19. Let $u = e^{3x}$ $\quad dv = \sin x\,dx$

$du = 3e^{3x}\,dx$ $\quad v = -\cos x$

$$\int e^{3x}\sin x\,dx = -e^{3x}\cos x + \int 3\cos x\,e^{3x}\,dx$$

Integrate by parts again.

Let $u = 3e^{3x}$ $\quad dv = \cos x\,dx$

$du = 9e^{3x}\,dx$ $\quad v = \sin x$

$$\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x - \int 9e^{3x}\sin x\,dx$$

$$10\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x + C$$

$$\int e^{3x}\sin x\,dx = \frac{1}{10}[-e^{3x}\cos x + 3e^{3x}\sin x] + C$$

$$= \left(\frac{3\sin x}{10} - \frac{\cos x}{10}\right)e^{3x} + C$$

20. Let $u = x^2$ $\quad dv = e^{-3x}\,dx$

$du = 2x\,dx$ $\quad v = -\frac{1}{3}e^{-3x}$

$$\int x^2e^{-3x}\,dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int e^{-3x}x\,dx$$

Let $u = x$ $\quad dv = e^{-3x}\,dx$

$du = dx$ $\quad v = -\frac{1}{3}e^{-3x}$

$$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx\right]$$

$$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}\,dx$$

$$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$$

$$= \left(-\frac{x^2}{3} - \frac{2x}{9} - \frac{2}{27}\right)e^{-3x} + C$$

21. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}$

$$dy = \left(1 + x + \frac{x^2}{2}\right)dx$$

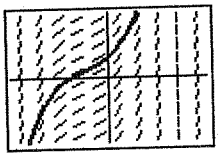
$$\int dy = \int\left(1 + x + \frac{x^2}{2}\right)dx$$

$$y = x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + C$$

$y(0) = C = 1$

$$y = \frac{x^3}{6} + \frac{x^2}{2} + x + 1$$

Graphical support:



[−4, 4] by [−3, 3]

22. $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$

$$dy = \left(x + \frac{1}{x}\right)^2 dx$$

$$\int dy = \int\left(x + \frac{1}{x}\right)^2 dx$$

$$y = \int\left(x^2 + 2 + \frac{1}{x^2}\right)dx$$

$$y = \frac{1}{3}x^3 + 2x - x^{-1} + C$$

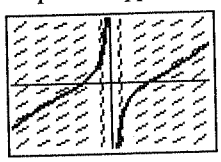
$$y(1) = \frac{1}{3} + 2 - 1 + C = 1$$

$$\frac{4}{3} + C = 1$$

$$C = -\frac{1}{3}$$

$$y = \frac{x^3}{3} + 2x - \frac{1}{x} - \frac{1}{3}$$

Graphical support:



[−2, 2] by [−10, 10]

23. $\frac{dy}{dt} = \frac{1}{t+4}$

$$dy = \frac{1}{t+4}\,dt$$

$$\int dy = \int\frac{1}{t+4}\,dt$$

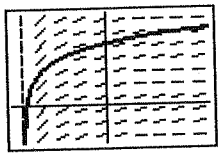
$$y = \ln|t+4| + C$$

$$y(-3) = \ln(1) + C = 2$$

$$C = 2$$

$$y = \ln(t+4) + 2$$

Graphical Support:



[−4.5, 5] by [−2, 5]

24. $\frac{dy}{d\theta} = \csc 2\theta\cot 2\theta$

$$dy = \csc 2\theta\cot 2\theta\,d\theta$$

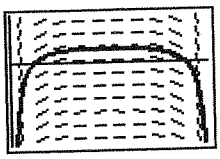
$$\int dy = \int\csc 2\theta\cot 2\theta\,d\theta$$

$$y = -\frac{1}{2}\csc 2\theta + C$$

$$y\left(\frac{\pi}{4}\right) = -\frac{1}{2} + C = 1$$

$$C = \frac{3}{2}$$

$$y = -\frac{1}{2}\csc 2\theta + \frac{3}{2}$$



[0, 1.57] by [−5, 3]