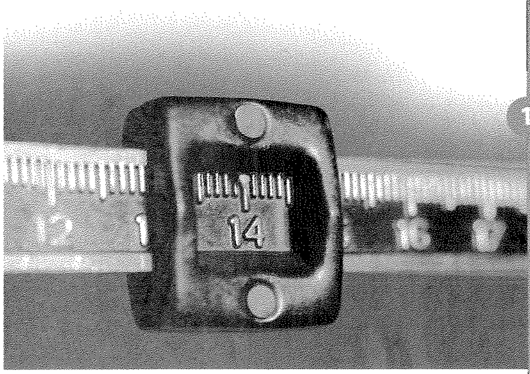
Problem-Solving 3.30 Solve Problem 30 with the help of an interactive guided tutorial.

with ChemASAP

Converting Between Units

In chemistry, as in many other subjects, you often need to express a measurement in a unit different from the one given or measured initially. **Problems in which a measurement with one unit is converted to an equivalent measurement with another unit are easily solved using dimensional analysis.**

Suppose that a laboratory experiment requires 7.5 dg of magnesium metal, and 100 students will do the experiment. How many grams of magnesium should your teacher have on hand? Multiplying 100 students by 7.5 dg/student gives you 750 dg. But then you must convert dg to grams. Sample Problem 3.7 shows you how to do the conversion.



SAMPLE PROBLEM 3.7

Converting Between Metric Units

Express 750 dg in grams.

1 Analyze ***List the knowns and the unknown.***

Knowns	Unknown
• mass = 750 dg	• mass = ? g
• 1 g = 10 dg	

The desired conversion is decigrams ⟶ grams. Using the expression relating the units, 10 dg = 1 g, multiply the given mass by the proper conversion factor.

2 Calculate ***Solve for the unknown.***

The correct conversion factor is shown below.

$$\frac{1\ \text{g}}{10\ \text{dg}}$$

Note that the known unit is in the denominator and the unknown unit is in the numerator.

$$750\ \text{dg} \times \frac{1\ \text{g}}{10\ \text{dg}} = 75\ \text{g}$$

3 Evaluate ***Does the result make sense?***

Because the unit gram represents a larger mass than the unit decigram, it makes sense that the number of grams is less than the given number of decigrams. The unit of the known (dg) cancels, and the answer has the correct unit (g). The answer also has the correct number of significant figures.

Practice Problems

32. Using tables from this chapter, convert the following.
- **a.** 0.044 km to meters
- **b.** 4.6 mg to grams
- **c.** 0.107 g to centigrams

33. Convert the following.
- **a.** 15 cm^3 to liters
- **b.** 7.38 g to kilograms
- **c.** 6.7 s to milliseconds
- **d.** 94.5 g to micrograms

Interactive Textbook

Problem-Solving 3.33 Solve Problem 33 with the help of an interactive guided tutorial.

with ChemASAP

Multistep Problems Many complex tasks in your everyday life are best handled by breaking them down into manageable parts. For example, if you were cleaning a car, you might first vacuum the inside, then wash the exterior, then dry the exterior, and finally put on a fresh coat of wax. Similarly, many complex word problems are more easily solved by breaking the solution down into steps.

When converting between units, it is often necessary to use more than one conversion factor. Sample Problem 3.8 illustrates the use of multiple conversion factors.

Checkpoint ***What problem-solving methods can help you solve complex word problems?***

SAMPLE PROBLEM 3.8

Converting Between Metric Units

What is 0.073 cm in micrometers?

1 Analyze ***List the knowns and the unknown.***

Knowns
- length = 0.073 cm = 7.3×10^{-2} cm
- 10^2 cm = 1 m
- 1 m = 10^6 μm

Unknown
- length = ? μm

The desired conversion is from centimeters to micrometers. The problem can be solved in a two-step conversion.

2 Calculate ***Solve for the unknown.***

First change centimeters to meters; then change meters to micrometers: centimeters $\longrightarrow$ meters $\longrightarrow$ micrometers. Each conversion factor is written so that the unit in the denominator cancels the unit in the numerator of the previous factor.

$$7.3 \times 10^{-2} \cancel{\text{cm}} \times \frac{1 \cancel{\text{m}}}{10^2 \cancel{\text{cm}}} \times \frac{10^6\ \mu\text{m}}{1 \cancel{\text{m}}} = 7.3 \times 10^2\ \mu\text{m}$$

3 Evaluate ***Does the result make sense?***

Because a micrometer is a much smaller unit than a centimeter, the answer should be numerically larger than the given measurement. The units have canceled correctly, and the answer has the correct number of significant figures.

Practice Problems

34. The radius of a potassium atom is 0.227 nm. Express this radius in the unit centimeters.

35. The diameter of Earth is 1.3×10^4 km. What is the diameter expressed in decimeters?

CHEMath

Scientific Notation

It is often convenient to express very large or very small numbers in scientific notation. The distance between the sun and Earth is 150,000,000 km, which can be written as 1.5×10^8 km. The diameter of a gold atom is 0.000 000 000 274 m, or 2.74×10^{-10} m.

When multiplying numbers written in scientific notation, add the exponents. When dividing numbers written in scientific notation, subtract the exponent in the denominator from the exponent in the numerator.

Math Handbook

For help with scientific notation, go to page R56.

Interactive Textbook

Problem-Solving 3.35 Solve Problem 35 with the help of an interactive guided tutorial.

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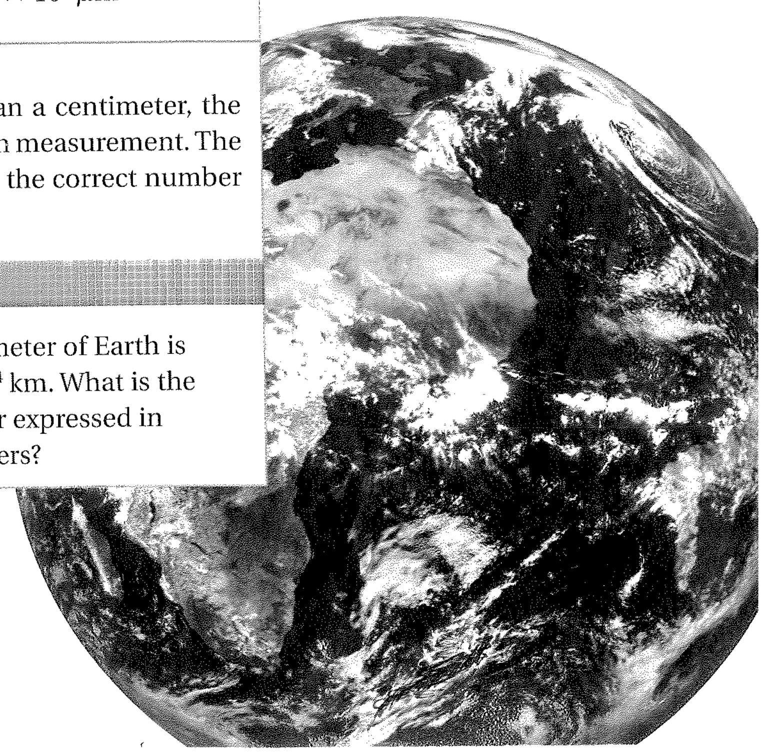


Converting Complex Units Many common measurements are expressed as a ratio of two units. For example, the results of international car races often give average lap speeds in kilometers per hour. You measure the densities of solids and liquids in grams per cubic centimeter. You measure the gas mileage in a car in miles per gallon of gasoline. If you use dimensional analysis, converting these complex units is just as easy as converting single units. It will just take multiple steps to arrive at an answer.

SAMPLE PROBLEM 3.9

Converting Ratios of Units

The mass per unit volume of a substance is a property called density. The density of manganese, a metallic element, is 7.21 g/cm^3. What is the density of manganese expressed in units kg/m^3?

1 Analyze ***List the knowns and the unknown.***

Knowns

- density of manganese = 7.21 g/cm^3
- 10^3 g = 1 kg
- 10^6 cm^3 = 1 m^3

Unknown

- density manganese = ? kg/m^3

The desired conversion is $g/cm^3 \longrightarrow kg/m^3$. The mass unit in the numerator must be changed from grams to kilograms: $g \longrightarrow kg$. In the denominator, the volume unit must be changed from cubic centimeters to cubic meters: $cm^3 \longrightarrow m^3$. Note that the relationship between cm^3 and m^3 was determined from the relationship between cm and m. Cubing the relationship 10^2 cm = 1 m yields $(10^2\ cm)^3 = (1\ m)^3$, or $10^6\ cm^3 = 1\ m^3$.

2 Calculate ***Solve for the unknown.***

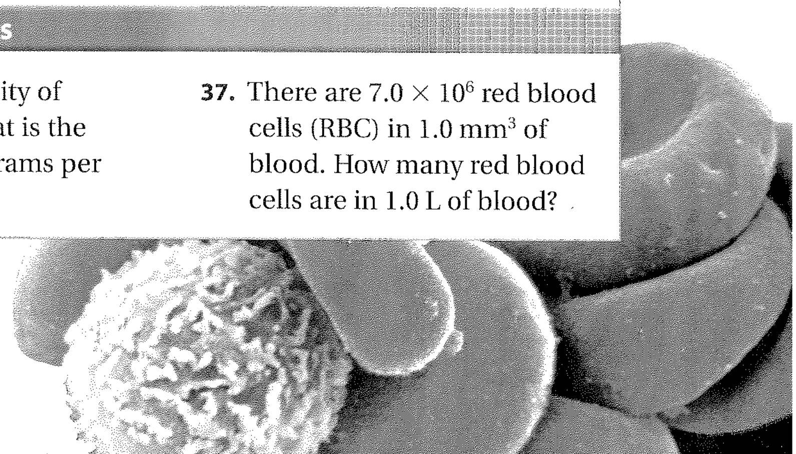
$$\frac{7.21\ \cancel{g}}{1\ \cancel{cm^3}} \times \frac{1\ kg}{10^3\ \cancel{g}} \times \frac{10^6\ \cancel{cm^3}}{1\ m^3} = 7.21 \times 10^3\ kg/m^3$$

3 Evaluate ***Does the result make sense?***
Because the physical size of the volume unit m^3 is so much larger than cm^3 (10^6 times), the calculated value of the density should be larger than the given value even though the mass unit is also larger (10^3 times). The units cancel, the conversion factors are correct, and the answer has the correct ratio of units.

Practice Problems

36. Gold has a density of 19.3 g/cm^3. What is the density in kilograms per cubic meter?

37. There are 7.0×10^6 red blood cells (RBC) in 1.0 mm^3 of blood. How many red blood cells are in 1.0 L of blood?



Math Handbook

For help with dimensional analysis, go to page R66.

Interactive Textbook

Problem-Solving 3.37 Solve Problem 37 with the help of an interactive guided tutorial.

with ChemASAP

Quick LAB

Dimensional Analysis

Purpose
To apply the problem-solving technique of dimensional analysis to conversion problems.

Materials
- 3 inch × 5 inch index cards or paper cut to approximately the same size
- pen

Procedure
A conversion factor is a ratio of equivalent measurements. For any relationship, you can write two ratios. On a conversion factor card you can write one ratio on each side of the card.

1. Make a conversion factor card for each metric relationship shown in Tables 3.3, 3.4, and 3.5. Show the inverse of the conversion factor on the back of each card.
2. Use the appropriate conversion factor cards to set up solutions to Sample Problems 3.7 and 3.8. Notice that in each solution, the unit in the denominator of the conversion factor cancels the unit in the numerator of the previous conversion factor.

Analyze and Conclude
1. What is the effect of multiplying a given measurement by one or more conversion factors?
2. Use your conversion factor cards to set up solutions to these problems.
   a. 78.5 cm = ? m
   b. 0.056 L = ? cm^3
   c. 77 kg = ? mg
   d. 0.098 nm = ? dm
   e. 0.96 cm = ? µm
   f. 0.0067 mm = ? nm

3.3 Section Assessment

38. **Key Concept** What happens to the numerical value of a measurement that is multiplied by a conversion factor? What happens to the actual size of the quantity?

39. **Key Concept** Why is dimensional analysis useful?

40. **Key Concept** What types of problems can be solved using dimensional analysis?

41. What conversion factor would you use to convert between these pairs of units?
   a. minutes to hours
   b. grams to milligrams
   c. cubic decimeters to milliliters

42. Make the following conversions. Express your answers in standard exponential form.
   a. 14.8 g to micrograms
   b. 3.72×10^{-3} kg to grams
   c. 66.3 L to cubic centimeters

43. An atom of gold has a mass of 3.271×10^{-22} g. How many atoms of gold are in 5.00 g of gold?

44. Convert the following. Express your answers in scientific notation.
   a. 7.5×10^4 J to kilojoules
   b. 3.9×10^5 mg to decigrams
   c. 2.21×10^{-4} dL to microliters

45. Light travels at a speed of 3.00×10^{10} cm/s. What is the speed of light in kilometers/hour?

Connecting Concepts

Problem-Solving Skills Reread the passage on solving numeric problems in Section 1.4. Explain how the three-step process might apply to conversion problems that involve dimensional analysis.

Interactive Textbook

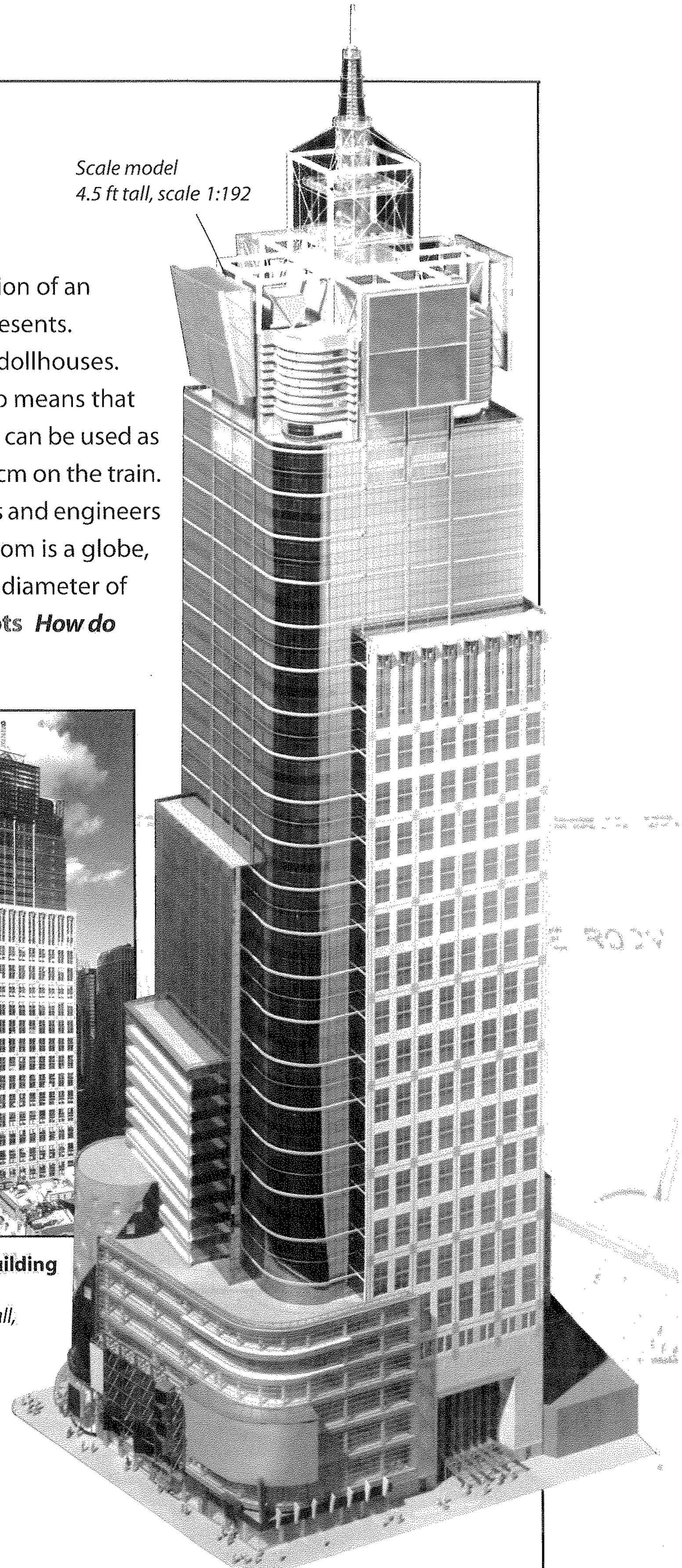
Assessment 3.3 Test yourself on the concepts in Section 3.3.
with ChemASAP

Technology & Society

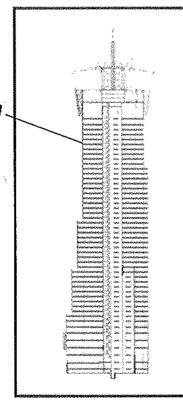
Scale Models

A scale model is a physical or conceptual representation of an object that is proportional in size to the object it represents. Examples include model trains, model airplanes, and dollhouses. Most model trains are built to a scale of 1:87. This ratio means that the model is $\frac{1}{87}$ the size of an actual train. This fraction can be used as a conversion factor. On the model, 1 cm represents 87 cm on the train.

Scale models aren't just for hobbyists—scientists and engineers use them, too. A simple scientific model in the classroom is a globe, which is a small-scale model of Earth. (A globe with a diameter of 30 cm has a scale of 1:42,500,000.) **Applying Concepts** ***How do you use the scale of a model as a conversion factor?***



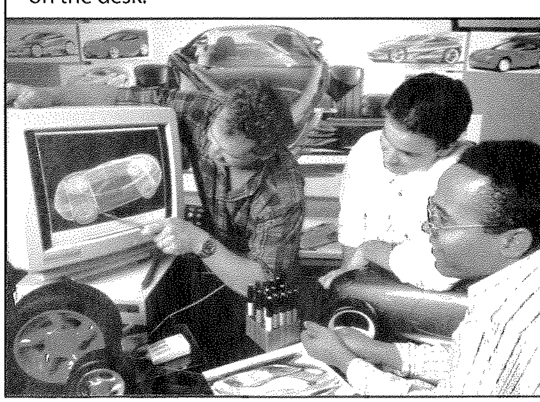
Scale model
4.5 ft tall, scale 1:192



Architectural drawing

Condé Nast Building
New York City
866 ft (264 m) tall, 48 floors

Computer modeling
By testing a model, engineers can make a product better before it is built. Engineers often design scale models on computers. These automotive engineers are using a computer-aided design (CAD) program to view a digital scale model of a car. Physical models of the car's wheels are on the desk.



Model building
Architects use both two-dimensional and three-dimensional scale models to design buildings. A common scale for floor plans is 1: 48.