CHAPTER 5

# Linear Functions



**Chapter Focus**

- Translate among different representations of linear functions.
- Find and interpret slopes and intercepts of linear equations that model real-world problems
- Solve real-world problems involving linear equations.

## Take *Flight*

You can use linear functions to describe patterns and relationships in flight times.

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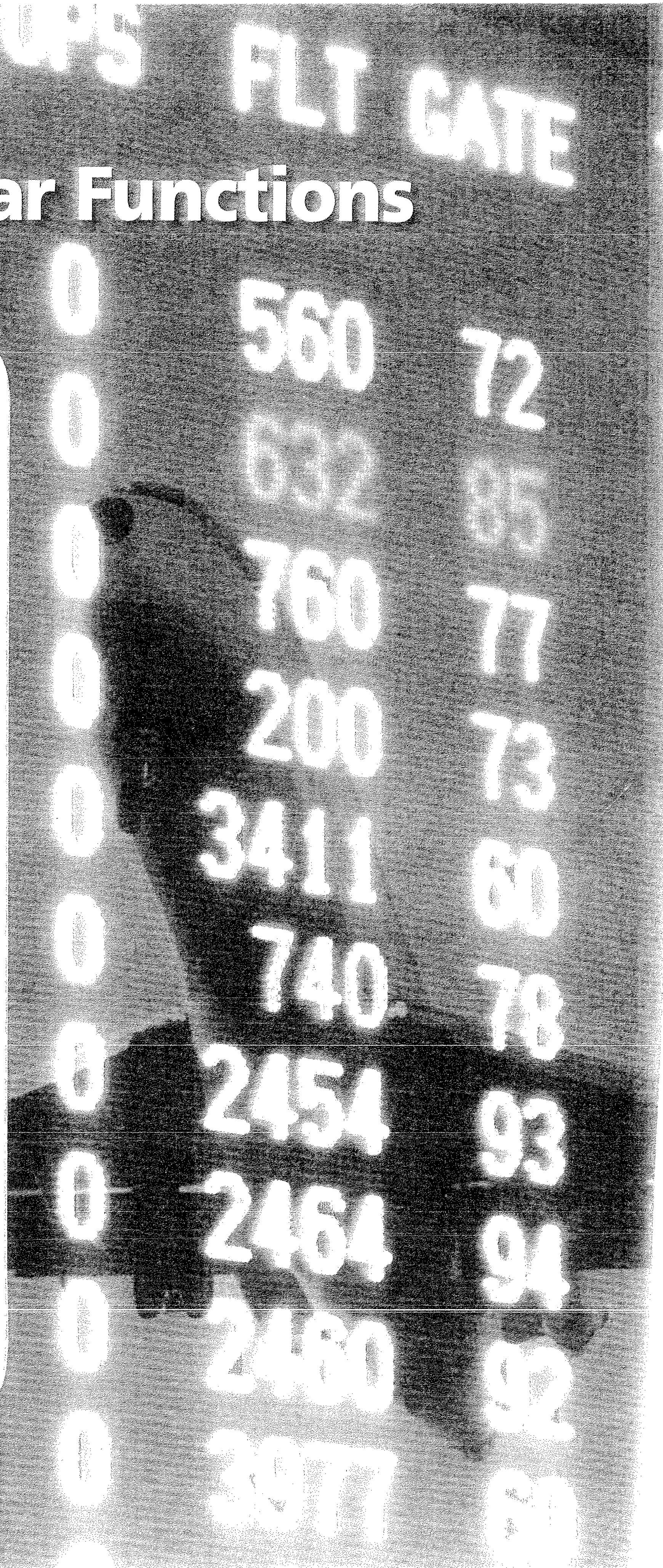
KEYWORD: MA7 ChProj



# ARE YOU READY?

## Vocabulary

**Match each term on the left with a definition on the right.**

1. coefficient
2. coordinate plane
3. transformation
4. perpendicular

A. a change in the size or position of a figure
B. forming right angles
C. a two-dimensional system formed by the intersection of a horizontal number line and a vertical number line
D. an ordered pair of numbers that gives the location of a point
E. a number that is multiplied by a variable

## Ordered Pairs

**Graph each point on the same coordinate plane.**

5. $A(2, 5)$
6. $B(-1, -3)$
7. $C(-5, 2)$
8. $D(4, -4)$
9. $E(-2, 0)$
10. $F(0, 3)$
11. $G(8, 7)$
12. $H(-8, -7)$

## Solve for a Variable

**Solve each equation for the indicated variable.**

13. $2x + y = 8$; $y$
14. $5y = 5x - 10$; $y$
15. $2y = 6x - 8$; $y$
16. $10x + 25 = 5y$; $y$

## Evaluate Expressions

**Evaluate each expression for the given value of the variable.**

17. $4g - 3$; $g = -2$
18. $8p - 12$; $p = 4$
19. $4x + 8$; $x = -2$
20. $-5t - 15$; $t = 1$

## Connect Words and Algebra

21. The value of a stock begins at \$0.05 and increases by \$0.01 each month. Write an equation representing the value of the stock $v$ in any month $m$.
22. Write a situation that could be modeled by the equation $b = 100 - s$.

## Rates and Unit Rates

**Find each unit rate.**

23. 322 miles on 14 gallons of gas
24. \$14.25 for 3 pounds of deli meat
25. 32 grams of fat in 4 servings
26. 120 pictures on 5 rolls of film

CHAPTER 5

# Study Guide: Preview

## Where You've Been

wrote equations in function notation.

graphed functions.

identified the domain and range of functions.

identified independent and dependent variables.

## In This Chapter

### You will study

- writing and graphing linear functions.
- identifying and interpreting the components of linear graphs, including the $x$-intercept, $y$-intercept, and slope.
- graphing and analyzing families of functions.

## Where You're Going

to solve systems of linear equations in Chapter 6.

to identify rates of change in linear data in biology and economics.

to make calculations and comparisons in your personal finances.

## Key Vocabulary/Vocabulario

| | |
|---|---|
| constant of variation | constante de variación |
| direct variation | variación directa |
| family of functions | familia de funciones |
| linear function | función lineal |
| parallel lines | líneas paralelas |
| perpendicular lines | líneas perpendiculares |
| slope | pendiente |
| transformation | transformación |
| $x$-intercept | intersección con el eje $x$ |
| $y$-intercept | intersección con el eje $y$ |

## Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What shape do you think is formed when a **linear function** is graphed on a coordinate plane?
2. The meaning of *intercept* is similar to the meaning of *intersection*. What do you think an **$x$-intercept** might be?
3. **Slope** is a word used in everyday life, as well as in mathematics. What is your understanding of the word *slope*?
4. A family is a group of related people. Use this concept to define **family of functions**.

# Reading and Writing Math

## Study Strategy: Use Multiple Representations

Representing a math concept in more than one way can help you understand it more clearly. As you read the explanations and example problems in your text, note the use of tables, lists, graphs, diagrams, and symbols, as well as words to explain a concept.

**From Lesson 4-4:**

In this example from Chapter 4, the given function is described using an equation, a table, ordered pairs, and a graph.

### Graphing Functions

**Graph each function.**

**A** $2x + 1 = y$ — Equation

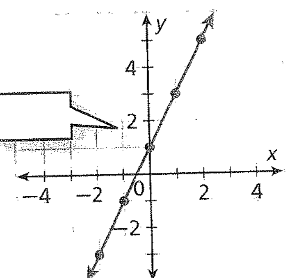
**Step 1** Choose several values of $x$ and generate ordered pairs. — Table

| $x$ | $2x + 1 = y$ | $(x, y)$ |
|---|---|---|
| $-3$ | $2(-3) + 1 = -5$ | $(-3, -5)$ |
| $-2$ | $2(-2) + 1 = -3$ | $(-2, -3)$ |
| $-1$ | $2(-1) + 1 = -1$ | $(-1, -1)$ |
| $0$ | $2(0) + 1 = 1$ | $(0, 1)$ |
| $1$ | $2(1) + 1 = 3$ | $(1, 3)$ |
| $2$ | $2(2) + 1 = 5$ | $(2, 5)$ |
| $3$ | $2(3) + 1 = 7$ | $(3, 7)$ |

Ordered Pairs

**Step 2** Plot enough points to see a pattern.

Graph

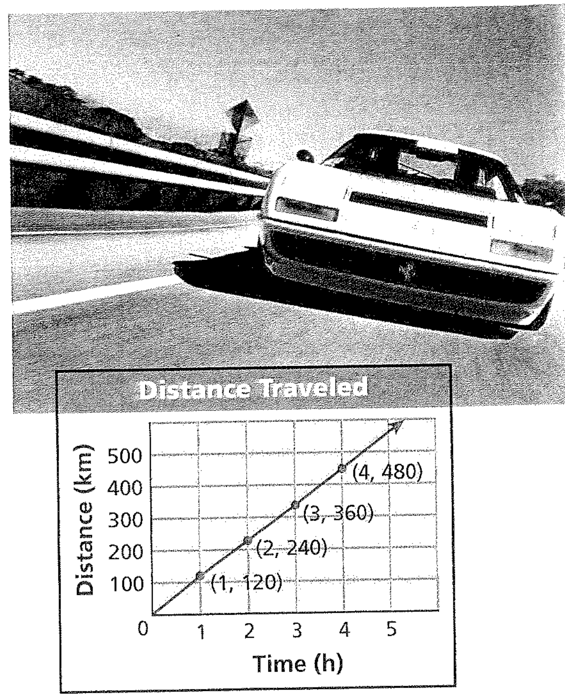



**Step 3** The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.

### Try This

1. If an employee earns $8.00 an hour, $y = 8x$ gives the total pay $y$ the employee will earn for working $x$ hours. For this equation, make a table of ordered pairs and a graph. Explain the relationships between the equation, the table, and the graph. How does each one describe the situation?
2. What situations might make one representation more useful than another?

# 5-1 Identifying Linear Functions




***Objectives***
Identify linear functions and linear equations.

Graph linear functions that represent real-world situations and give their domain and range.

***Vocabulary***
linear function
linear equation

**Why learn this?**
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.

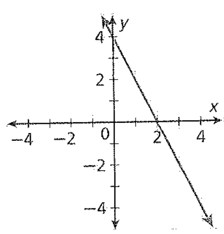
Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers $y$ that the car would travel in $x$ hours. Solutions are shown in the graph.

The graph represents a function because each domain value ($x$-value) is paired with exactly one range value ($y$-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.

## EXAMPLE 1 Identifying a Linear Function by Its Graph

**Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

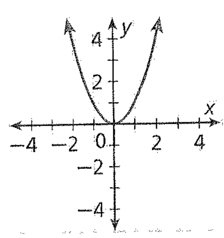
**A**



*Each domain value is paired with exactly one range value. The graph forms a line.*

linear function

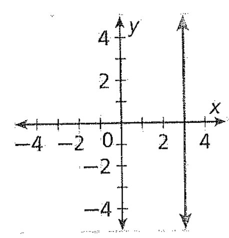
**B**



*Each domain value is paired with exactly one range value. The graph is not a line.*

not a linear function

**C**

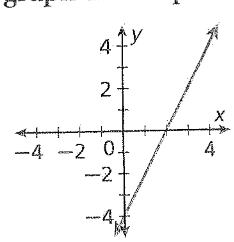


*The only domain value, 3, is paired with many different range values.*

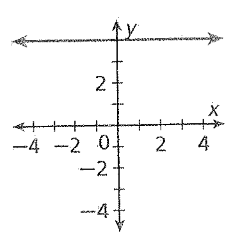
not a function

**CHECK IT OUT!** **Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

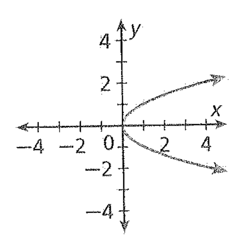
**1a.**



**1b.**



**1c.**



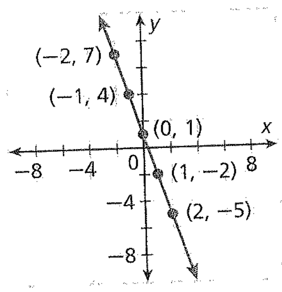
You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in $x$ corresponds to a constant change in $y$.

> **Caution!**
>
> If you find a constant change in the $y$-values, check for a constant change in the $x$-values. Both need to be constant for the function to be linear.

| $x$ | $y$ |
|---|---|
| −2 | 7 |
| −1 | 4 |
| 0 | 1 |
| 1 | −2 |
| 2 | −5 |

In this table, a constant change of $+1$ in $x$ corresponds to a constant change of $-3$ in $y$. These points satisfy a linear function.

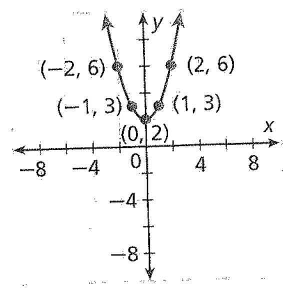
*The points from this table lie on a line.*




| $x$ | $y$ |
|---|---|
| −2 | 6 |
| −1 | 3 |
| 0 | 2 |
| 1 | 3 |
| 2 | 6 |

In this table, a constant change of $+1$ in $x$ does *not* correspond to a constant change in $y$. These points do *not* satisfy a linear function.

*The points from this table do not lie on a line.*




## EXAMPLE 2 Identifying a Linear Function by Using Ordered Pairs

**Tell whether each set of ordered pairs satisfies a linear function. Explain.**

**A** $\{(2, 4), (5, 3), (8, 2), (11, 1)\}$

| $x$ | $y$ |
|---|---|
| 2 | 4 |
| 5 | 3 |
| 8 | 2 |
| 11 | 1 |

*Write the ordered pairs in a table. Look for a pattern.*

*A constant change of +3 in x corresponds to a constant change of −1 in y.*

These points satisfy a linear function.

**B** $\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}$

| $x$ | $y$ |
|---|---|
| −10 | 10 |
| −5 | 4 |
| 0 | 2 |
| 5 | 0 |

*Write the ordered pairs in a table. Look for a pattern.*

*A constant change of +5 in x corresponds to different changes in y.*

These points do not satisfy a linear function.

**CHECK IT OUT!** 2. Tell whether the set of ordered pairs $\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}$ satisfies a linear function. Explain.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation*. A **linear equation** is any equation that can be written in the *standard form* shown below.

Know it! Note

**Standard Form of a Linear Equation**

$Ax + By = C$ where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both 0

Notice that when a linear equation is written in standard form

- $x$ and $y$ both have exponents of 1.
- $x$ and $y$ are not multiplied together.
- $x$ and $y$ do not appear in denominators, exponents, or radical signs.

| Linear | | Not Linear | |
|---|---|---|---|
| $3x + 2y = 10$ | *Standard form* | $3xy + x = 1$ | *x and y are multiplied.* |
| $y - 2 = 3x$ | *Can be written as* $3x - y = -2$ | $x^3 + y = -1$ | *x has an exponent other than 1.* |
| $-y = 5x$ | *Can be written as* $5x + y = 0$ | $x + \frac{6}{y} = 12$ | *y is in a denominator.* |

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

## EXAMPLE 3 Graphing Linear Functions

**Tell whether each function is linear. If so, graph the function.**

**A** $y = x + 3$

$$\begin{aligned} y &= x + 3 && \text{Write the equation in standard form.} \\ -x & \quad -x && \text{Subtraction Property of Equality} \\ y - x &= 3 \\ -x + y &= 3 && \text{The equation is in standard form } (A = -1, B = 1, C = 3). \end{aligned}$$

**Remember!**

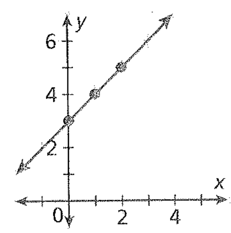
- $y - x = y + (-x)$
- $y + (-x) = -x + y$
- $-x = -1x$
- $y = 1y$

The equation can be written in standard form, so the function is linear.

To graph, choose three values of $x$, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

| $x$ | $y = x + 3$ | $(x, y)$ |
|---|---|---|
| 0 | $y = 0 + 3 = 3$ | $(0, 3)$ |
| 1 | $y = 1 + 3 = 4$ | $(1, 4)$ |
| 2 | $y = 2 + 3 = 5$ | $(2, 5)$ |

Plot the points and connect them with a straight line.




**B** $y = x^2$

This is not linear, because $x$ has an exponent other than 1.

CHECK IT OUT! **Tell whether each function is linear. If so, graph the function.**

**3a.** $y = 5x - 9$ **3b.** $y = 12$ **3c.** $y = 2^x$

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

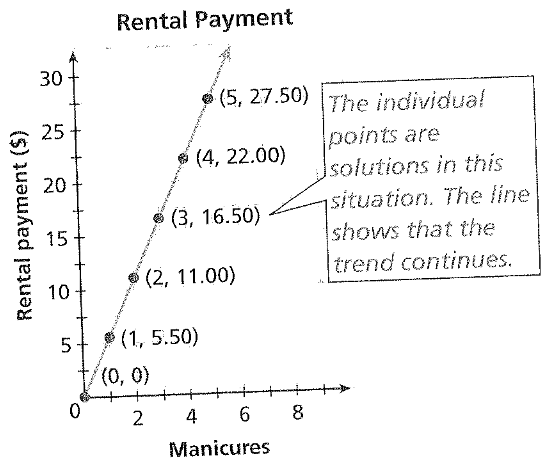
**EXAMPLE 4** ***Career Application***

**Sue rents a manicure station in a salon and pays the salon owner \$5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where $x$ is the number of manicures. Graph this function and give its domain and range.**

*Choose several values of x and make a table of ordered pairs.*

| $x$ | $f(x) = 5.50x$ |
|---|---|
| 0 | $f(0) = 5.50(0) = 0$ |
| 1 | $f(1) = 5.50(1) = 5.50$ |
| 2 | $f(2) = 5.50(2) = 11.00$ |
| 3 | $f(3) = 5.50(3) = 16.50$ |
| 4 | $f(4) = 5.50(4) = 22.00$ |
| 5 | $f(5) = 5.50(5) = 27.50$ |

*Graph the ordered pairs.*




The number of manicures must be a whole number, so the domain is $\{0, 1, 2, 3, \ldots\}$. The range is $\{0, 5.50, 11.00, 16.50, \ldots\}$.

**Remember!**

$f(x) = y$, so in Example 4, graph the function values (dependent variable) on the $y$-axis.

**CHECK IT OUT!** 4. **What if...?** At another salon, Sue can rent a station for \$10.00 per day plus \$3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where $x$ is the number of manicures. Graph this function and give its domain and range.

## THINK AND DISCUSS

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?
2. In Example 4, why is every point on the line not a solution?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.

**Know it! Note**

Determining Whether a Function Is Linear

| From its graph | From its equation | From a list of ordered pairs |
|---|---|---|
| | | |

# 5-1 Exercises

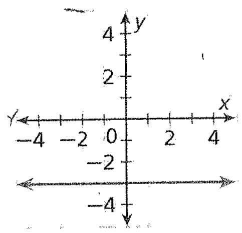


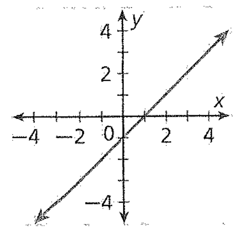
## GUIDED PRACTICE

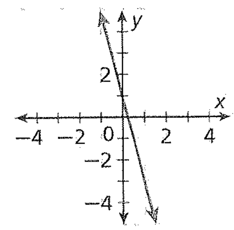
1. **Vocabulary** Is the *linear equation* $3x - 2 = y$ in standard form? Explain.

SEE EXAMPLE 1 p. 300

**Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

2. 

3. 

4. 

SEE EXAMPLE 2 p. 301

**Tell whether the given ordered pairs satisfy a linear function. Explain.**

5.

| x | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| y | 0 | 2 | 4 | 6 | 8 |

6.

| x | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|
| y | 1 | 2 | 3 | 4 | 5 |

7. $\{(0, 5), (-2, 3), (-4, 1), (-6, -1), (-8, -3)\}$

8. $\{(2, -2), (-1, 0), (-4, 1), (-7, 3), (-10, 6)\}$

SEE EXAMPLE 3 p. 302

**Tell whether each function is linear. If so, graph the function.**

9. $2x + 3y = 5$
10. $2y = 8$
11. $\dfrac{x^2 + 3}{5} = y$
12. $\dfrac{x}{5} = \dfrac{y}{3}$

SEE EXAMPLE 4 p. 303

13. **Transportation** A train travels at a constant speed of 75 mi/h. The function $f(x) = 75x$ gives the distance that the train travels in $x$ hours. Graph this function and give its domain and range.

14. **Entertainment** A movie rental store charges a \$6.00 membership fee plus \$2.50 for each movie rented. The function $f(x) = 2.50x + 6$ gives the cost of renting $x$ movies. Graph this function and give its domain and range.

## PRACTICE AND PROBLEM SOLVING

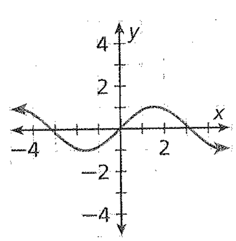
**Independent Practice**

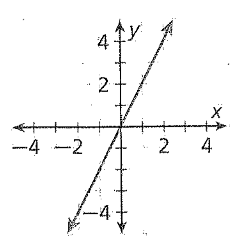
| For Exercises | See Example |
|---|---|
| 15–17 | 1 |
| 18–20 | 2 |
| 21–24 | 3 |
| 25 | 4 |

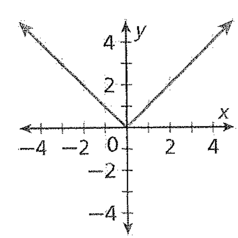
**Extra Practice**

Skills Practice p. S12
Application Practice p. S32

**Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

15. 

16. 

17. 

**Tell whether the given ordered pairs satisfy a linear function. Explain.**

18.

| x | −3 | 0 | 3 | 6 | 9 |
|---|---|---|---|---|---|
| y | −2 | −1 | 0 | 2 | 4 |

19.

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | −3 | −2 | −1 | 0 | 1 |

20. $\{(3, 4), (0, 2), (-3, 0), (-6, -2), (-9, -4)\}$

Tell whether each function is linear. If so, graph the function.

**21.** $y = 5$ **22.** $4y - 2x = 0$ **23.** $\frac{3}{x} + 4y = 10$ **24.** $5 + 3y = 8$

**25. Transportation** The gas tank in Tony's car holds 15 gallons, and the car can travel 25 miles for each gallon of gas. When Tony begins with a full tank of gas, the function $f(x) = -\frac{1}{25}x + 15$ gives the amount of gas $f(x)$ that will be left in the tank after traveling $x$ miles (if he does not buy more gas). Graph this function and give its domain and range.

Tell whether the given ordered pairs satisfy a function. If so, is it a linear function?

**26.** $\{(2, 5), (2, 4), (2, 3), (2, 2), (2, 1)\}$

**27.** $\{(-8, 2), (-6, 0), (-4, -2), (-2, -4), (0, -6)\}$

**28.**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −10 | −6 | −2 | 2 | 4 |
| $y$ | 0 | 0.25 | 0.50 | 0.75 | 1 |

**29.**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −5 | −1 | 3 | 7 | 11 |
| $y$ | 1 | 1 | 1 | 1 | 1 |

Tell whether each equation is linear. If so, write the equation in standard form and give the values of $A$, $B$, and $C$.

**30.** $2x - 8y = 16$ **31.** $y = 4x + 2$ **32.** $2x = \frac{y}{3} - 4$ **33.** $\frac{4}{x} = y$

**34.** $\frac{x+4}{2} = \frac{y-4}{3}$ **35.** $x = 7$ **36.** $xy = 6$ **37.** $3x - 5 + y = 2y - 4$

**38.** $y = -x + 2$ **39.** $5x = 2y - 3$ **40.** $2y = -6$ **41.** $y = \sqrt{x}$

Graph each linear function.

**42.** $y = 3x + 7$ **43.** $y = x + 25$ **44.** $y = 8 - x$ **45.** $y = 2x$

**46.** $-2y = -3x + 6$ **47.** $y - x = 4$ **48.** $y + 2x = -3$ **49.** $x = 5 + y$

**50. Measurement** One inch is equal to approximately 2.5 centimeters. Let $x$ represent inches and $y$ represent centimeters. Write an equation in standard form relating $x$ and $y$. Give the values of $A$, $B$, and $C$.

**51. Wages** Molly earns \$8.00 an hour at her job.

**a.** Let $x$ represent the number of hours that Molly works. Write a function using $x$ and $f(x)$ that describes Molly's pay for working $x$ hours.

**b.** Graph this function and give its domain and range.

**52. Write About It** For $y = 2x - 1$, make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.

**53. Critical Thinking** Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.

**MULTI-STEP TEST PREP**

**54.** This problem will prepare you for the Multi-Step Test Prep on page 342.

**a.** Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.

**b.** Create a graph of the data.

**c.** How can you tell from the graph that the relationship is linear?

| Time (min) | Calories |
|---|---|
| 3 | 27 |
| 6 | 54 |
| 9 | 81 |
| 12 | 108 |
| 15 | 135 |
| 18 | 162 |
| 21 | 189 |

55. **Physical Science** A ball was dropped from a height of 100 meters. Its height above the ground in meters at different times after its release is given in the table. Do these ordered pairs satisfy a linear function? Explain.

| Time (s) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Height (m) | 100 | 90.2 | 60.8 | 11.8 |

56. **Critical Thinking** Is the equation $x = 9$ a linear equation? Does it describe a linear function? Explain.

## TEST PREP

57. Which is NOT a linear function?

Ⓐ $y = 8x$ Ⓑ $y = x + 8$ Ⓒ $y = \frac{8}{x}$ Ⓓ $y = 8 - x$

58. The speed of sound in 0 °C air is about 331 feet per second. Which function could be used to describe the distance in feet $d$ that sound will travel in air in $s$ seconds?

Ⓕ $d = s + 331$ Ⓖ $d = 331s$ Ⓗ $s = 331d$ Ⓙ $s = 331 - d$

59. **Extended Response** Write your own linear function. Show that it is a linear function in at least three different ways. Explain any connections you see between your three methods.

## CHALLENGE AND EXTEND

60. What equation describes the $x$-axis? the $y$-axis? Do these equations represent linear functions?

**Geometry** **Copy and complete each table below. Then tell whether the table shows a linear relationship.**

61.

| Perimeter of a Square | |
|---|---|
| **Side Length** | **Perimeter** |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

62.

| Area of a Square | |
|---|---|
| **Side Length** | **Area** |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

63.

| Volume of a Cube | |
|---|---|
| **Side Length** | **Volume** |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

## SPIRAL REVIEW

**Simplify each expression.** *(Lesson 1-4)*

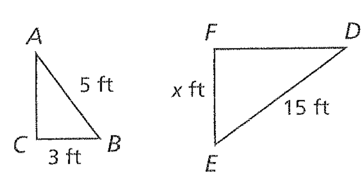
64. $8^2$ 65. $(-1)^3$ 66. $(-4)^4$ 67. $\left(\frac{1}{3}\right)^2$

**Solve each equation. Check your answer.** *(Lesson 2-4)*

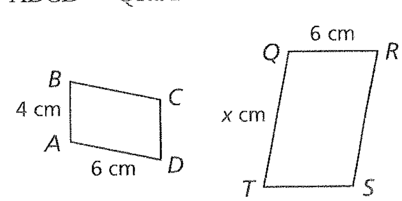
68. $6m + 5 = 3m - 4$ 69. $2(t - 4) = 3 - (3t + 1)$ 70. $9y + 5 - 2y = 2y + 5 - y + 3$

**Find the value of $x$ in each diagram.** *(Lesson 2-8)*

71. $\triangle ABC \sim \triangle DEF$




72. $ABCD \sim QRST$



# 5-2 Using Intercepts

**Objectives**

Find $x$- and $y$-intercepts and interpret their meanings in real-world situations.

Use $x$- and $y$-intercepts to graph lines.

**Vocabulary**

$y$-intercept
$x$-intercept

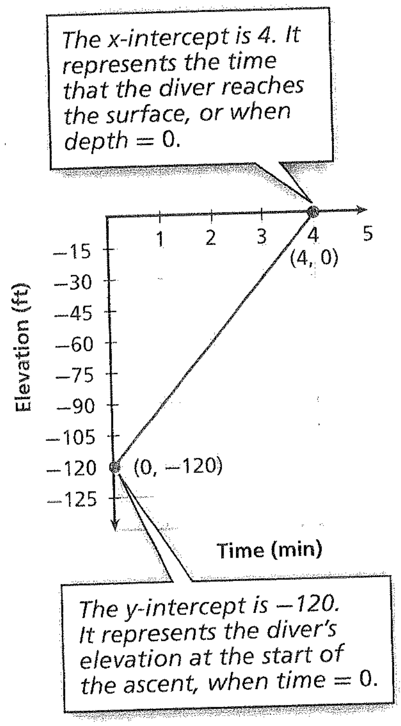
**Who uses this?**

Divers can use intercepts to determine the time a safe ascent will take.

A diver explored the ocean floor 120 feet below the surface and then ascended at a rate of 30 feet per minute. The graph shows the diver's elevation below sea level during the ascent.

The **$y$-intercept** is the $y$-coordinate of the point where the graph intersects the $y$-axis. The $x$-coordinate of this point is always 0.

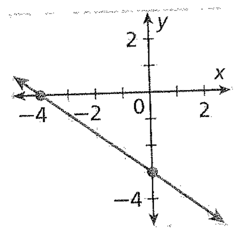
The **$x$-intercept** is the $x$-coordinate of the point where the graph intersects the $x$-axis. The $y$-coordinate of this point is always 0.




**EXAMPLE 1** **Finding Intercepts**

**Find the $x$- and $y$-intercepts.**

**A**




*The graph intersects the x-axis at (−4, 0).*
The $x$-intercept is $-4$.

*The graph intersects the y-axis at (0, −3).*
The $y$-intercept is $-3$.

**B** $3x - 2y = 12$

To find the $x$-intercept, replace $y$ with 0 and solve for $x$.

$$3x - 2y = 12$$
$$3x - 2(0) = 12$$
$$3x - 0 = 12$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

The $x$-intercept is 4.

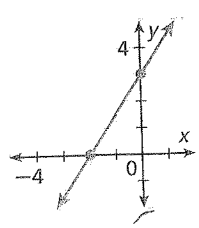
To find the $y$-intercept, replace $x$ with 0 and solve for $y$.

$$3x - 2y = 12$$
$$3(0) - 2y = 12$$
$$0 - 2y = 12$$
$$-2y = 12$$
$$\frac{-2y}{-2} = \frac{12}{-2}$$
$$y = -6$$

The $y$-intercept is $-6$.

**CHECK IT OUT!** **Find the $x$- and $y$-intercepts.**

**1a.**




**1b.** $-3x + 5y = 30$

**1c.** $4x + 2y = 16$

## Student to Student

### *Finding Intercepts*

**Madison Stewart**
Jefferson High School

*I use the "cover-up" method to find intercepts. To use this method, make sure the equation is in standard form first.*

*If I have $4x - 3y = 12$:*

*First, I cover $4x$ with my finger and solve the equation I can still see.*

$$-3y = 12$$
$$y = -4$$

*The y-intercept is $-4$.*

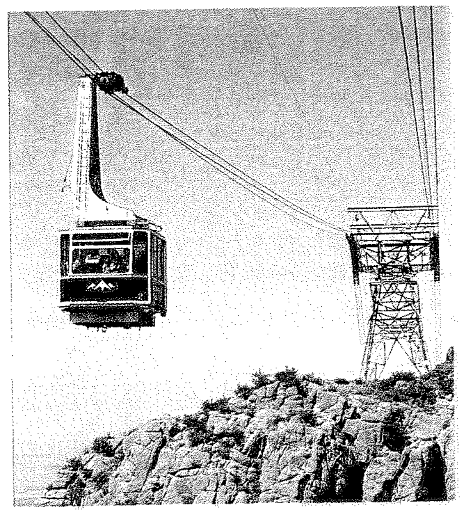
*Then I cover $-3y$ with my finger and do the same thing.*

$$4x = 12$$
$$x = 3$$

*The x-intercept is 3.*

**EXAMPLE 2** ***Travel Application***

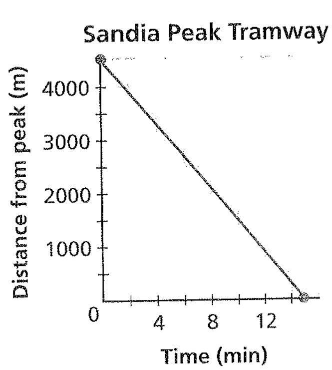
**The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function $f(x) = 4500 - 300x$ gives the tram's distance in meters from the top of the peak after $x$ minutes. Graph this function and find the intercepts. What does each intercept represent?**



*Neither time nor distance can be negative, so choose several nonnegative values for x. Use the function to generate ordered pairs.*

| $x$ | 0 | 2 | 5 | 10 | 15 |
|---|---|---|---|---|---|
| $f(x) = 4500 - 300x$ | 4500 | 3900 | 3000 | 1500 | 0 |

*Graph the ordered pairs. Connect the points with a line.*




- $y$-intercept: 4500. This is the starting distance from the top (time = 0).
- $x$-intercept: 15. This the time when the tram reaches the peak (distance = 0).

**Caution!**

The graph is not the path of the tram. Even though the line is descending, the graph describes the distance from the peak as the tram goes *up* the mountain.

**CHECK IT OUT!**

2. The school store sells pens for \$2.00 and notebooks for \$3.00. The equation $2x + 3y = 60$ describes the number of pens $x$ and notebooks $y$ that you can buy for \$60.
   **a.** Graph the function and find its intercepts.
   **b.** What does each intercept represent?

Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

## EXAMPLE 3 Graphing Linear Equations by Using Intercepts

**Use intercepts to graph the line described by each equation.**

**A** $2x - 4y = 8$

**Step 1** Find the intercepts.

***x*-intercept:**

$$2x - 4y = 8$$
$$2x - 4(0) = 8$$
$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

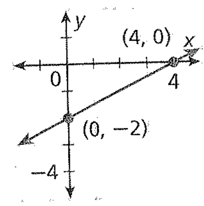
***y*-intercept:**

$$2x - 4y = 8$$
$$2(0) - 4y = 8$$
$$-4y = 8$$
$$\frac{-4y}{-4} = \frac{8}{-4}$$
$$y = -2$$

**Step 2** Graph the line.

*Plot (4, 0) and (0, −2).*

*Connect with a straight line.*




**B** $\frac{2}{3}y = 4 - \frac{1}{2}x$

**Step 1** Write the equation in standard form.

$$6\left(\frac{2}{3}y\right) = 6\left(4 - \frac{1}{2}x\right)$$ *Multiply both sides by 6, the LCD of the fractions, to clear the fractions.*

$$4y = 24 - 3x$$

$$3x + 4y = 24$$ *Write the equation in standard form.*

**Step 2** Find the intercepts.

***x*-intercept:**

$$3x + 4y = 24$$
$$3x + 4(0) = 24$$
$$3x = 24$$
$$\frac{3x}{3} = \frac{24}{3}$$
$$x = 8$$

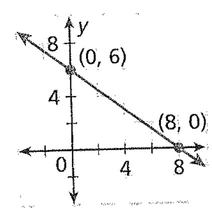
***y*-intercept:**

$$3x + 4y = 24$$
$$3(0) + 4y = 24$$
$$4y = 24$$
$$\frac{4y}{4} = \frac{24}{4}$$
$$y = 6$$

**Step 3** Graph the line.

*Plot (8, 0) and (0, 6).*

*Connect with a straight line.*




> **Helpful Hint**
>
> You can use a third point to check your line. Either choose a point from your graph and check it in the equation, or use the equation to generate a point and check that it is on your graph.

**CHECK IT OUT!** **Use intercepts to graph the line described by each equation.**

**3a.** $-3x + 4y = -12$

**3b.** $y = \frac{1}{3}x - 2$

## THINK AND DISCUSS

**1.** A function has $x$-intercept 4 and $y$-intercept 2. Name two points on the graph of this function.

**2.** What is the $y$-intercept of $2.304x + y = 4.318$? What is the $x$-intercept of $x - 92.4920y = -21.5489$?

Know it! Note

**3. GET ORGANIZED** Copy and complete the graphic organizer.

Graphing $Ax + By = C$ Using Intercepts

1. Find the $x$-intercept by __?__. → 2. Find the $y$-intercept by __?__. → 3. Graph the line by __?__.

# 5-2 Exercises

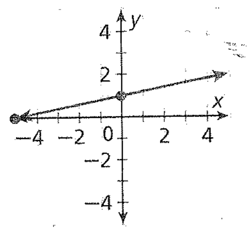
go.hrw.com
**Homework Help Online**
KEYWORD: MA7 5-2
**Parent Resources Online**
KEYWORD: MA7 Parent

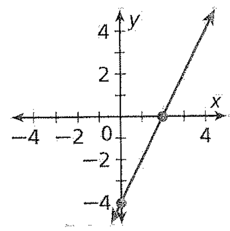
## GUIDED PRACTICE

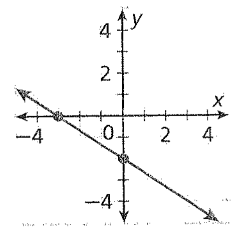
1. **Vocabulary** The __?__ is the $y$-coordinate of the point where a graph crosses the $y$-axis. ($x$-*intercept* or $y$-*intercept*)

SEE EXAMPLE 1
p. 307

**Find the $x$- and $y$-intercepts.**

2. 


3. 


4. 


5. $2x - 4y = 4$
6. $-2y = 3x - 6$
7. $4y + 5x = 2y - 3x + 16$

SEE EXAMPLE 2
p. 308

8. **Biology** To thaw a specimen stored at −25 °C, the temperature of a refrigeration tank is raised 5 °C every hour. The temperature in the tank after $x$ hours can be described by the function $f(x) = -25 + 5x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?

SEE EXAMPLE 3
p. 309

**Use intercepts to graph the line described by each equation.**

9. $4x - 5y = 20$
10. $y = 2x + 4$
11. $\frac{1}{3}x - \frac{1}{4}y = 2$
12. $-5y + 2x = -10$

## PRACTICE AND PROBLEM SOLVING

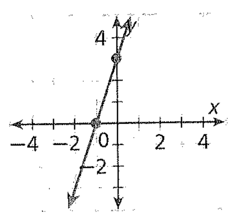
**Independent Practice**

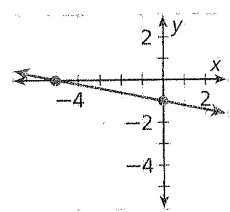
| For Exercises | See Example |
|---|---|
| 13–21 | 1 |
| 22–23 | 2 |
| 24–29 | 3 |

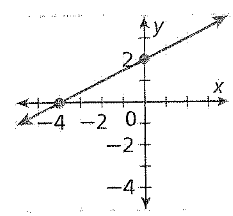
**Extra Practice**

Skills Practice p. S12
Application Practice p. S32

**Find the $x$- and $y$-intercepts.**

13. 


14. 


15. 


16. $6x + 3y = 12$
17. $4y - 8 = 2x$
18. $-2y + x = 2y - 8$
19. $4x + y = 8$
20. $y - 3x = -15$
21. $2x + y = 10x - 1$

22. **Environmental Science** A fishing lake was stocked with 300 bass. Each year, the population decreases by 25. The population of bass in the lake after $x$ years is represented by the function $f(x) = 300 - 25x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?

23. **Sports** Julie is running a 5-kilometer race. She runs 1 kilometer every 5 minutes. Julie's distance from the finish line after $x$ minutes is represented by the function $f(x) = 5 - \frac{1}{5}x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?

**Use intercepts to graph the line described by each equation.**

**24.** $4x - 6y = 12$ **25.** $2x + 3y = 18$ **26.** $\frac{1}{2}x - 4y = 4$

**27.** $y - x = -1$ **28.** $5x + 3y = 15$ **29.** $x - 3y = -1$

LINK

**Biology**

Bamboo is the world's fastest-growing woody plant. Some varieties can grow more than 30 centimeters a day and up to 40 meters tall.

**30. Biology** A bamboo plant is growing 1 foot per day. When you first measure it, it is 4 feet tall.

a. Write an equation to describe the height $y$, in feet, of the bamboo plant $x$ days after you measure it.

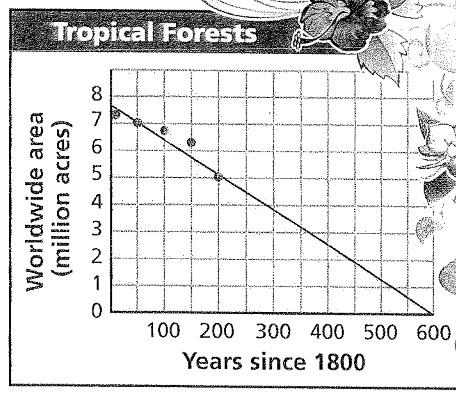
b. What is the $y$-intercept?

c. What is the meaning of the $y$-intercept in this problem?

**31. Estimation** Look at the scatter plot and trend line.

a. Estimate the $x$- and $y$-intercepts.

b. What is the real-world meaning of each intercept?




**32. Personal Finance** A bank employee notices an abandoned checking account with a balance of \$412. If the bank charges a \$4 monthly fee for the account, the function $b = 412 - 4m$ shows the balance $b$ in the account after $m$ months.

a. Graph the function and give its domain and range. (*Hint:* The bank will keep charging the monthly fee even after the account is empty.)

b. Find the intercepts. What does each intercept represent?

c. When will the bank account balance be 0?

**33. Critical Thinking** Complete the following to learn about intercepts and horizontal and vertical lines.

a. Graph $x = -6$, $x = 1$, and $x = 5$. Find the intercepts.

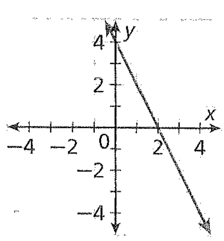
b. Graph $y = -3$, $y = 2$, and $y = 7$. Find the intercepts.

c. Write a rule describing the intercepts of linear equations whose graphs are horizontal and vertical lines.

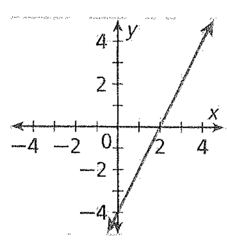
**Match each equation with a graph.**

**34.** $-2x - y = 4$ **35.** $y = 4 - 2x$ **36.** $2y + 4x = 8$ **37.** $4x - 2y = 8$

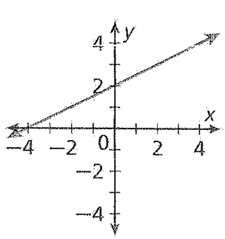
A.



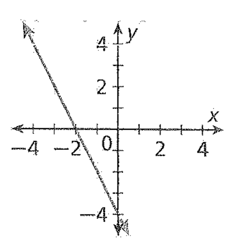

B.




C.




D.



**MULTI-STEP TEST PREP**

38. This problem will prepare you for the Multi-Step Test Prep on page 342.

Kristyn rode a stationary bike at the gym. She programmed the timer for 20 minutes. The display counted backward to show how much time remained in her workout. It also showed her mileage.

a. What are the intercepts?

b. What do the intercepts represent?

| Time Remaining (min) | Distance Covered (mi) |
|---|---|
| 20 | 0 |
| 16 | 0.35 |
| 12 | 0.70 |
| 8 | 1.05 |
| 4 | 1.40 |
| 0 | 1.75 |

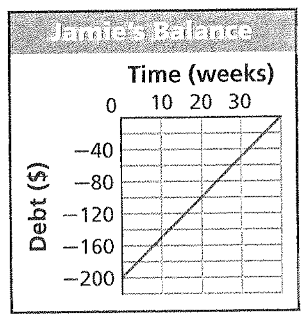
39. **Write About It** Write a real-world problem that could be modeled by a linear function whose $x$-intercept is 5 and whose $y$-intercept is 60.

**TEST PREP**

40. Which is the $x$-intercept of $-2x = 9y - 18$?

Ⓐ $-9$ Ⓑ $-2$ Ⓒ $2$ Ⓓ $9$

41. Which of the following situations could be represented by the graph?




Ⓕ Jamie owed her uncle \$200. Each week for 40 weeks she paid him \$5.

Ⓖ Jamie owed her uncle \$200. Each week for 5 weeks she paid him \$40.

Ⓗ Jamie owed her uncle \$40. Each week for 5 weeks she paid him \$200.

Ⓙ Jamie owed her uncle \$40. Each week for 200 weeks she paid him \$5.

42. **Gridded Response** What is the $y$-intercept of $60x + 55y = 660$?

## CHALLENGE AND EXTEND

**Use intercepts to graph the line described by each equation.**

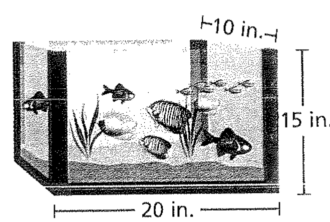
43. $\frac{1}{2}x + \frac{1}{5}y = 1$ 44. $0.5x - 0.2y = 0.75$ 45. $y = \frac{3}{8}x + 6$

46. For any linear equation $Ax + By = C$, what are the intercepts?

47. Find the intercepts of $22x - 380y = 20{,}900$. Explain how to use the intercepts to determine appropriate scales for the graph.

## SPIRAL REVIEW

48. Marlon's fish tank is 80% filled with water. Based on the measurements shown, what volume of the tank is NOT filled with water? *(Lesson 2-9)*




**Solve each inequality and graph the solutions.** *(Lesson 3-3)*

49. $3c > 12$ 50. $-4 \geq \frac{t}{2}$ 51. $\frac{1}{2}m \geq -3$ 52. $-2w > 14$

**Tell whether the given ordered pairs satisfy a linear function. Explain.** *(Lesson 5-1)*

53. $\{(-2, 0), (0, 3), (2, 6), (4, 9), (6, 12)\}$ 54. $\{(0, 0), (1, 1), (4, 2), (9, 3), (16, 4)\}$

Connecting Algebra to Geometry

# Area in the Coordinate Plane

Lines in the coordinate plane can form the sides of polygons. You can use points on these lines to help you find the areas of these polygons.

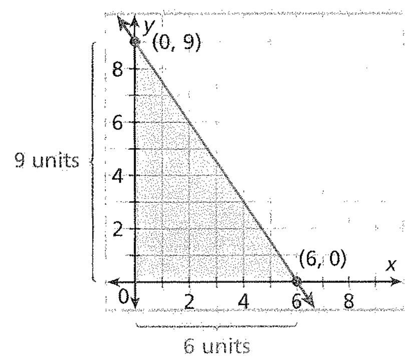
## Example

**Find the area of the triangle formed by the $x$-axis, the $y$-axis, and the line described by $3x + 2y = 18$.**

**Step 1** Find the intercepts of $3x + 2y = 18$.

| **$x$-intercept:** | **$y$-intercept:** |
|---|---|
| $3x + 2y = 18$ | $3x + 2y = 18$ |
| $3x + 2(0) = 18$ | $3(0) + 2y = 18$ |
| $3x = 18$ | $2y = 18$ |
| $x = 6$ | $y = 9$ |

**Step 2** Use the intercepts to graph the line. The $x$-intercept is 6, so plot $(6, 0)$. The $y$-intercept is 9, so plot $(0, 9)$. Connect with a straight line. Then shade the triangle formed by the line and the axes, as described.




**Step 3** Recall that the area of a triangle is given by $A = \frac{1}{2}bh$.

- The length of the base is 6.
- The height is 9.

**Step 4** Substitute these values into the formula.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(9) \qquad \textit{Substitute into the area formula.}$$

$$= \frac{1}{2}(54) \qquad \textit{Simplify.}$$

$$= 27$$

The area of the triangle is 27 square units.

## Try This

1. Find the area of the triangle formed by the $x$-axis, the $y$-axis, and the line described by $3x + 2y = 12$.
2. Find the area of the triangle formed by the $x$-axis, the $y$-axis, and the line described by $y = 6 - x$.
3. Find the area of the polygon formed by the $x$-axis, the $y$-axis, the line described by $y = 6$, and the line described by $x = 4$.

# 5-3 Rate of Change and Slope


**Objectives**

Find rates of change and slopes.

Relate a constant rate of change to the slope of a line.

**Vocabulary**

rate of change
rise
run
slope

**Why learn this?**

Rates of change can be used to find how quickly costs have increased.

In 1985, the cost of sending a 1-ounce letter was 22 cents. In 1988, the cost was 25 cents. How fast did the cost change from 1985 to 1988? In other words, at what *rate* did the cost change?

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

**EXAMPLE 1** ***Consumer Application***

**The table shows the cost of mailing a 1-ounce letter in different years. Find the rate of change in cost for each time interval. During which time interval did the cost increase at the greatest rate?**

| Year | 1988 | 1990 | 1991 | 2004 | 2008 |
|---|---|---|---|---|---|
| Cost (¢) | 25 | 25 | 29 | 37 | 42 |

**Step 1** Identify the dependent and independent variables.

dependent: cost    independent: year

**Step 2** Find the rates of change.

1988 to 1990 $\quad \frac{\text{change in cost}}{\text{change in years}} = \frac{25-25}{1990-1988} = \frac{0}{2} = 0 \quad \frac{0 \text{ cents}}{\text{year}}$

1990 to 1991 $\quad \frac{\text{change in cost}}{\text{change in years}} = \frac{29-25}{1991-1990} = \frac{4}{1} = 4 \quad \frac{4 \text{ cents}}{\text{year}}$

1991 to 2004 $\quad \frac{\text{change in cost}}{\text{change in years}} = \frac{37-29}{2004-1991} = \frac{8}{13} \approx 0.62 \approx \frac{0.62 \text{ cents}}{\text{year}}$

2004 to 2008 $\quad \frac{\text{change in cost}}{\text{change in years}} = \frac{42-37}{2008-2004} = \frac{5}{4} = 1.25 \quad \frac{1.25 \text{ cents}}{\text{year}}$

The cost increased at the greatest rate from 1990 to 1991.

**Caution!**

A rate of change of 1.25 cents per year for a 4-year period means that the *average* change was 1.25 cents per year. The *actual* change in each year may have been different.

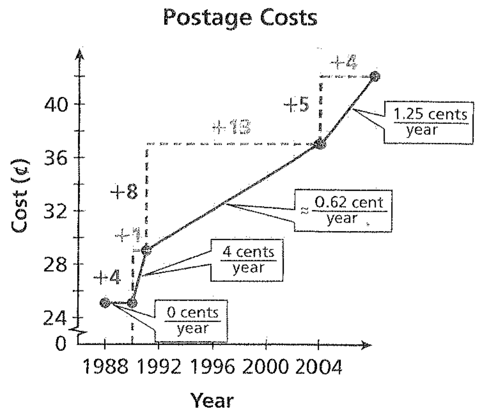
**CHECK IT OUT!**

1. The table shows the balance of a bank account on different days of the month. Find the rate of change for each time interval. During which time interval did the balance decrease at the greatest rate?

| Day | 1 | 6 | 16 | 22 | 30 |
|---|---|---|---|---|---|
| Balance ($) | 550 | 285 | 210 | 210 | 175 |

## EXAMPLE 2 Finding Rates of Change from a Graph

**Graph the data from Example 1 and show the rates of change.**




*Graph the ordered pairs. The vertical blue segments show the changes in the dependent variable, and the horizontal green segments show the changes in the independent variable.*

*Notice that the greatest rate of change is represented by the steepest of the red line segments.*

*Also notice that between 1988 and 1990, when the cost did not change, the red line segment is horizontal.*

**CHECK IT OUT!** **2.** Graph the data from Check It Out Problem 1 and show the rates of change.

If all of the connected segments have the same rate of change, then they all have the same steepness and together form a straight line. The constant rate of change of a nonvertical line is called the *slope* of the line.

**Know it! Note**

### Slope of a Line

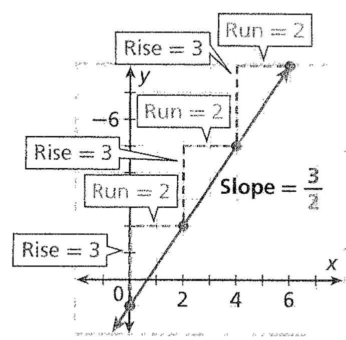
The **rise** is the difference in the $y$-values of two points on a line.

The **run** is the difference in the $x$-values of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that $y$ is the dependent variable and $x$ is the independent variable.)

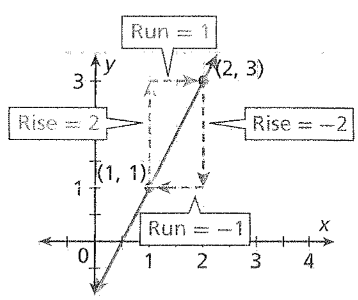



## EXAMPLE 3 Finding Slope

**Caution!**

Pay attention to the scales on the axes. One square on the grid may not represent 1 unit. In Example 3, each square represents $\frac{1}{2}$ unit.

**Find the slope of the line.**




*Begin at one point and count vertically to find the rise.*

*Then count horizontally to the second point to find the run.* $\text{slope} = \frac{2}{1} = 2$

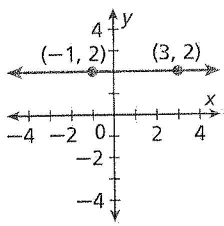
*It does not matter which point you start with. The slope is the same.* $\text{slope} = \frac{-2}{-1} = 2$

**CHECK IT OUT!** **3.** Find the slope of the line that contains $(0, -3)$ and $(5, -5)$.

## EXAMPLE 4 Finding Slopes of Horizontal and Vertical Lines

**Find the slope of each line.**

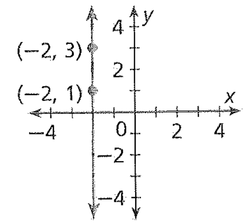
**A**




$\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0$

The slope is 0.

**B**

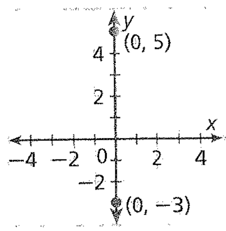



$\frac{\text{rise}}{\text{run}} = \frac{2}{0}$ *You cannot divide by 0.*

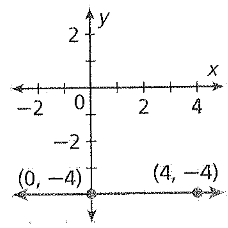
The slope is undefined.

**CHECK IT OUT!** Find the slope of each line.

**4a.**

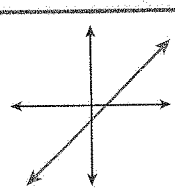
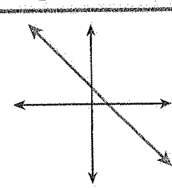
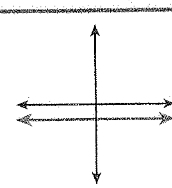
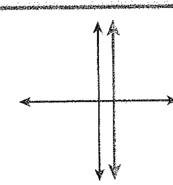



**4b.**




As shown in the previous examples, slope can be positive, negative, zero, or undefined. You can tell which of these is the case by looking at the graph of a line—you do not need to calculate the slope.

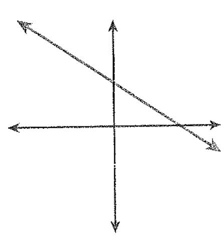
**Know it! Note**

| Positive Slope | Negative Slope | Zero Slope | Undefined Slope |
|---|---|---|---|
|  |  |  |  |
| Line rises from left to right. | Line falls from left to right. | Horizontal line | Vertical line |

## EXAMPLE 5 Describing Slope

**Tell whether the slope of each line is positive, negative, zero, or undefined.**

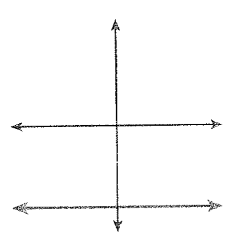
**A**



*The line falls from left to right.*

The slope is negative.

**B**

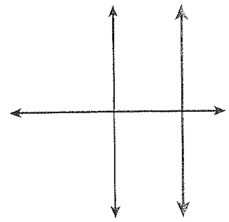


*The line is horizontal.*

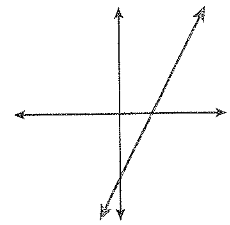
The slope is 0.

**CHECK IT OUT!** **Tell whether the slope of each line is positive, negative, zero, or undefined.**

**5a.**

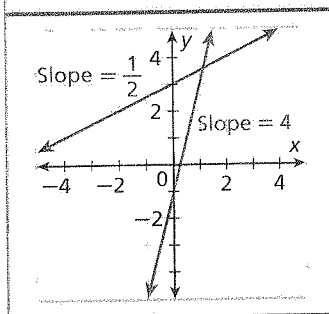


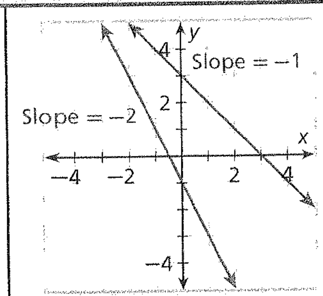
**5b.**

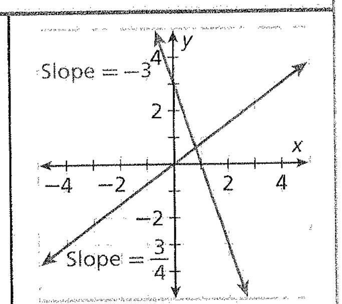


A line's slope is a measure of its steepness. Some lines are steeper than others. As the absolute value of the slope increases, the line becomes steeper. As the absolute value of the slope decreases, the line becomes less steep.

**Comparing Slopes**

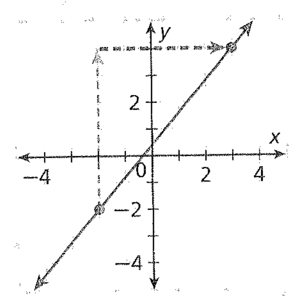


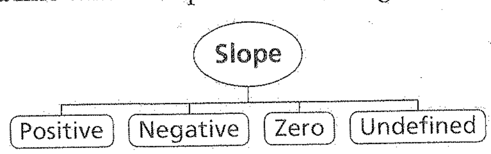

| | | |
|---|---|---|
| The line with slope 4 is steeper than the line with slope $\frac{1}{2}$. | The line with slope −2 is steeper than the line with slope −1. | The line with slope −3 is steeper than the line with slope $\frac{3}{4}$. |
| $\lvert 4 \rvert > \left\lvert \frac{1}{2} \right\rvert$ | $\lvert -2 \rvert > \lvert -1 \rvert$ | $\lvert -3 \rvert > \left\lvert \frac{3}{4} \right\rvert$ |

## THINK AND DISCUSS




1. What is the rise shown in the graph? What is the run? What is the slope?
2. The rate of change of the profits of a company over one year is negative. How have the profits of the company changed over that year?
3. Would you rather climb a hill with a slope of 4 or a hill with a slope of $\frac{5}{2}$? Explain your answer.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a line whose slope matches the given description.

Know it! Note



# 5-3 Exercises



## GUIDED PRACTICE

1. **Vocabulary** The *slope* of any nonvertical line is ___?___. (*positive* or *constant*)

SEE EXAMPLE 1 p. 314

2. The table shows the volume of gasoline in a gas tank at different times. Find the rate of change for each time interval. During which time interval did the volume decrease at the greatest rate?

| Time (h) | 0 | 1 | 3 | 6 | 7 |
|---|---|---|---|---|---|
| Volume (gal) | 12 | 9 | 5 | 1 | 1 |

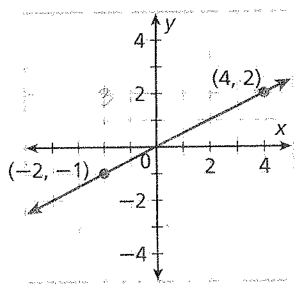
SEE EXAMPLE 2 p. 315

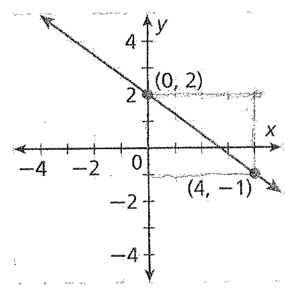
3. The table shows a person's heart rate over time. Graph the data and show the rates of change.

| Time (min) | 0 | 2 | 5 | 7 | 10 |
|---|---|---|---|---|---|
| Heart Rate (beats/min) | 64 | 92 | 146 | 84 | 64 |

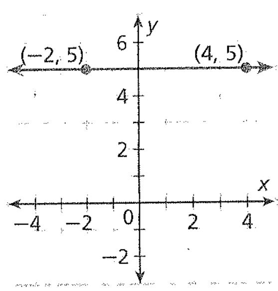
**Find the slope of each line.**

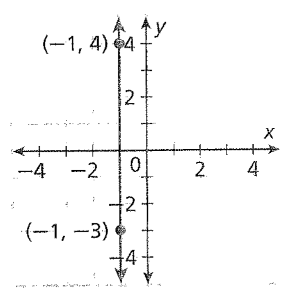
SEE EXAMPLE 3 p. 315

4.



5.


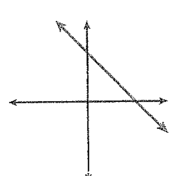

SEE EXAMPLE 4 p. 316

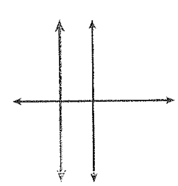
6.


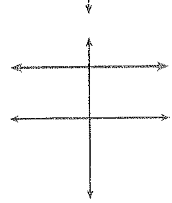

7.


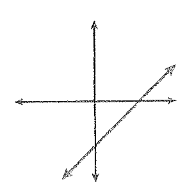

SEE EXAMPLE 5 p. 316

**Tell whether the slope of each line is positive, negative, zero, or undefined.**

8.


9.


10.


11.


## PRACTICE AND PROBLEM SOLVING

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 12 | 1 |
| 13 | 2 |
| 14–15 | 3 |
| 16–17 | 4 |
| 18–19 | 5 |

**Extra Practice**

Skills Practice p. S12
Application Practice p. S32

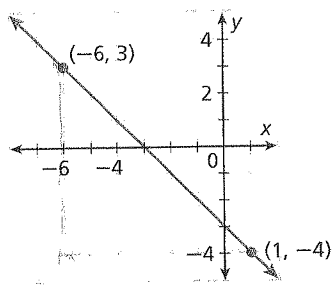
12. The table shows the length of a baby at different ages. Find the rate of change for each time interval. Round your answers to the nearest tenth. During which time interval did the baby have the greatest growth rate?

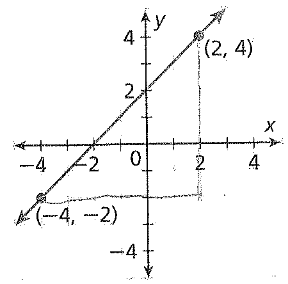
| Age (mo) | 3 | 9 | 18 | 26 | 33 |
|---|---|---|---|---|---|
| Length (in.) | 23.5 | 27.5 | 31.6 | 34.5 | 36.7 |

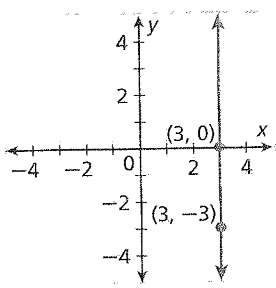
13. The table shows the distance of an elevator from the ground floor at different times. Graph the data and show the rates of change.

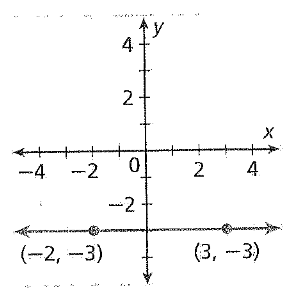
| Time (s) | 0 | 15 | 23 | 30 | 35 |
|---|---|---|---|---|---|
| Distance (m) | 30 | 70 | 0 | 45 | 60 |

**Find the slope of each line.**

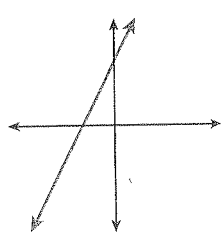
14.


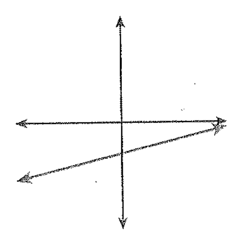

15.



16.



17.



**Tell whether the slope of each line is positive, negative, zero, or undefined.**

18.


19.


**LINK**

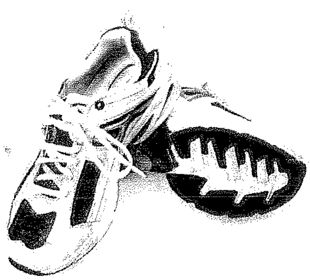
**Travel**

The Incline Railway's climb up Lookout Mountain has been called "America's Most Amazing Mile." A round-trip on the railway lasts about 1.5 hours.

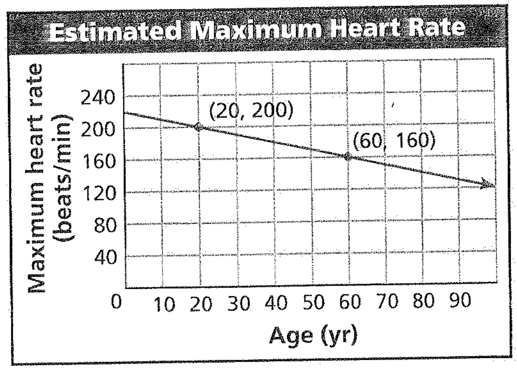
20. **Travel** The Lookout Mountain Incline Railway in Chattanooga, Tennessee, is the steepest passenger railway in the world. A section of the railway has a slope of about 0.73. In this section, a vertical change of 1 unit corresponds to a horizontal change of what length? Round your answer to the nearest hundredth.

21. **Critical Thinking** In Lesson 5-1, you learned that in a linear function, a constant change in $x$ corresponds to a constant change in $y$. How is this related to slope?

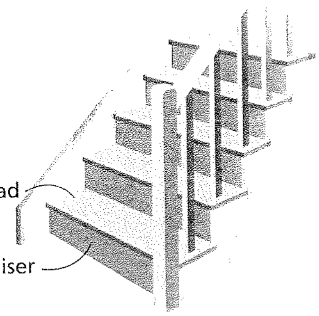
**MULTI-STEP TEST PREP**



22. This problem will prepare you for the Multi-Step Test Prep on page 342.
    a. The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the slope.
    b. Describe the rate of change in this situation.




23. **Construction** Most staircases in use today have 9-inch treads and $8\frac{1}{2}$-inch risers. What is the slope of a staircase with these measurements?




24. A ladder is leaned against a building. The bottom of the ladder is 9 feet from the building. The top of the ladder is 16 feet above the ground.
    a. Draw a diagram to represent this situation.
    b. What is the slope of the ladder?

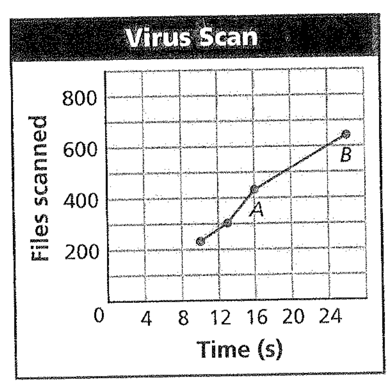
25. **Write About It** Why will the slope of any horizontal line be 0? Why will the slope of any vertical line be undefined?

26. The table shows the distance traveled by a car during a five-hour road trip.

| Time (h) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance (mi) | 0 | 40 | 80 | 80 | 110 | 160 |

    a. Graph the data and show the rates of change.
    b. The rate of change represents the average speed. During which hour was the car's average speed the greatest?

27. **Estimation** The graph shows the number of files scanned by a computer virus detection program over time.
    a. Estimate the coordinates of point $A$.
    b. Estimate the coordinates of point $B$.
    c. Use your answers from parts **a** and **b** to estimate the rate of change (in files per second) between points $A$ and $B$.



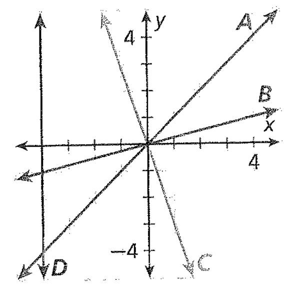

28. **Data Collection** Use a graphing calculator and a motion detector for the following. Set the equipment so that the graph shows distance on the $y$-axis and time on the $x$-axis.
    a. Experiment with walking in front of the motion detector. How must you walk to graph a straight line? Explain.
    b. Describe what you must do differently to graph a line with a positive slope vs. a line with a negative slope.
    c. How can you graph a line with slope 0? Explain.

## TEST PREP

29. The slope of which line has the greatest absolute value?

Ⓐ line $A$ Ⓒ line $C$

Ⓑ line $B$ Ⓓ line $D$

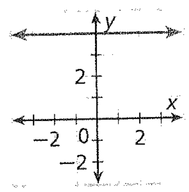


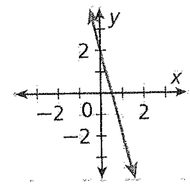

30. For which line is the run equal to 0?

Ⓐ line $A$ Ⓒ line $C$

Ⓑ line $B$ Ⓓ line $D$

31. Which line has a slope of 4?

Ⓕ



Ⓗ
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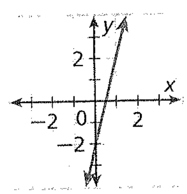


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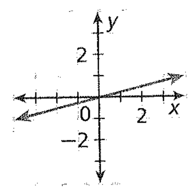



## CHALLENGE AND EXTEND

32. **Recreation** Tara and Jade are hiking up a hill. Each has a different stride. The run for Tara's stride is 32 inches, and the rise is 8 inches. The run for Jade's stride is 36 inches. What is the rise of Jade's stride?

33. **Economics** The table shows cost in dollars charged by an electric company for various amounts of energy in kilowatt-hours.

| Energy (kWh) | 0 | 200 | 400 | 600 | 1000 | 2000 |
|---|---|---|---|---|---|---|
| Cost ($) | 3 | 3 | 31 | 59 | 115 | 150 |

a. Graph the data and show the rates of change.

b. Compare the rates of change for each interval. Are they all the same? Explain.

c. What do the rates of change represent?

d. Describe in words the electric company's billing plan.

## SPIRAL REVIEW

**Add or subtract.** *(Lesson 1-2)*

34. $-5 + 15$

35. $9 - 11$

36. $-5 - (-25)$

**Find the domain and range of each relation, and tell whether the relation is a function.** *(Lesson 4-2)*

37. $\{(3, 4), (3, 2), (3, 0), (3, -2)\}$

38.

| $x$ | 0 | 2 | 4 | −2 | −4 |
|---|---|---|---|---|---|
| $y$ | 0 | 2 | 4 | 2 | 4 |

**Find the $x$- and $y$-intercepts.** *(Lesson 5-2)*

39. $2x + y = 6$

40. $y = -3x - 9$

41. $2y = -4x + 1$

5-3 Algebra LAB

# Explore Constant Changes

*Use with Lesson 5-3*

There are many real-life situations in which the amount of change is constant. In these activities, you will explore what happens when

- a quantity increases by a constant amount.
- a quantity decreases by a constant amount.

## Activity 1

Janice has read 7 books for her summer reading club. She plans to read 2 books each week for the rest of the summer. The table shows the total number of books that Janice will have read after different numbers of weeks have passed.

| Janice's Summer Reading | |
|---|---|
| **Week** | **Total Books Read** |
| 0 | 7 |
| 1 | 9 |
| 2 | 11 |
| 3 | 13 |
| 4 | 15 |
| 5 | 17 |

1. What number is added to the number of books in each row to get the number of books in the next row?
2. What does your answer to Problem 1 represent in Janice's situation? Describe the meaning of the constant change.
3. Graph the ordered pairs from the table. Describe how the points are related.
4. Look again at your answer to Problem 1. Explain how this number affects your graph.

## Try This

At a particular college, a full-time student must take at least 12 credit hours per semester and may take up to 18 credit hours per semester. Tuition costs $200 per credit hour.

| Tuition Costs | |
|---|---|
| **Credit Hours** | **Cost ($)** |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |

1. Copy and complete the table by using the information above.
2. What number is added to the cost in each row to get the cost in the next row?
3. What does your answer to Problem 2 above represent in the situation? Describe the meaning of the constant change.
4. Graph the ordered pairs from the table. Describe how the points are related.
5. Look again at your answer to Problem 2. Explain how this number affects your graph.
6. Compare your graphs from Activity 1 and Problem 4. How are they alike? How are they different?
7. **Make a Conjecture** Describe the graph of any situation that involves repeated addition of a positive number. Why do you think your description is correct?

## Activity 2

An airplane is 3000 miles from its destination. The plane is traveling at a rate of 540 miles per hour. The table shows how far the plane is from its destination after various amounts of time have passed.

| Airplane's Distance | |
|---|---|
| **Time (h)** | **Distance to Destination (mi)** |
| 0 | 3000 |
| 1 | 2460 |
| 2 | 1920 |
| 3 | 1380 |
| 4 | 840 |

1. What number is subtracted from the distance in each row to get the distance in the next row?
2. What does your answer to Problem 1 represent in the situation? Describe the meaning of the constant change.
3. Graph the ordered pairs from the table. Describe how the points are related.
4. Look again at your answer to Problem 1. Explain how this number affects your graph.

## Try This

A television game show begins with 20 contestants. Each week, the players vote 2 contestants off the show.

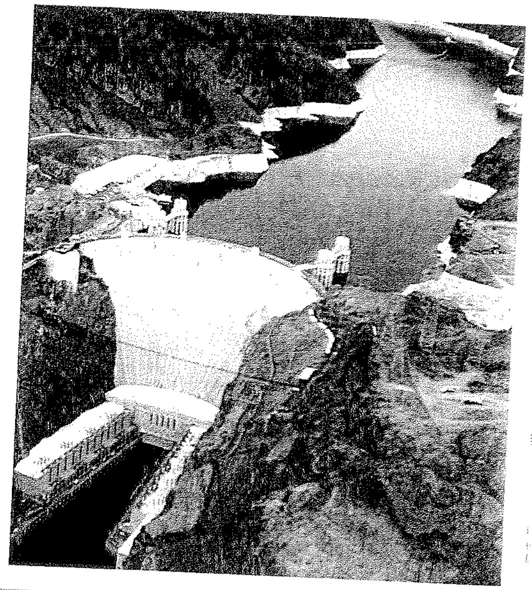
| Game Show | |
|---|---|
| **Week** | **Contestants Remaining** |
| 0 | 20 |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

8. Copy and complete the table by using the information above.
9. What number is subtracted from the number of contestants in each row to get the number of contestants in the next row?
10. What does your answer to Problem 9 represent in the situation? Describe the meaning of the constant change.
11. Graph the ordered pairs from the table. Describe how the points are related.
12. Look again at your answer to Problem 9. Explain how this number affects your graph.
13. Compare your graphs from Activity 2 and Problem 11. How are they alike? How are they different?
14. **Make a Conjecture** Describe the graph of any situation that involves repeated subtraction of a positive number. Why do you think your description is correct?
15. Compare your two graphs from Activity 1 with your two graphs from Activity 2. How are they alike? How are they different?
16. **Make a Conjecture** How are graphs of situations involving repeated subtraction different from graphs of situations involving repeated addition? Explain your answer.

# 5-4 The Slope Formula

**Objective**
Find slope by using the slope formula.

**Why learn this?**
You can use the slope formula to find how quickly a quantity, such as the amount of water in a reservoir, is changing. (See Example 3.)



In Lesson 5-3, slope was described as the constant rate of change of a line. You saw how to find the slope of a line by using its graph.

There is also a formula you can use to find the slope of a line, which is usually represented by the letter $m$. To use this formula, you need the coordinates of two different points on the line.

**Know it! Note**

**Slope Formula**

| WORDS | FORMULA | EXAMPLE |
|---|---|---|
| The slope of a line is the ratio of the difference in $y$-values to the difference in $x$-values between any two different points on the line. | If $(x_1, y_1)$ and $(x_2, y_2)$ are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$. | If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4-(-3)}{1-2} = \frac{7}{-1} = -7$. |

**EXAMPLE 1** **Finding Slope by Using the Slope Formula**

**Find the slope of the line that contains $(4, -2)$ and $(-1, 2)$.**

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \textit{Use the slope formula.}$$

$$= \frac{2-(-2)}{-1-4} \quad \textit{Substitute } (4, -2) \textit{ for } (x_1, y_1) \textit{ and } (-1, 2) \textit{ for } (x_2, y_2).$$

$$= \frac{4}{-5} \quad \textit{Simplify.}$$

$$= -\frac{4}{5}$$

The slope of the line that contains $(4, -2)$ and $(-1, 2)$ is $-\frac{4}{5}$.

**Reading Math**

The small numbers to the bottom right of the variables are called subscripts. Read $x_1$ as "$x$ sub one" and $y_2$ as "$y$ sub two."

**CHECK IT OUT!**

**1a.** Find the slope of the line that contains $(-2, -2)$ and $(7, -2)$.

**1b.** Find the slope of the line that contains $(5, -7)$ and $(6, -4)$.

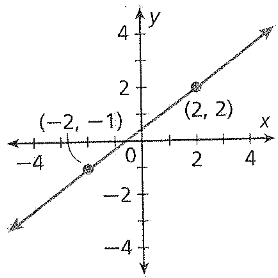
**1c.** Find the slope of the line that contains $\left(\frac{3}{4}, \frac{7}{5}\right)$ and $\left(\frac{1}{4}, \frac{2}{5}\right)$.

Sometimes you are not given two points to use in the formula. You might have to choose two points from a graph or a table.

## EXAMPLE 2 Finding Slope from Graphs and Tables

**Each graph or table shows a linear relationship. Find the slope.**

**A**




*Let (2, 2) be $(x_1, y_1)$ and (−2, −1) be $(x_2, y_2)$.*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

$$= \frac{-1 - 2}{-2 - 2}$$ *Substitute (2, 2) for $(x_1, y_1)$ and (−2, −1) for $(x_2, y_2)$.*

$$= \frac{-3}{-4}$$ *Simplify.*

$$= \frac{3}{4}$$

**B**

| | | | | |
|---|---|---|---|---|
| x | 2 | 2 | 2 | 2 |
| y | 0 | 1 | 3 | 5 |

**Step 1** Choose any two points from the table. Let $(2, 0)$ be $(x_1, y_1)$ and $(2, 3)$ be $(x_2, y_2)$.

**Step 2** Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

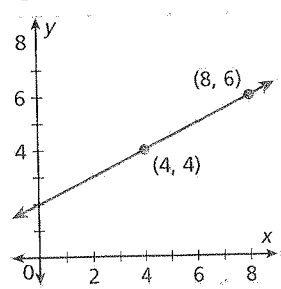
$$= \frac{3 - 0}{2 - 2}$$ *Substitute (2, 0) for $(x_1, y_1)$ and (2, 3) for $(x_2, y_2)$.*

$$= \frac{3}{0}$$ *Simplify.*

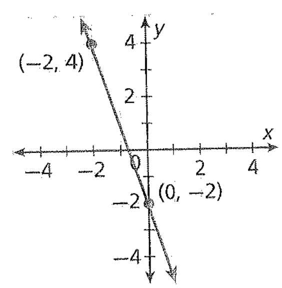
The slope is undefined.

**CHECK IT OUT!** **Each graph or table shows a linear relationship. Find the slope.**

**2a.**




**2b.**




**2c.**

| | | | | |
|---|---|---|---|---|
| x | 0 | 2 | 5 | 6 |
| y | 1 | 5 | 11 | 13 |

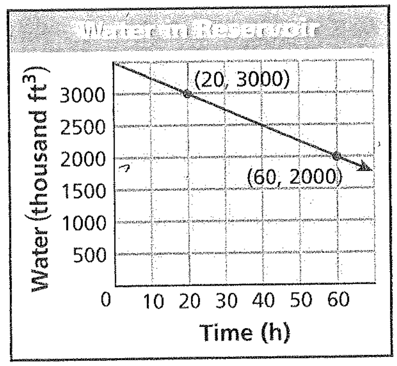
**2d.**

| | | | | |
|---|---|---|---|---|
| x | −2 | 0 | 2 | 4 |
| y | 3 | 0 | −3 | −6 |

Remember that slope is a rate of change. In real-world problems, finding the slope can give you information about how a quantity is changing.

EXAMPLE 3 *Environmental Science Application*

**The graph shows how much water is in a reservoir at different times. Find the slope of the line. Then tell what the slope represents.**




**Step 1** Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2000 - 3000}{60 - 20}$$

$$= \frac{-1000}{40} = -25$$

**Step 2** Tell what the slope represents.

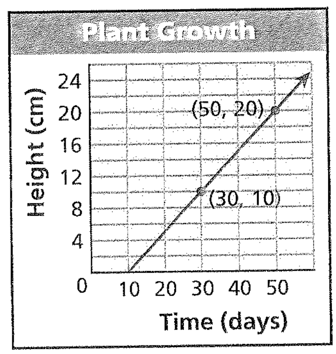
In this situation, $y$ represents volume of water and $x$ represents time.

So slope represents $\frac{\text{change in volume}}{\text{change in time}}$ in units of $\frac{\text{thousands of cubic feet}}{\text{hours}}$.

A slope of $-25$ means the amount of water in the reservoir is decreasing (negative change) at a rate of 25 thousand cubic feet each hour.

CHECK IT OUT!

**3.** The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.




If you know the equation that describes a line, you can find its slope by using any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

EXAMPLE 4 **Finding Slope from an Equation**

**Find the slope of the line described by $6x - 5y = 30$.**

**Step 1** Find the $x$-intercept.

$$6x - 5y = 30$$

$$6x - 5(0) = 30 \quad \textit{Let } y = 0.$$

$$6x = 30$$

$$\frac{6x}{6} = \frac{30}{6}$$

$$x = 5$$

**Step 2** Find the $y$-intercept.

$$6x - 5y = 30$$

$$6(0) - 5y = 30 \quad \textit{Let } x = 0.$$

$$-5y = 30$$

$$\frac{-5y}{-5} = \frac{30}{-5}$$

$$y = -6$$

**Step 3** The line contains $(5, 0)$ and $(0, -6)$. Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 5} = \frac{-6}{-5} = \frac{6}{5}$$

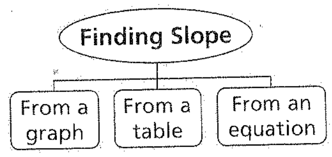
CHECK IT OUT!

**4.** Find the slope of the line described by $2x + 3y = 12$.

## THINK AND DISCUSS

1. The slope of a line is the difference of the ___?___ divided by the difference of the ___?___ for any two points on the line.
2. Two points lie on a line. When you substitute their coordinates into the slope formula, the value of the denominator is 0. Describe this line.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find slope using the given method.

Know it! Note




# 5-4 Exercises

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## GUIDED PRACTICE

SEE EXAMPLE 1 p. 324

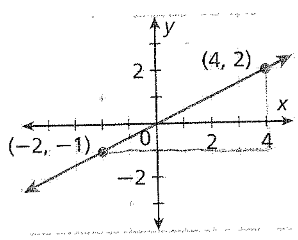
Find the slope of the line that contains each pair of points.

1. $(3, 6)$ and $(6, 9)$
2. $(2, 7)$ and $(4, 4)$
3. $(-1, -5)$ and $(-9, -1)$

SEE EXAMPLE 2 p. 325

Each graph or table shows a linear relationship. Find the slope.

4.



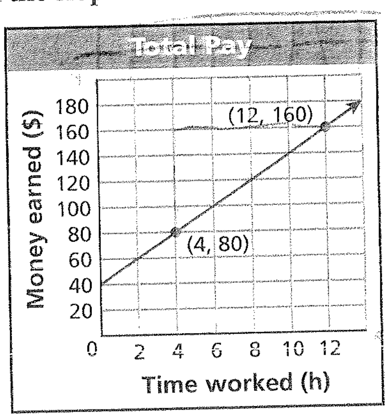

5.

| $x$ | $y$ |
|---|---|
| 0 | 25 |
| 2 | 45 |
| 4 | 65 |
| 6 | 85 |

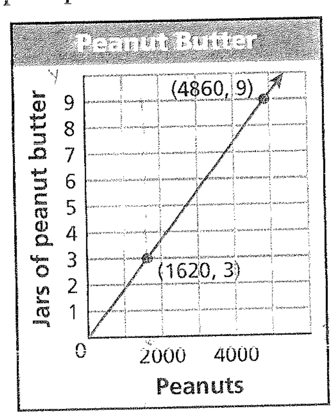
SEE EXAMPLE 3 p. 326

Find the slope of each line. Then tell what the slope represents.

6.




7.




SEE EXAMPLE 4 p. 326

Find the slope of the line described by each equation.

8. $8x + 2y = 96$
9. $5x = 90 - 9y$
10. $5y = 160 + 9x$

# PRACTICE AND PROBLEM SOLVING

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 11–13 | 1 |
| 14–15 | 2 |
| 16–17 | 3 |
| 18–20 | 4 |

**Extra Practice**

Skills Practice p. S12
Application Practice p. S32

**Find the slope of the line that contains each pair of points.**

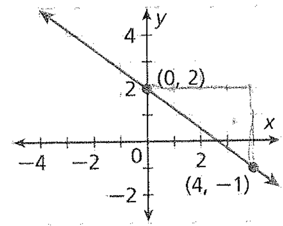
**11.** (2, 5) and (3, 1) **12.** (−9, −5) and (6, −5) **13.** (3, 4) and (3, −1)

**Each graph or table shows a linear relationship. Find the slope.**

**14.**

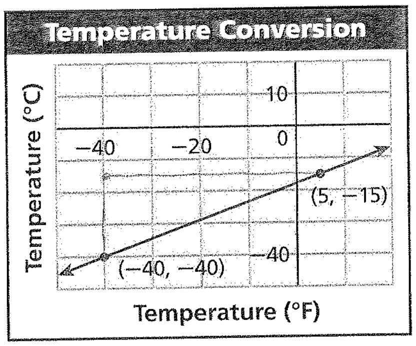
| $x$ | $y$ |
|---|---|
| 1 | 18.5 |
| 2 | 22 |
| 3 | 25.5 |
| 4 | 29 |

**15.**

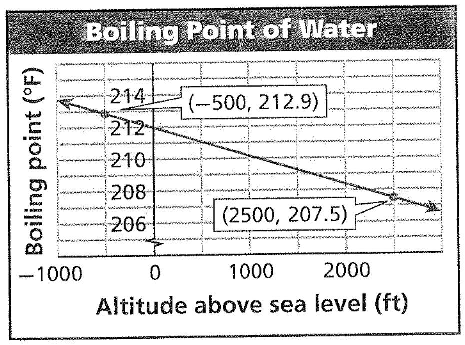



**Find the slope of each line. Then tell what the slope represents.**

**16.**




**17.**




**Find the slope of the line described by each equation.**

**18.** $7x + 13y = 91$ **19.** $5y = 130 - 13x$ **20.** $7 - 3y = 9x$

**21.** **ERROR ANALYSIS** Two students found the slope of the line that contains $(-6, 3)$ and $(2, -1)$. Who is incorrect? Explain the error.

(A) $m = \dfrac{-1-3}{2-(-6)} = \dfrac{-4}{8} = -\dfrac{1}{2}$

(B) $m = \dfrac{-1-3}{-6-2} = \dfrac{-4}{-8} = \dfrac{1}{2}$

**22.** **Environmental Science** The table shows how the number of cricket chirps per minute changes with the air temperature.

| Temperature (°F) | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| Chirps per minute | 0 | 40 | 80 | 120 | 160 | 200 |

a. Find the rates of change.

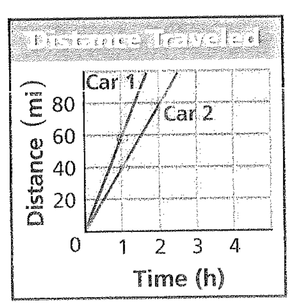
b. Is the graph of the data a line? If so, what is the slope? If not, explain why not.

**23.** **Critical Thinking** The graph shows the distance traveled by two cars.

a. Which car is going faster? How much faster?

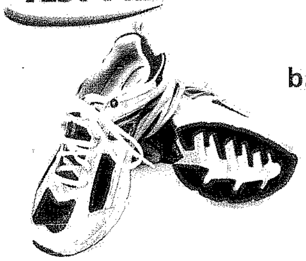
b. How are the speeds related to slope?

c. At what rate is the distance between the cars changing?




**24.** **Write About It** You are given the coordinates of two points on a line. Describe two different ways to find the slope of that line.

MULTI-STEP TEST PREP



25. This problem will prepare you for the Multi-Step Test Prep on page 342.

 a. One way to estimate your maximum heart rate is to subtract your age from 220. Write a function to describe the relationship between maximum heart rate $y$ and age $x$.

 b. The graph of this function is a line. Find its slope. Then tell what the slope represents.

TEST PREP

26. The equation $2y + 3x = -6$ describes a line with what slope?

 (A) $\frac{3}{2}$  (B) 0  (C) $\frac{1}{2}$  (D) $-\frac{3}{2}$

27. A line with slope $-\frac{1}{3}$ could pass through which of the following pairs of points?

 (F) $\left(0, -\frac{1}{3}\right)$ and $(1, 1)$  (H) $(0, 0)$ and $\left(-\frac{1}{3}, -\frac{1}{3}\right)$

 (G) $(-6, 5)$ and $(-3, 4)$  (J) $(5, -6)$ and $(4, 3)$

28. **Gridded Response** Find the slope of the line that contains $(-1, 2)$ and $(5, 5)$.

## CHALLENGE AND EXTEND

**Find the slope of the line that contains each pair of points.**

29. $(a, 0)$ and $(0, b)$

30. $(2x, y)$ and $(x, 3y)$

31. $(x, y)$ and $(x + 2, 3 - y)$

**Find the value of $x$ so that the points lie on a line with the given slope.**

32. $(x, 2)$ and $(-5, 8)$, $m = -1$

33. $(4, x)$ and $(6, 3x)$, $m = \frac{1}{2}$

34. $(1, -3)$ and $(3, x)$, $m = -1$

35. $(-10, -4)$ and $(x, x)$, $m = \frac{1}{7}$

36. A line contains the point $(1, 2)$ and has a slope of $\frac{1}{2}$. Use the slope formula to find another point on this line.

37. The points $(-2, 4)$, $(0, 2)$, and $(3, x - 1)$ all lie on the same line. What is the value of $x$? (*Hint:* Remember that the slope of a line is constant for any two points on the line.)

## SPIRAL REVIEW

**Solve each inequality and graph the solutions.** *(Lesson 3-7)*

38. $|x| + 5 < 16$

39. $|x + 8| < 3$

40. $3|x| \le 12$

41. $|x| - 11 \ge -4$

42. $|x - 6| > 10$

43. $|x + 1| \ge 7$

**Tell whether the given ordered pairs satisfy a linear function.** *(Lesson 5-1)*

44. $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$

45. $\{(9, 0), (8, -5), (5, -20), (3, -30)\}$

**Use the intercepts to graph the line described by each equation.** *(Lesson 5-2)*

46. $x - y = 5$

47. $3x + y = 9$

48. $y = 5x + 10$