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Quick Review 5.3

1. $\frac{dy}{dx} = \sin x$

2. $\frac{dy}{dx} = \cos x$

3. $\frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$

4. $\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

5. $\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x$

6. $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x - 1 = \ln x$

7. $\frac{dy}{dx} = \frac{(n+1)x^n}{n+1} = x^n$

8. $\frac{dy}{dx} = -\frac{1}{(2^x+1)^2} \cdot (\ln 2)2^x = -\frac{2^x \ln 2}{(2^x+1)^2}$

9. $\frac{dy}{dx} = xe^x + e^x$

10. $\frac{dy}{dx} = \frac{1}{x^2+1}$

Section 5.3 Exercises

1. (a) $\int_2^2 g(x)\,dx = 0$

(b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3(-4) = -12$

(d) $\int_2^5 f(x)\,dx = \int_2^1 f(x)\,dx + \int_1^5 f(x)\,dx$
$= -\int_1^2 f(x)\,dx + \int_1^5 f(x)\,dx$
$= 4 + 6 = 10$

(e) $\int_1^5 [f(x) - g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$
$= 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)]\,dx = \int_1^5 4f(x)\,dx - \int_1^5 g(x)\,dx$
$= 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$
$= 4(6) - 8 = 16$

2. (a) $\int_1^9 -2f(x)\,dx = -2\int_1^9 f(x)\,dx = -2(-1) = 2$

(b) $\int_7^9 [f(x) + h(x)]\,dx = \int_7^9 f(x)\,dx + \int_7^9 h(x)\,dx$
$= 5 + 4 = 9$

(c) $\int_7^9 [2f(x) - 3h(x)]\,dx = \int_7^9 2f(x)\,dx - \int_7^9 3h(x)\,dx$
$= 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)\,dx$
$= 2(5) - 3(4) = -2$

(d) $\int_9^1 f(x)\,dx = -\int_1^9 f(x)\,dx = 1$

(e) $\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx + \int_9^7 f(x)\,dx$
$= \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx$
$= -1 - 5 = -6$

(f) $\int_9^7 [h(x) - f(x)]\,dx = \int_9^7 h(x)\,dx - \int_9^7 f(x)\,dx$
$= -\int_7^9 h(x)\,dx + \int_7^9 f(x)\,dx$
$= -4 + 5 = 1$

3. (a) $\int_1^2 f(u)\,du = 5$

(b) $\int_1^2 \sqrt{3}f(z)\,dz = \sqrt{3}\int_1^2 f(z)\,dz = 5\sqrt{3}$

(c) $\int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt = -5$

(d) $\int_1^2 [-f(x)]\,dx = -\int_1^2 f(x)\,dx = -5$

4. (a) $\int_0^{-3} g(t)\,dt = -\int_{-3}^0 g(t)\,dt = -\sqrt{2}$

(b) $\int_{-3}^0 g(u)\,du = \sqrt{2}$

(c) $\int_{-3}^0 [-g(x)]\,dx = -\int_{-3}^0 g(x)\,dx = -\sqrt{2}$

(d) $\int_{-3}^0 \frac{g(r)}{\sqrt{2}}\,dr = \frac{1}{\sqrt{2}}\int_{-3}^0 g(r)\,dr = 1$

5. (a) $\int_3^4 f(z)\,dz = \int_3^0 f(z)\,dz + \int_0^4 f(z)\,dz$
$= -\int_0^3 f(z)\,dz + \int_0^4 f(z)\,dz$
$= -3 + 7 = 4$

(b) $\int_4^3 f(t)\,dt = \int_4^0 f(t)\,dt + \int_0^3 f(t)\,dt$
$= -\int_0^4 f(t)\,dt + \int_0^3 f(t)\,dt$
$= -7 + 3 = -4$

6. (a) $\int_1^3 h(r)\,dr = \int_1^{-1} h(r)\,dr + \int_{-1}^3 h(r)\,dr$
$= -\int_{-1}^1 h(r)\,dr + \int_{-1}^3 h(r)\,dr = 6$

(b) $-\int_3^1 h(u)\,du = -\int_3^{-1} h(u)\,du - \int_{-1}^1 h(u)\,du$
$= \int_{-1}^3 h(u)\,du - \int_{-1}^1 h(u)\,du = 6$

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7. An antiderivative of 7 is $F(x) = 7x$.

$$\int_3^1 7\,dx = F(1) - F(3) = 7 - 21 = -14$$

8. An antiderivative of $5x$ is $F(x) = \frac{5}{2}x^2$

$$\int_0^2 5x\,dx = F(2) - F(0) = 10 - 0 = 10$$

9. An antiderivative of $\frac{x}{8}$ is $F(x) = \frac{x^2}{16}$.

$$\int_3^5 \frac{x}{8}\,dx = F(5) - F(3) = \frac{25}{16} - \frac{9}{16} = 1$$

10. An antiderivative of $2t - 3$ is $F(t) = t^2 - 3t$.

$$\int_0^2 (2t - 3)\,dt = F(2) - F(0) = -2 - 0 = -2$$

11. An antiderivative of $t - \sqrt{2}$ is $F(t) = \frac{1}{2}t^2 - t\sqrt{2}$.

$$\int_0^{\sqrt{2}} (t - \sqrt{2})\,dt = F(\sqrt{2}) - F(0) = -1 - 0 = -1$$

12. An antiderivative of $1 + \frac{z}{2}$ is $F(z) = z + \frac{z^2}{4}$.

$$\int_2^1 \left(1 + \frac{z}{2}\right) dz = F(1) - F(2) = \frac{5}{4} - 3 = -\frac{7}{4}$$

13. An antiderivative of $\frac{1}{1 + x^2}$ is $F(x) = \tan^{-1} x$.

$$\int_{-1}^1 \frac{1}{1 + x^2}\,dx = F(1) - F(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

14. An antiderivative of $\frac{1}{\sqrt{1 - x^2}}$ is $F(x) = \sin^{-1} x$.

$$\int_{-1/2}^{1/2} \frac{dx}{\sqrt{1 - x^2}} = F\left(\frac{1}{2}\right) - F\left(-\frac{1}{2}\right) = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

15. An antiderivative of e^x is $F(x) = e^x$.

$$\int_0^2 e^x\,dx = F(2) - F(0) = e^2 - 1 \approx 6.389$$

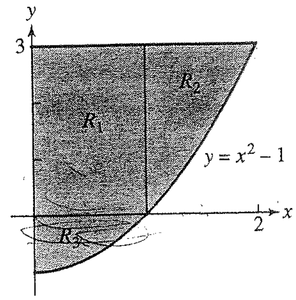
16. An antiderivative of $\frac{3}{x + 1}$ is $F(x) = 3 \ln |x + 1|$.

$$\int_0^3 \frac{3\,dx}{x + 1} = F(3) - F(0)$$

$$= 3 \ln 4 - 0$$

$$= 3 \ln 4 \approx 4.159$$

17. Divide the shaded area as follows.



Note that an antiderivative of $x^2 - 1$ is $F(x) = \frac{1}{3}x^3 - x$.

Area of $R_1 = 3(1) = 3$

$$\text{Area of } R_2 = (3)(1) - \int_1^2 (x^2 - 1)\,dx$$

$$= 3 - [F(2) - F(1)]$$

$$= 3 - \left[\left(\frac{2}{3}\right) - \left(-\frac{2}{3}\right)\right] = \frac{5}{3}$$

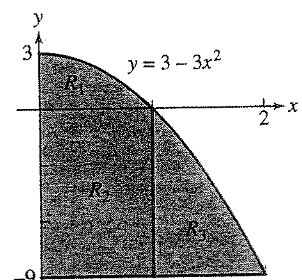
$$\text{Area of } R_3 = -\int_0^1 (x^2 - 1)\,dx$$

$$= -[F(1) - F(0)]$$

$$= -\left[\left(-\frac{2}{3}\right) - 0\right] = \frac{2}{3}$$

Total shaded area $= 3 + \frac{5}{3} + \frac{2}{3} = \frac{16}{3}$

18. Divide the shaded area as follows.



Note that an antiderivative of $3 - 3x^2$ is $F(x) = 3x - x^3$.

$$\text{Area of } R_1 = \int_0^1 (3 - 3x^2)\,dx = F(1) - F(0) = 2$$

Area of $R_2 = (9)(1) = 9$

$$\text{Area of } R_3 = (9)(1) + \int_1^2 (3 - 3x^2)\,dx$$

$$= 9 + [F(2) - F(1)]$$

$$= 9 + (-2 - 2) = 5$$

Total shaded area $= 2 + 9 + 5 = 16$

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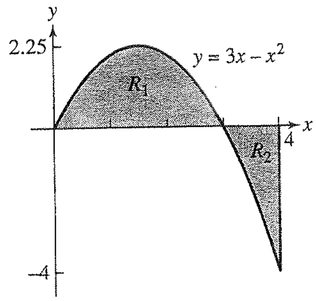
19. Divide the shaded area as follows.



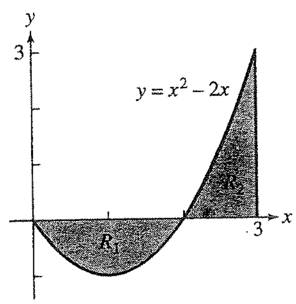
Note that an antiderivative of $3x - x^2$ is $F(x) = \frac{3}{2}x^2 - \frac{1}{3}x^3$.

$$\text{Area of } R_1 = \int_0^3 (3x - x^2)\,dx = F(3) - F(0) = \frac{9}{2} - 0 = \frac{9}{2}$$

$$\begin{aligned}\text{Area of } R_2 &= -\int_3^4 (3x - x^2)\,dx \\ &= -[F(4) - F(3)] \\ &= -\left(\frac{8}{3} - \frac{9}{2}\right) = \frac{11}{6}\end{aligned}$$

$$\text{Total shaded area} = \frac{9}{2} + \frac{11}{6} = \frac{19}{3}$$

20. Divide the shaded area as follows.



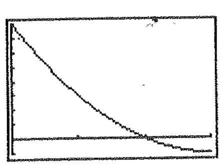
Note that an antiderivative of $x^2 - 2x$ is $F(x) = \frac{1}{3}x^3 - x^2$.

$$\begin{aligned}\text{Area of } R_1 &= -\int_0^2 (x^2 - 2x)\,dx \\ &= -[F(2) - F(0)] \\ &= -\left(-\frac{4}{3} - 0\right) = \frac{4}{3}\end{aligned}$$

$$\begin{aligned}\text{Area of } R_2 &= \int_2^3 (x^2 - 2x)\,dx \\ &= F(3) - F(2) \\ &= 0 - \left(-\frac{4}{3}\right) = \frac{4}{3}\end{aligned}$$

$$\text{Total shaded area} = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$$

21.



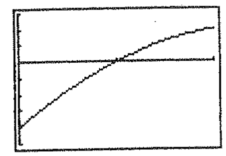
[0, 3] by [−1, 8]

An antiderivative of $x^2 - 6x + 8$ is $F(x) = \frac{1}{3}x^3 - 3x^2 + 8x$.

(a) $\int_0^3 (x^2 - 6x + 8)\,dx = F(3) - F(0) = 6 - 0 = 6$

(b) $$\begin{aligned}\text{Area} &= \int_0^2 (x^2 - 6x + 8)\,dx - \int_2^3 (x^2 - 6x + 8)\,dx \\ &= [F(2) - F(0)] - [F(3) - F(2)] \\ &= \left(\frac{20}{3} - 0\right) - \left(6 - \frac{20}{3}\right) = \frac{22}{3}\end{aligned}$$

22.



[0, 2] by [−5, 3]

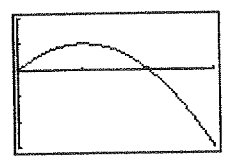
An antiderivative of $-x^2 + 5x - 4$ is

$F(x) = -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x.$

(a) $\int_0^2 (-x^2 + 5x - 4)\,dx = F(2) - F(0) = -\frac{2}{3} - 0 =$

(b) $$\begin{aligned}\text{Area} &= -\int_0^1 (-x^2 + 5x - 4)\,dx + \int_1^2 (-x^2 + 5x - \\ &= -[F(1) - F(0)] + [F(2) - F(1)] \\ &= -\left(-\frac{11}{6} - 0\right) + \left[-\frac{2}{3} - \left(-\frac{11}{6}\right)\right] = 3\end{aligned}$$

23.



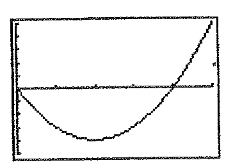
[0, 3] by [−3, 2]

An antiderivative of $2x - x^2$ is $F(x) = x^2 - \frac{1}{3}x^3$.

(a) $\int_0^3 (2x - x^2)\,dx = F(3) - F(0) = 0 - 0 = 0$

(b) $$\begin{aligned}\text{Area} &= \int_0^2 (2x - x^2)\,dx - \int_2^3 (2x - x^2)\,dx \\ &= [F(2) - F(0)] - [F(3) - F(2)] \\ &= \left(\frac{4}{3} - 0\right) - \left(0 - \frac{4}{3}\right) = \frac{8}{3}\end{aligned}$$

24.



[0, 5] by [−5, 5]

An antiderivative of $x^2 - 4x$ is $F(x) = \frac{1}{3}x^3 - 2x^2$.

(a) $\int_0^5 (x^2 - 4x)\,dx = F(5) - F(0) = -\frac{25}{3} - 0 = -\frac{25}{3}$

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21. **(a)** No, $f(x) = \tan x$ is discontinuous at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

(b) The integral does not have a value. If $0 < b < \frac{\pi}{2}$, then $\int_0^b \tan x\, dx = \left[-\ln|\cos x|\right]_0^b = -\ln|\cos b|$ since the Fundamental Theorem applies for $[0, b]$. As $b \to \frac{\pi}{2}^-$, $\cos b \to 0^+$ so $-\ln|\cos b| \to \infty$ or $\int_0^b \tan x\, dx \to \infty$.

Hence the integral does not exist over a subinterval of $[0, 2\pi]$, so it doesn't exist over $[0, 2\pi]$.

22. **(a)** No, $f(x) = \frac{x+1}{x^2-1}$ is discontinuous at $x = 1$.

(b) The integral does not have a value. If $0 < b < 1$, then $\int_0^b \frac{x+1}{x^2-1}\, dx = \int_0^b \frac{1}{x-1}\, dx = \left[\ln|x-1|\right]_0^b = \ln|b-1|$, since $\frac{x+1}{x^2-1} = \frac{1}{x-1}$ and the Fundamental Theorem applies for $[0, b]$. As $b \to 1^-$, $\ln|b-1| \to -\infty$ or $\int_0^b \frac{x+1}{x^2-1}\, dx \to -\infty$. Hence the integral does not exist over a subinterval of $[0, 2]$, so it does not exist over $[0, 2]$.

23. **(a)** No, $f(x) = \frac{\sin x}{x}$ is discontinuous at $x = 0$.

(b) $\text{NINT}\left(\frac{\sin x}{x}, x, -1, 2\right) \approx 2.55$. The integral exists since the area is finite because $\frac{\sin x}{x}$ is bounded.

24. **(a)** No, $f(x) = \frac{1-\cos x}{x^2}$ is discontinuous at $x = 0$.

(b) $\text{NINT}\left(\frac{1-\cos x}{x^2}, x, -2, 3\right) \approx 2.08$. The integral exists since the area is finite because $\frac{1-\cos x}{x^2}$ is bounded.

25. First, find the area under the graph of $y = x^2$.

$$\int_0^1 x^2\, dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$$

Next find the area under the graph of $y = 2 - x$.

$$\int_1^2 (2 - x)\, dx = \left[2x - \frac{1}{2}x^2\right]_1^2 = 2 - \frac{3}{2} = \frac{1}{2}$$

Area of the shaded region $= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

26. First find the area under the graph of $y = \sqrt{x}$.

$$\int_0^1 x^{1/2}\, dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$$

Next find the area under the graph of $y = x^2$.

$$\int_1^2 x^2\, dx = \left[\frac{1}{3}x^3\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Area of the shaded region $= \frac{2}{3} + \frac{7}{3} = 3$

27. First find the area under the graph of $y = 1 + \cos x$.

$$\int_0^\pi (1 + \cos x)\, dx = \left[x + \sin x\right]_0^\pi = \pi$$

The area of the rectangle is 2π.

Area of the shaded region $= 2\pi - \pi = \pi$.

28. First, find the area of the region between $y = \sin x$ and the x-axis for $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

$$\int_{\pi/6}^{5\pi/6} \sin x\, dx = \left[-\cos x\right]_{\pi/6}^{5\pi/6} = \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

The area of the rectangle is $\left(\sin \frac{\pi}{6}\right)\left(\frac{2\pi}{3}\right) = \frac{\pi}{3}$

Area of the shaded region $= \sqrt{3} - \frac{\pi}{3}$

29. $\text{NINT}\left(\frac{1}{3 + 2\sin x}, x, 0, 10\right) \approx 3.802$

30. $\text{NINT}\left(\frac{2x^4 - 1}{x^4 - 1}, x, -0.8, 0.8\right) \approx 1.427$

31. $\frac{1}{2}\text{NINT}(\sqrt{\cos x}, x, -1, 1) \approx 0.914$

32. $\sqrt{8 - 2x^2} \geq 0$ between $x = -2$ and $x = 2$

$\text{NINT}(\sqrt{8 - 2x^2}, x, -2, 2) \approx 8.886$

33. Plot $y_1 = \text{NINT}(e^{-t^2}, t, 0, x)$, $y_2 = 0.6$ in a $[0, 1]$ by $[0,1]$ window, then use the intersect function to find $x \approx 0.699$.

34. When $y = 0$, $x = 1$.

$y^3 = 1 - x^3$

$y = \sqrt[3]{1 - x^3}$

$\text{NINT}(\sqrt[3]{1 - x^3}, x, 0, 1) \approx 0.883$

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12. (a) $\int_0^{10} x^3\,dx$

(b) $\int_0^{10} x \sin x\,dx$

(c) $\int_0^{10} x(3x-2)^2\,dx$

(d) $\int_0^{10} \frac{1}{1+x^2}\,dx$

(e) $\int_0^{10} \pi\left(9 - \sin^2 \frac{\pi x}{10}\right) dx$

13. The graph is above the x-axis for $0 \le x < 4$ and below the x-axis for $4 < x \le 6$

$$\text{Total area} = \int_0^4 (4-x)\,dx - \int_4^6 (4-x)\,dx$$
$$= \left[4x - \frac{1}{2}x^2\right]_0^4 - \left[4x - \frac{1}{2}x^2\right]_4^6$$
$$= [8-0] - [6-8] = 10$$

14. The graph is above the x-axis for $0 \le x < \frac{\pi}{2}$ and below the x-axis for $\frac{\pi}{2} < x \le \pi$

$$\text{Total area} = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx$$
$$= \left[\sin x\right]_0^{\pi/2} - \left[\sin x\right]_{\pi/2}^{\pi}$$
$$= (1-0) - (0-1) = 2$$

15. $\int_{-2}^{2} 5\,dx = \left[5x\right]_{-2}^{2} = 10 - (-10) = 20$

16. $\int_2^5 4x\,dx = \left[2x^2\right]_2^5 = 50 - 8 = 42$

17. $\int_0^{\pi/4} \cos x\,dx = \left[\sin x\right]_0^{\pi/4} = \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$

18. $\int_{-1}^{1} (3x^2 - 4x + 7)\,dx = \left[x^3 - 2x^2 + 7x\right]_{-1}^{1}$
$$= 6 - (-10) = 16$$

19. $\int_0^1 (8s^3 - 12s^2 + 5)\,ds = \left[2s^4 - 4s^3 + 5s\right]_0^1 = 3 - 0 = 3$

20. $\int_1^2 \frac{4}{x^2}\,dx = \left[-\frac{4}{x}\right]_1^2 = -2 - (-4) = 2$

21. $\int_1^{27} y^{-4/3}\,dy = \left[-3y^{-1/3}\right]_1^{27} = -1 - (-3) = 2$

22. $\int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 t^{-3/2}\,dt = \left[-2t^{-1/2}\right]_1^4 = -1 - (-2) = 1$

23. $\int_0^{\pi/3} \sec^2\theta\,d\theta = \left[\tan\theta\right]_0^{\pi/3} = \sqrt{3} - 0 = \sqrt{3}$

24. $\int_1^e \frac{1}{x}\,dx = \left[\ln|x|\right]_1^e = 1 - 0 = 1$

25. $\int_0^1 \frac{36}{(2x+1)^3}\,dx = \int_0^1 36(2x+1)^{-3}\,dx$
$$= \left[-9(2x+1)^{-2}\right]_0^1$$
$$= -1 - (-9) = 8$$

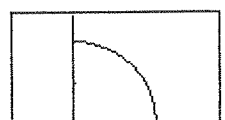
26. $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \int_1^2 (x + x^{-2})\,dx$
$$= \left[\frac{1}{2}x^2 - x^{-1}\right]_1^2$$
$$= \frac{3}{2} - \left(-\frac{1}{2}\right) = 2$$

27. $\int_{-\pi/3}^{0} \sec x \tan x\,dx = \left[\sec x\right]_{-\pi/3}^{0} = 1 - 2 = -1$

28. $\int_{-1}^{1} 2x \sin(1-x^2)\,dx = \left[\cos(1-x^2)\right]_{-1}^{1} = 1 - 1 = 0$

29. $\int_0^2 \frac{2}{y+1}\,dy = \left[2\ln|y+1|\right]_0^2 = 2\ln 3 - 0 = 2\ln 3$

30. Graph $y = \sqrt{4-x^2}$ on $[0, 2]$.

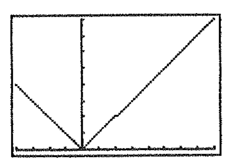


$[-1.35, 3.35]$ by $[-0.5, 2.6]$

The region under the curve is a quarter of a circle of radius 2.

$$\int_0^2 \sqrt{4-x^2}\,dx = \frac{1}{4}\pi(2)^2 = \pi$$

31. Graph $y = |x|\,dx$ on $[-4, 8]$.

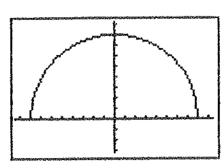


$[-4, 8]$ by $[0, 8]$

The region under the curve consists of two triangles.

$$\int_{-4}^{8} |x|\,dx = \frac{1}{2}(4)(4) + \frac{1}{2}(8)(8) = 40$$

32. Graph $y = \sqrt{64-x^2}$ on $[-8, 8]$.



$[-9.4, 9.4]$ by $[-3.2, 9.2]$

The region under the curve $y = \sqrt{64-x^2}$ is half a circle of radius 8.

$$\int_{-8}^{8} 2\sqrt{64-x^2}\,dx = 2\int_{-8}^{8} \sqrt{64-x^2}\,dx = 2\left[\frac{1}{2}\pi(8)^2\right] = 64\pi$$