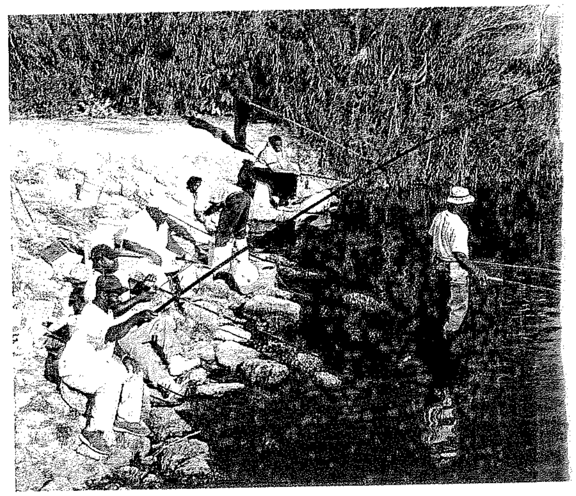
# 5-5 The Midpoint and Distance Formulas

***Objectives***
Apply the formula for midpoint.

Use the Distance Formula to find the distance between two points.

***Vocabulary***
midpoint

**Why learn this?**
You can use the coordinate plane to model and solve problems involving distances, such as the distance across a lake. (See Example 4.)



In Lesson 5-4, you used the coordinates of points to determine the slope of lines. You can also use coordinates to determine the *midpoint* of a line segment on the coordinate plane.

The **midpoint** of a line segment is the point that divides the segment into two congruent segments. *Congruent segments* are segments that have the same length.

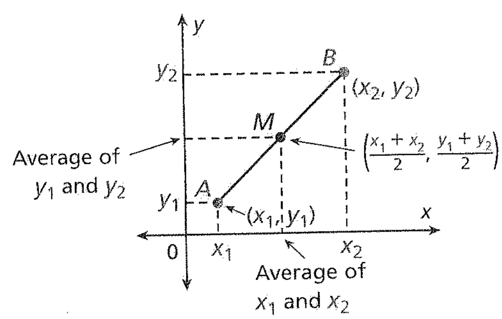
You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the $x$-coordinates and the average of the $y$-coordinates of the endpoints.

Know it! Note

**Midpoint Formula**

The midpoint $M$ of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



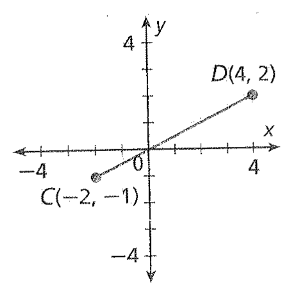

EXAMPLE 1 **Finding the Coordinates of a Midpoint**

**Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(-2, -1)$ and $D(4, 2)$.**

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *Write the formula.*

$M\left(\frac{-2 + 4}{2}, \frac{-1 + 2}{2}\right)$ *Substitute.*

$M\left(\frac{2}{2}, \frac{1}{2}\right) = M\left(1, \frac{1}{2}\right)$ *Simplify.*




**Remember!**
A segment is named by its endpoints. The notation $\overline{CD}$ is read "segment $CD$."

CHECK IT OUT! **1.** Find the coordinates of the midpoint of $\overline{EF}$ with endpoints $E(-2, 3)$ and $F(5, -3)$.

## EXAMPLE 2 Finding the Coordinates of an Endpoint

**$M$ is the midpoint of $\overline{AB}$. $A$ has coordinates $(2, 2)$, and $M$ has coordinates $(4, -3)$. Find the coordinates of $B$.**

**Step 1** Let the coordinates of $B$ equal $(x, y)$.

**Step 2** Use the Midpoint Formula.

$$(4, -3) = \left(\frac{2 + x}{2}, \frac{2 + y}{2}\right)$$

**Step 3** Find the $x$-coordinate.

$$4 = \frac{2 + x}{2}$$ *Set the coordinates equal.*

$$2(4) = 2\left(\frac{2 + x}{2}\right)$$ *Multiply both sides by 2.*

$$8 = 2 + x$$ *Simplify.*

$$\begin{array}{rr} -2 & -2 \\ \hline 6 = & x \end{array}$$ *Subtract 2 from both sides.* *Simplify.*

Find the $y$-coordinate.

$$-3 = \frac{2 + y}{2}$$

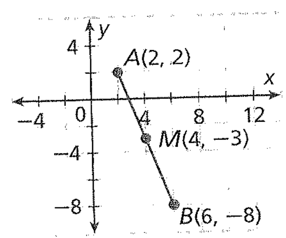
$$2(-3) = 2\left(\frac{2 + y}{2}\right)$$

$$-6 = 2 + y$$

$$\begin{array}{rr} -2 & -2 \\ \hline -8 = & y \end{array}$$

The coordinates of $B$ are $(6, -8)$.

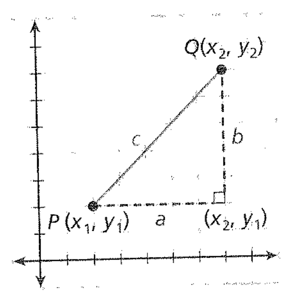
***Check*** Graph points $A$ and $B$ and midpoint $M$.




*Point $M$ appears to be the midpoint of $\overline{AB}$.*

**CHECK IT OUT!** **2.** $S$ is the midpoint of $\overline{RT}$. $R$ has coordinates $(-6, -1)$, and $S$ has coordinates $(-1, 1)$. Find the coordinates of $T$.

You can also use coordinates to find the distance between two points or the length of a line segment. To find the length of segment $PQ$, draw a horizontal segment from $P$ and a vertical segment from $Q$ to form a right triangle.




**Remember!**

The *Pythagorean Theorem* states that if a right triangle has legs of lengths $a$ and $b$ and a hypotenuse of length $c$, then $a^2 + b^2 = c^2$.

$$c^2 = a^2 + b^2$$ *Pythagorean Theorem*

$$c = \sqrt{a^2 + b^2}$$ *Solve for $c$. Use the positive square root to represent distance.*

$$PQ = \sqrt{\underbrace{(x_2 - x_1)^2}_{\text{Length of horizontal segment}} + \underbrace{(y_2 - y_1)^2}_{\text{Length of vertical segment}}}$$

This equation represents the Distance Formula.

**Know it! Note**

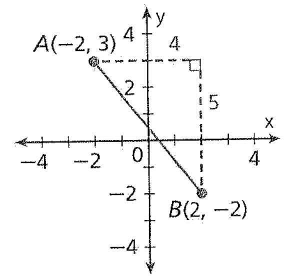
**Distance Formula**

In a coordinate plane, the distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 **Finding Distance in the Coordinate Plane**

**Use the Distance Formula to find the distance, to the nearest hundredth, from $A(-2, 3)$ to $B(2, -2)$.**




$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$d = \sqrt{[2 - (-2)]^2 + (-2 - 3)^2}$ *Substitute $(-2, 3)$ for $(x_1, y_1)$ and $(2, -2)$ for $(x_2, y_2)$.*

$d = \sqrt{4^2 + (-5)^2}$ *Subtract.*

$d = \sqrt{16 + 25}$ *Simplify powers.*

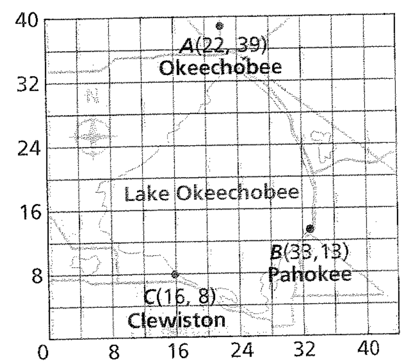
$d = \sqrt{41}$ *Add.*

$d \approx 6.40$ *Find the square root to the nearest hundredth.*

CHECK IT OUT! **3.** Use the Distance Formula to find the distance, to the nearest hundredth, from $R(3, 2)$ to $S(-3, -1)$.

EXAMPLE 4 ***Geography Application***

**Each unit on the map of Lake Okeechobee represents 1 mile. Delia and her father plan to travel from point $A$ near the town of Okeechobee to point $B$ at Pahokee. To the nearest tenth of a mile, how far do Delia and her father plan to travel?**




$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(33 - 22)^2 + (13 - 39)^2}$ *Substitute.*

$d = \sqrt{11^2 + (-26)^2}$ *Subtract.*

$d = \sqrt{121 + 676}$ *Simplify powers.*

$d = \sqrt{797}$ *Add.*

$d \approx 28.2$ *Find the square root to the nearest tenth.*

Delia and her father plan to travel about 28.2 miles.

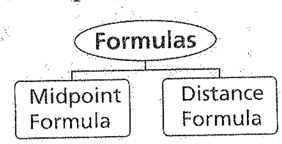
CHECK IT OUT! **4.** Jacob takes a boat from Pahokee to Clewiston. To the nearest tenth of a mile, how far does he travel?

## THINK AND DISCUSS

**1.** If you use the formula $M\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$ to find the midpoint of a segment, will you get the correct coordinates for the midpoint? Explain.

Know it! Note

**2. GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a formula. Then make a sketch that will illustrate the formula.



# 5-5 Exercises



## GUIDED PRACTICE

1. **Vocabulary** In your own words, describe the *midpoint* of a line segment.

SEE EXAMPLE 1 p. 330

**Find the coordinates of the midpoint of each segment.**

2. $\overline{AB}$ with endpoints $A(4, -6)$ and $B(-4, 2)$
3. $\overline{CD}$ with endpoints $C(0, -8)$ and $D(3, 0)$
4. $\overline{EF}$ with endpoints $E(-8, 17)$ and $F(-12, -16)$

SEE EXAMPLE 2 p. 331

5. $M$ is the midpoint of $\overline{LN}$. $L$ has coordinates $(-3, -1)$, and $M$ has coordinates $(0, 1)$. Find the coordinates of $N$.
6. $B$ is the midpoint of $\overline{AC}$. $A$ has coordinates $(-3, 4)$, and $B$ has coordinates $\left(-1\frac{1}{2}, 1\right)$. Find the coordinates of $C$.

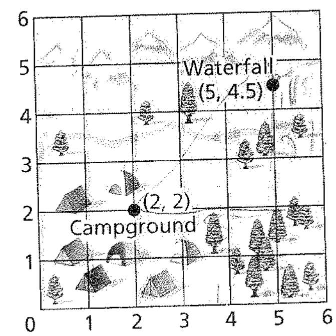
SEE EXAMPLE 3 p. 332

**Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.**

7. $A(1, -2)$ and $B(-4, -4)$
8. $X(-2, 7)$ and $Y(-2, -8)$
9. $V(2, -1)$ and $W(-4, 8)$

SEE EXAMPLE 4 p. 332

10. **Recreation** Each unit on the map of a public park represents 1 kilometer. To the nearest tenth of a kilometer, what is the distance from the campground to the waterfall?




## PRACTICE AND PROBLEM SOLVING

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 11–12 | 1 |
| 13–15 | 2 |
| 16–19 | 3 |
| 20 | 4 |

**Extra Practice**

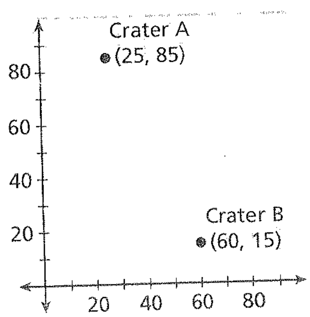
Skills Practice p. S12
Application Practice p. S32

**Find the coordinates of the midpoint of each segment.**

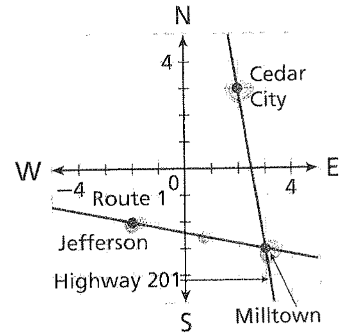
11. $\overline{XY}$ with endpoints $X(-3, -7)$ and $Y(-1, 1)$
12. $\overline{MN}$ with endpoints $M(12, -7)$ and $N(-5, -2)$
13. $M$ is the midpoint of $\overline{QR}$. $Q$ has coordinates $(-3, 5)$, and $M$ has coordinates $(7, -9)$. Find the coordinates of $R$.
14. $D$ is the midpoint of $\overline{CE}$. $E$ has coordinates $(-3, -2)$, and $D$ has coordinates $\left(2\frac{1}{2}, 1\right)$. Find the coordinates of $C$.
15. $Y$ is the midpoint of $\overline{XZ}$. $X$ has coordinates $(0, -1)$, and $Y$ has coordinates $\left(1, -4\frac{1}{2}\right)$. Find the coordinates of $Z$.

**Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.**

16. $U(0, 1)$ and $V(-3, -9)$
17. $M(10, -1)$ and $N(2, -5)$
18. $P(-10, 1)$ and $Q(5, 5)$
19. $F(6, 15)$ and $G(4, 24)$
20. **Astronomy** Each unit on the map of a section of a moon represents 1 kilometer. To the nearest tenth of a kilometer, what is the distance between the two craters?



For Exercises 21 and 22, use the map, and round your answers to the nearest tenth of a mile. Each unit on the map represents 1 mile.




**21.** How far is it from Cedar City to Milltown along Highway 201?

**22.** A car breaks down on Route 1 halfway between Jefferson and Milltown. A tow truck is sent out from Jefferson. How far does the truck travel to reach the car?

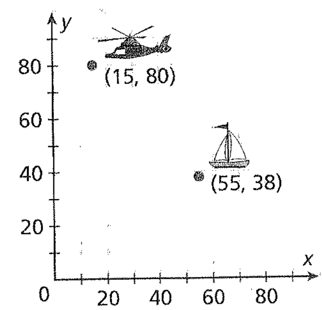
**23. Estimation** Estimate the distance between the points $(-5.21, 1.84)$ and $(16.62, -23.19)$. Explain how you determined your answer.

**24. Geometry** The coordinates of the vertices of $\Delta ABC$ are $A(1, 4)$, $B(-2, -1)$, and $C(3, -2)$. Find the perimeter of $\Delta ABC$ to the nearest whole number.

**Find the distance, to the nearest hundredth, between each pair of points.**

**25.** $J(0, 3)$ and $K(-6, -9)$ **26.** $L(-5, 2)$ and $M(-8, 10)$ **27.** $N(4, -6)$ and $P(-2, 7)$

**28. Aviation** A Coast Guard helicopter receives a distress signal from a boat. The units on the map represent miles. To the nearest minute, how long will it take the helicopter to reach the boat if the helicopter travels at an average speed of 75 miles per hour?




**29. Geometry** A diameter of a circle has endpoints $(-2, -5)$ and $(2, 1)$.

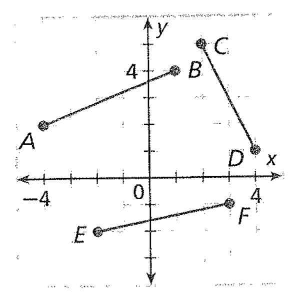
a. Find the length of the diameter to the nearest tenth.

b. Find the coordinates of the center of the circle.

c. Find the circumference of the circle to the nearest whole number.

**30. Travel** A group of tourists is traveling by camel in the desert. They are following a straight path from point $(2, 4)$ to point $(8, 12)$ on a map. Each unit on the map represents 1 mile. An oasis lies at the midpoint of the path. The group has already traveled 3.2 miles. How much farther do they need to go to reach the oasis?

**31. Multi-Step** Use the Distance Formula to order $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ from shortest to longest.




**32. Critical Thinking** Rebecca found the $x$-coordinate of the midpoint of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ by dividing the difference between $x_2$ and $x_1$ by 2 and then adding the quotient to $x_1$. Did this method give the correct $x$-coordinate for the midpoint? Explain.

**33. Write About It** Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or vertical line.

**MULTI-STEP TEST PREP**

**34.** This problem will help you prepare for the Multi-Step Test Prep on page 342.

On a map of a city park, the ordered pairs $(3, 5)$ and $(8, 17)$ mark the starting and ending points of a straight jogging trail. Each unit on the map represents 0.1 mile.

a. What is the length of the trail in miles?

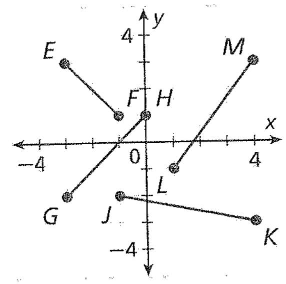
b. How many back-and-forth trips would Marisol need to make on the trail in order to run at least 7 miles?

## TEST PREP

35. Which segment has a length closest to 4 units?

Ⓐ $\overline{EF}$  Ⓒ $\overline{JK}$

Ⓑ $\overline{GH}$  Ⓓ $\overline{LM}$




36. What is the distance between the points (7, −3) and (−5, 6)?

Ⓕ 4  Ⓗ 15

Ⓖ 9  Ⓙ 21

37. A coordinate plane is placed over the map of a town. A library is located at (−5, 1), and a museum is located at (3, 5). What is the distance, to the nearest tenth, from the library to the museum?

Ⓐ 4.5 units  Ⓑ 5.7 units  Ⓒ 6.3 units  Ⓓ 8.9 units

38. **Short Response** Brian is driving along a straight highway. His truck can travel 22 miles per gallon of gasoline, and it has 2 gallons of gas remaining. On a map, the truck's current location is (7, 12), and the nearest gas station on the highway is located at (16, 52). Each unit on the map represents 1 mile. Will the truck reach the gas station before running out of gas? Support your answer.

## CHALLENGE AND EXTEND

39. **Geometry** Find the area of a trapezoid with vertices $A(-4, 2)$, $B(0, 4)$, $C(3, 3)$, and $D(-3, 0)$. (*Hint:* The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.)

40. $X$ has coordinates $(a, 3a)$, and $Y$ has coordinates $(-5a, 0)$. Find the coordinates of the midpoint of $\overline{XY}$.

41. The coordinates of $P$ are $(a - 5, 0)$. The coordinates of $Q$ are $(a + 1, a)$. The distance between $P$ and $Q$ is 10 units. Find the value of $a$.

42. Find two points on the $y$-axis that are a distance of 5 units from (4, 2).

43. The coordinates of $S$ are (4, 5). The coordinates of the midpoint of $\overline{ST}$ are (−1, −7). Find the length of $\overline{ST}$.

## SPIRAL REVIEW

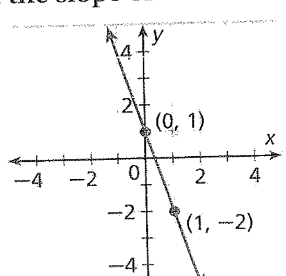
44. In the 2007–2008 season, the Orlando Magic won 52 games and lost 30 games. What percent of its games did the team win? Round your answer to the nearest percent. *(Lesson 2-9)*

**Solve each inequality and graph the solutions.** *(Lesson 3-5)*

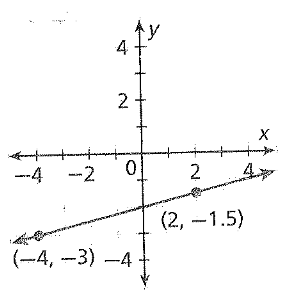
45. $4x < 7x + 6$  46. $4y + 3 \geq 5y + 8$  47. $6(z + 2) > 3z$

**Find the slope of each line.** *(Lesson 5-3)*

48.




49.



# 5-6 Direct Variation

***Objective***
Identify, write, and graph direct variation.

***Vocabulary***
direct variation
constant of variation

**Who uses this?**
Chefs can use direct variation to determine ingredients needed for a certain number of servings.

Paella is a rice dish that originated in Valencia, Spain.

A recipe for paella calls for 1 cup of rice to make 5 servings. In other words, a chef needs 1 cup of rice for every 5 servings.

| Rice (c) $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Servings $y$ | 5 | 10 | 15 | 20 |

The equation $y = 5x$ describes this relationship. In this relationship, the number of servings *varies directly* with the number of cups of rice.

A **direct variation** is a special type of linear relationship that can be written in the form $y = kx$, where $k$ is a nonzero constant called the **constant of variation**.

## EXAMPLE 1 Identifying Direct Variations from Equations

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

**A** $y = 4x$

This equation represents a direct variation because it is in the form $y = kx$. The constant of variation is 4.

**B** $-3x + 5y = 0$

$$\begin{aligned} -3x + 5y &= 0 && \textit{Solve the equation for y.} \\ \underline{+3x} \quad & \underline{+3x} && \textit{Since } -3x \textit{ is added to } 5y\textit{, add } 3x \textit{ to both sides.} \\ 5y &= 3x \\ \frac{5y}{5} &= \frac{3x}{5} && \textit{Since y is multiplied by 5, divide both sides by 5.} \\ y &= \frac{3}{5}x \end{aligned}$$

This equation represents a direct variation because it can be written in the form $y = kx$. The constant of variation is $\frac{3}{5}$.

**C** $2x + y = 10$

$$\begin{aligned} 2x + y &= 10 && \textit{Solve the equation for y.} \\ \underline{-2x} \quad & \underline{-2x} && \textit{Since } 2x \textit{ is added to y, subtract } 2x \textit{ from both sides.} \\ y &= -2x + 10 \end{aligned}$$

This equation does not represent a direct variation because it cannot be written in the form $y = kx$.

**CHECK IT OUT!** **Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

**1a.** $3y = 4x + 1$ **1b.** $3x = -4y$ **1c.** $y + 3x = 0$

What happens if you solve $y = kx$ for $k$?

$$y = kx$$

$$\frac{y}{x} = \frac{kx}{x} \qquad \textit{Divide both sides by } x\ (x \neq 0).$$

$$\frac{y}{x} = k$$

So, in a direct variation, the ratio $\frac{y}{x}$ is equal to the constant of variation. Another way to identify a direct variation is to check whether $\frac{y}{x}$ is the same for each ordered pair (except where $x = 0$).

## EXAMPLE 2 Identifying Direct Variations from Ordered Pairs

**Tell whether each relationship is a direct variation. Explain.**

**A**

| $x$ | 1 | 3 | 5 |
|---|---|---|---|
| $y$ | 6 | 18 | 30 |

**Method 1** Write an equation.

$y = 6x$ *Each y-value is 6 times the corresponding x-value.*

This is a direct variation because it can be written as $y = kx$, where $k = 6$.

**Method 2** Find $\frac{y}{x}$ for each ordered pair.

$\frac{6}{1} = 6 \qquad \frac{18}{3} = 6 \qquad \frac{30}{5} = 6$

This is a direct variation because $\frac{y}{x}$ is the same for each ordered pair.

**B**

| $x$ | 2 | 4 | 8 |
|---|---|---|---|
| $y$ | −2 | 0 | 4 |

**Method 1** Write an equation.

$y = x - 4$ *Each y-value is 4 less than the corresponding x-value.*

This is not a direct variation because it cannot be written as $y = kx$.

**Method 2** Find $\frac{y}{x}$ for each ordered pair.

$\frac{-2}{2} = -1 \qquad \frac{0}{4} = 0 \qquad \frac{4}{8} = \frac{1}{2}$

This is not a direct variation because $\frac{y}{x}$ is not the same for all ordered pairs.

**CHECK IT OUT!** **Tell whether each relationship is a direct variation. Explain.**

**2a.**

| $x$ | $y$ |
|---|---|
| −3 | 0 |
| 1 | 3 |
| 3 | 6 |

**2b.**

| $x$ | $y$ |
|---|---|
| 2.5 | −10 |
| 5 | −20 |
| 7.5 | −30 |

**2c.**

| $x$ | $y$ |
|---|---|
| −2 | 5 |
| 1 | 3 |
| 4 | 1 |

If you know one ordered pair that satisfies a direct variation, you can write the equation. You can also find other ordered pairs that satisfy the direct variation.

## EXAMPLE 3 Writing and Solving Direct Variation Equations

**The value of $y$ varies directly with $x$, and $y = 6$ when $x = 12$. Find $y$ when $x = 27$.**

**Method 1** Find the value of $k$ and then write the equation.

| | |
|---|---|
| $y = kx$ | *Write the equation for a direct variation.* |
| $6 = k(12)$ | *Substitute 6 for y and 12 for x. Solve for k.* |
| $\frac{1}{2} = k$ | *Since k is multiplied by 12, divide both sides by 12.* |

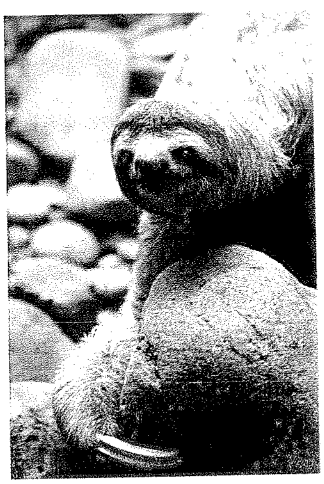
The equation is $y = \frac{1}{2}x$. When $x = 27$, $y = \frac{1}{2}(27) = 13.5$.

**Method 2** Use a proportion.

| | |
|---|---|
| $\frac{6}{12} = \frac{y}{27}$ | *In a direct variation, $\frac{y}{x}$ is the same for all values of x and y.* |
| $12y = 162$ | *Use cross products.* |
| $y = 13.5$ | *Since y is multiplied by 12, divide both sides by 12.* |

**CHECK IT OUT!** **3.** The value of $y$ varies directly with $x$, and $y = 4.5$ when $x = 0.5$. Find $y$ when $x = 10$.

## EXAMPLE 4 Graphing Direct Variations



**The three-toed sloth is an extremely slow animal. On the ground, it travels at a speed of about 6 feet per minute. Write a direct variation equation for the distance $y$ a sloth will travel in $x$ minutes. Then graph.**

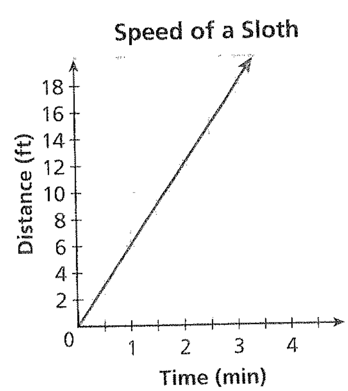
**Step 1** Write a direct variation equation.

| distance | = | 6 feet per minute | times | number of minutes |
|---|---|---|---|---|
| $y$ | = | 6 | • | $x$ |

**Step 2** Choose values of $x$ and generate ordered pairs.

| $x$ | $y = 6x$ | $(x, y)$ |
|---|---|---|
| 0 | $y = 6(0) = 0$ | $(0, 0)$ |
| 1 | $y = 6(1) = 6$ | $(1, 6)$ |
| 2 | $y = 6(2) = 12$ | $(2, 12)$ |

**Step 3** Graph the points and connect.




**CHECK IT OUT!** **4.** The perimeter $y$ of a square varies directly with its side length $x$. Write a direct variation equation for this relationship. Then graph.

Look at the graph in Example 4. It passes through $(0, 0)$ and has a slope of 6. The graph of any direct variation $y = kx$

- is a line through $(0, 0)$.
- has a slope of $k$.

## THINK AND DISCUSS

1. How do you know that a direct variation is linear?
2. How does the graph of a direct variation differ from the graphs of other types of linear relationships?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how you can use the given information to identify a direct variation.

Know it! Note

| Recognizing a Direct Variation | | |
|---|---|---|
| From an Equation | From Ordered Pairs | From a Graph |

# 5-6 Exercises

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### GUIDED PRACTICE

1. **Vocabulary** If $x$ varies directly with $y$, then the relationship between the two variables is said to be a __?__. (*direct variation* or *constant of variation*)

SEE EXAMPLE 1 p. 336

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

2. $y = 4x + 9$
3. $2y = -8x$
4. $x + y = 0$

SEE EXAMPLE 2 p. 337

**Tell whether each relationship is a direct variation. Explain.**

5.

| | | | |
|---|---|---|---|
| $x$ | 10 | 5 | 2 |
| $y$ | 12 | 7 | 4 |

6.

| | | | |
|---|---|---|---|
| $x$ | 3 | −1 | −4 |
| $y$ | −6 | 2 | 8 |

SEE EXAMPLE 3 p. 338

7. The value of $y$ varies directly with $x$, and $y = -3$ when $x = 1$. Find $y$ when $x = -6$.
8. The value of $y$ varies directly with $x$, and $y = 6$ when $x = 18$. Find $y$ when $x = 12$.

SEE EXAMPLE 4 p. 338

9. **Wages** Cameron earns $7 per hour at her after-school job. The total amount of her paycheck varies directly with the amount of time she works. Write a direct variation equation for the amount of money $y$ that she earns for working $x$ hours. Then graph.

### PRACTICE AND PROBLEM SOLVING

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

10. $y = \frac{1}{6}x$
11. $4y = x$
12. $x = 2y - 12$

**Tell whether each relationship is a direct variation. Explain.**

13.

| | | | |
|---|---|---|---|
| $x$ | 6 | 9 | 17 |
| $y$ | 13.2 | 19.8 | 37.4 |

14.

| | | | |
|---|---|---|---|
| $x$ | −6 | 3 | 12 |
| $y$ | 4 | −2 | −8 |

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 10–12 | 1 |
| 13–14 | 2 |
| 15–16 | 3 |
| 17 | 4 |

**Extra Practice**

Skills Practice p. S13
Application Practice p. S32

15. The value of $y$ varies directly with $x$, and $y = 8$ when $x = -32$. Find $y$ when $x = 64$.
16. The value of $y$ varies directly with $x$, and $y = \frac{1}{2}$ when $x = 3$. Find $y$ when $x = 1$.
17. While on his way to school, Norman saw that the cost of gasoline was $2.50 per gallon. Write a direct variation equation to describe the cost $y$ of $x$ gallons of gas. Then graph.

**Tell whether each relationship is a direct variation. Explain your answer.**

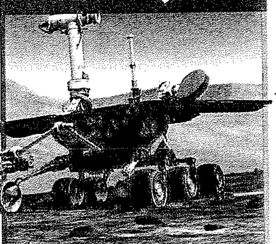
18. The equation $-15x + 4y = 0$ relates the length of a videotape in inches $x$ to its approximate playing time in seconds $y$.
19. The equation $y - 2.00x = 2.50$ relates the cost $y$ of a taxicab ride to distance $x$ of the cab ride in miles.

**Each ordered pair is a solution of a direct variation. Write the equation of direct variation. Then graph your equation and show that the slope of the line is equal to the constant of variation.**

| | | | |
|---|---|---|---|
| 20. $(2, 10)$ | 21. $(-3, 9)$ | 22. $(8, 2)$ | 23. $(1.5, 6)$ |
| 24. $(7, 21)$ | 25. $(1, 2)$ | 26. $(2, -16)$ | 27. $\left(\frac{1}{7}, 1\right)$ |
| 28. $(-2, 9)$ | 29. $(9, -2)$ | 30. $(4, 6)$ | 31. $(3, 4)$ |
| 32. $(5, 1)$ | 33. $(1, -6)$ | 34. $\left(-1, \frac{1}{2}\right)$ | 35. $(7, 2)$ |

**LINK**

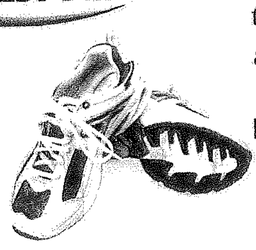
**Astronomy**



The Mars rover *Spirit* landed on Mars in January 2004 and immediately began sending photos of the planet's surface back to Earth.

36. **Astronomy** Weight varies directly with gravity. A Mars lander weighed 767 pounds on Earth but only 291 pounds on Mars. Its accompanying Mars rover weighed 155 pounds on Mars. How much did it weigh on Earth? Round your answer to the nearest pound.
37. **Environment** Mischa bought an energy-efficient washing machine. She will save about 15 gallons of water per wash load.
    a. Write an equation of direct variation to describe how many gallons of water $y$ Mischa saves for $x$ loads of laundry she washes.
    b. Graph your direct variation from part **a.** Is every point on the graph a solution in this situation? Why or why not?
    c. If Mischa does 2 loads of laundry per week, how many gallons of water will she have saved at the end of a year?
38. **Critical Thinking** If you double an $x$-value in a direct variation, will the corresponding $y$-value double? Explain.
39. **Write About It** In a direct variation $y = kx$, $k$ is sometimes called the "constant of proportionality." How are proportions related to direct variations?

**MULTI-STEP TEST PREP**



40. This problem will prepare you for the Multi-Step Test Prep on page 342.

    Rhea exercised on a treadmill at the gym. When she was finished, the display showed that she had walked at an average speed of 3 miles per hour.

    a. Write an equation that gives the number of miles $y$ that Rhea would cover in $x$ hours if she walked at this speed.
    b. Explain why this is a direct variation and find the value of $k$. What does this value represent in Rhea's situation?

## TEST PREP

41. Which equation does NOT represent a direct variation?

Ⓐ $y = \frac{1}{3}x$ Ⓑ $y = -2x$ Ⓒ $y = 4x + 1$ Ⓓ $6x - y = 0$

42. Identify which set of data represents a direct variation.

Ⓕ

| | | | |
|---|---|---|---|
| x | 1 | 2 | 3 |
| y | 1 | 2 | 3 |

Ⓗ

| | | | |
|---|---|---|---|
| x | 1 | 2 | 3 |
| y | 3 | 5 | 7 |

Ⓖ

| | | | |
|---|---|---|---|
| x | 1 | 2 | 3 |
| y | 0 | 1 | 2 |

Ⓙ

| | | | |
|---|---|---|---|
| x | 1 | 2 | 3 |
| y | 3 | 4 | 5 |

43. Two yards of fabric cost \$13, and 5 yards of fabric cost \$32.50. Which equation relates the cost of the fabric $c$ to its length $\ell$?

Ⓐ $c = 2.6\ell$ Ⓑ $c = 6.5\ell$ Ⓒ $c = 13\ell$ Ⓓ $c = 32.5\ell$

44. **Gridded Response** A car is traveling at a constant speed. After 3 hours, the car has traveled 180 miles. If the car continues to travel at the same constant speed, how many hours will it take to travel a total of 270 miles?

## CHALLENGE AND EXTEND

45. **Transportation** The function $y = 20x$ gives the number of miles $y$ that a sport-utility vehicle (SUV) can travel on $x$ gallons of gas. The function $y = 60x$ gives the number of miles $y$ that a hybrid car can travel on $x$ gallons of gas.
    a. If you drive 120 miles, how much gas will you save by driving the hybrid instead of the SUV?
    b. Graph both functions on the same coordinate plane. Will the lines ever meet other than at the origin? Explain.
    c. **What if...?** Shannon drives 15,000 miles in one year. How many gallons of gas will she use if she drives the SUV? the hybrid?

46. Suppose the equation $ax + by = c$, where $a$, $b$, and $c$ are real numbers, describes a direct variation. What do you know about the value of $c$?

## SPIRAL REVIEW

**Solve for the indicated variable.** *(Lesson 2-5)*

47. $p + 4q = 7;\ p$ 48. $\frac{s-5}{t} = 2;\ s$ 49. $xy + 2y = 4;\ x$

**Determine a relationship between the $x$- and $y$-values and write an equation.** *(Lesson 4-3)*

50.

| x | y |
|---|---|
| 1 | −5 |
| 2 | −4 |
| 3 | −3 |
| 4 | −2 |

51.

| x | y |
|---|---|
| 1 | −2 |
| 2 | −4 |
| 3 | −6 |
| 4 | −8 |

52.

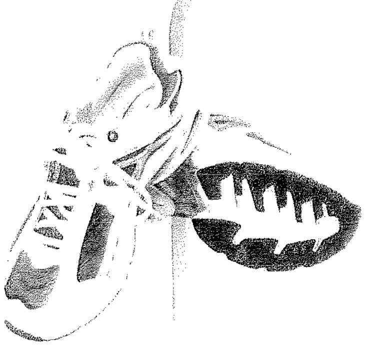
| x | y |
|---|---|
| −3 | 9 |
| −2 | 6 |
| −1 | 3 |
| 0 | 0 |

**Find the slope of the line described by each equation.** *(Lesson 5-4)*

53. $4x + y = -9$ 54. $6x - 3y = -9$ 55. $5x = 10y - 5$

# MULTI-STEP TEST PREP





## Characteristics of Linear Functions

**Heart Health** People who exercise need to be aware of their maximum heart rate.

1. One way to estimate your maximum heart rate $m$ is to subtract 85% of your age in years from 217. Create a table of values that shows the maximum heart rates for people ages 13 to 18. Then write an equation to describe the data in the table.
2. Use your table from Problem 1 to graph the relationship between age and maximum heart rate. What are the intercepts? What is the slope?
3. What do the intercepts represent in this situation?
4. What does the slope represent? Explain why the slope is negative.
5. Another formula for estimating maximum heart rate is $m = 206.3 - 0.711a$, where $a$ represents age in years. Describe how this equation is different from your equation in Problem 1. Include slope and intercepts in your description.
6. Which equation gives a higher maximum heart rate for people ages 75 and younger?
7. To be exercising in your *aerobic training zone* means that your heart rate is 70% to 80% of your maximum heart rate. Write two equations that someone could use to estimate the range of heart rates that are within his or her aerobic training zone. Use your equation for maximum heart rate from Problem 1.

# READY TO GO ON?

## Quiz for Lessons 5-1 Through 5-6

### 5-1 Identifying Linear Functions

Tell whether the given ordered pairs satisfy a linear function. Explain.

1.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 1 | 0 | 1 | 4 | 9 |

2. $\{(-3, 8), (-2, 6), (-1, 4), (0, 2), (1, 0)\}$

### 5-2 Using Intercepts

Use intercepts to graph the line described by each equation.

3. $2x - 4y = 16$

4. $-3y + 6x = -18$

5. $y = -3x + 3$

### 5-3 Rate of Change and Slope

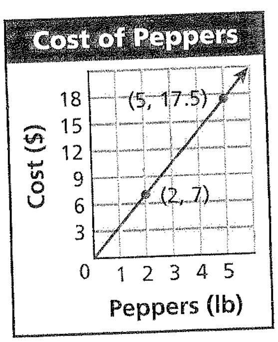
6. The chart gives the amount of water in a rain gauge in inches at various times. Graph the data and show the rates of change.

| Time (h) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Rain (in.) | 0.2 | 0.4 | 0.7 | 0.8 | 1.0 |

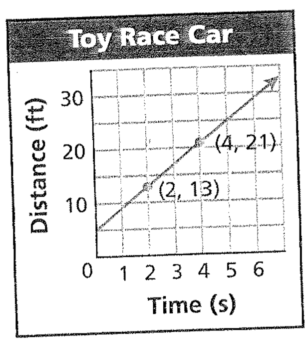
### 5-4 The Slope Formula

Find the slope of each line. Then tell what the slope represents.

7.




8.

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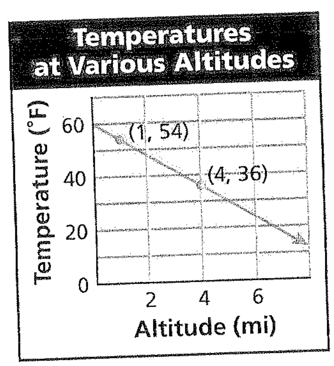



### 5-5 The Midpoint and Distance Formulas

10. Find the coordinates of the midpoint of $\overline{XY}$ with endpoints $X(-4, 6)$ and $Y(3, 8)$.

11. On a treasure map, the coordinates of a crooked palm tree are (3, 6), and the coordinates of a buried treasure chest are (12, 18). Each unit on the map represents 10 feet. What is the distance in feet between the palm tree and the treasure chest?

### 5-6 Direct Variation

Tell whether each relationship is a direct variation. If so, identify the constant of variation.

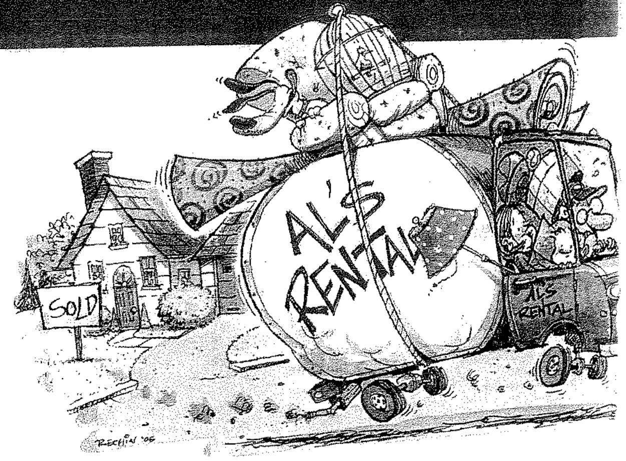
12.

| $x$ | 1 | 4 | 8 | 12 |
|---|---|---|---|---|
| $y$ | 3 | 6 | 10 | 14 |

13.

| $x$ | −6 | −2 | 0 | 3 |
|---|---|---|---|---|
| $y$ | −3 | −1 | 0 | 1.5 |

# 5-7 Slope-Intercept Form




*Objectives*
Write a linear equation in slope-intercept form.
Graph a line using slope-intercept form.

**Who uses this?**
Consumers can use slope-intercept form to model and calculate costs, such as the cost of renting a moving van. (See Example 4.)

You have seen that you can graph a line if you know two points on the line. Another way is to use the slope of the line and the point that contains the $y$-intercept.

**Writing Math**
Any integer can be written as a fraction with 1 in the denominator.
$-2 = \frac{-2}{1}$

**EXAMPLE 1 Graphing by Using Slope and $y$-intercept**

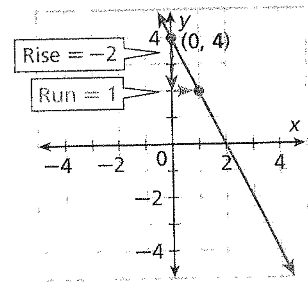
**Graph the line with slope $-2$ and $y$-intercept 4.**

**Step 1** The $y$-intercept is 4, so the line contains $(0, 4)$. Plot $(0, 4)$.

**Step 2** Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{1}$

Count 2 units down and 1 unit right from $(0, 4)$ and plot another point.

**Step 3** Draw the line through the two points.




**CHECK IT OUT!** **Graph each line given the slope and $y$-intercept.**

**1a.** slope $= 2$, $y$-intercept $= -3$ **1b.** slope $= -\frac{2}{3}$, $y$-intercept $= 1$

If you know the slope of a line and the $y$-intercept, you can write an equation that describes the line.

**Step 1** If a line has slope 2 and the $y$-intercept is 3, then $m = 2$ and $(0, 3)$ is on the line. Substitute these values into the slope formula.

*Slope formula* $\rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad 2 = \frac{y - 3}{x - 0}$ ←*Since you don't know $(x_2, y_2)$, use $(x, y)$.*

**Step 2** Solve for $y$: $2 = \frac{y - 3}{x - 0}$

$2 = \frac{y-3}{x}$ *Simplify the denominator.*

$2 \cdot x = \left(\frac{y-3}{x}\right) \cdot x$ *Multiply both sides by x.*

$2x = y - 3$

$\underline{+3} \quad \underline{+3}$ *Add 3 to both sides.*

$2x + 3 = y$, or $y = 2x + 3$

Know it! Note

## Slope-Intercept Form of a Linear Equation

If a line has slope $m$ and the $y$-intercept is $b$, then the line is described by the equation $y = mx + b$.

Any linear equation can be written in slope-intercept form by solving for $y$ and simplifying. In this form, you can immediately see the slope and $y$-intercept. Also, you can quickly graph a line when the equation is written in slope-intercept form.

### EXAMPLE 2 Writing Linear Equations in Slope-Intercept Form

**Write the equation that describes each line in slope-intercept form.**

**A** slope $= \frac{1}{3}$, $y$-intercept $= 6$

$y = mx + b$ *Substitute the given values for m and b.*

$y = \frac{1}{3}x + 6$ *Simplify if necessary.*

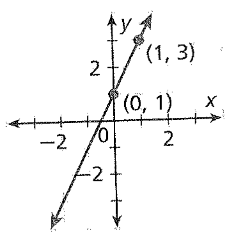
**B** slope $= 0$, $y$-intercept $= -5$

$y = mx + b$

$y = 0x + (-5)$

$y = -5$

**C**




**Step 1** Find the $y$-intercept. The graph crosses the $y$-axis at $(0, 1)$, so $b = 1$.

**Step 2** Find the slope. The line contains the points $(0, 1)$ and $(1, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$m = \frac{3-1}{1-0} = \frac{2}{1} = 2$ *Substitute (0, 1) for ($x_1$, $y_1$) and (1, 3) for ($x_2$, $y_2$).*

**Step 3** Write the equation.

$y = mx + b$ *Write the slope-intercept form.*

$y = 2x + 1$ *Substitute 2 for m and 1 for b.*

**D** slope $= 4$, $(2, 5)$ is on the line

**Step 1** Find the $y$-intercept.

$y = mx + b$ *Write the slope-intercept form.*

$5 = 4(2) + b$ *Substitute 4 for m, 2 for x, and 5 for y.*

$5 = 8 + b$ *Solve for b. Since 8 is added to b, subract 8 from both sides to undo the addition.*

$\underline{-8 \quad -8}$

$-3 = b$

**Step 2** Write the equation.

$y = mx + b$ *Write the slope-intercept form.*

$y = 4x + (-3)$ *Substitute 4 for m and –3 for b.*

$y = 4x - 3$

CHECK IT OUT! **Write the equation that describes each line in slope-intercept form.**

**2a.** slope $= -12$, $y$-intercept $= -\frac{1}{2}$

**2b.** slope $= 1$, $y$-intercept $= 0$

**2c.** slope $= 8$, $(-3, 1)$ is on the line.

**EXAMPLE 3** **Using Slope-Intercept Form to Graph**

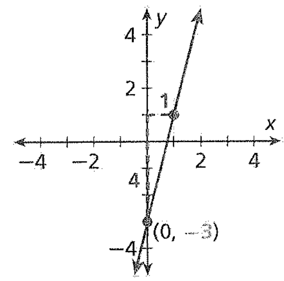
**Write each equation in slope-intercept form. Then graph the line described by the equation.**

**A** $y = 4x - 3$

$y = 4x - 3$ is in the form $y = mx + b$.

slope: $m = 4 = \frac{4}{1}$

$y$-intercept: $b = -3$




**Step 1** Plot $(0, -3)$.

**Step 2** Count 4 units up and 1 unit right and plot another point.

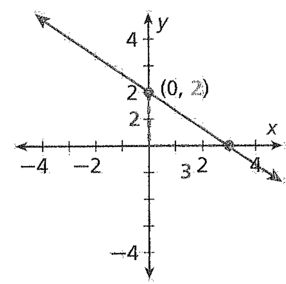
**Step 3** Draw the line connecting the two points.

**B** $y = -\frac{2}{3}x + 2$

$y = -\frac{2}{3}x + 2$ is in the form $y = mx + b$.

slope: $m = -\frac{2}{3} = \frac{-2}{3}$

$y$-intercept: $b = 2$




**Step 1** Plot $(0, 2)$

**Step 2** Count 2 units down and 3 units right and plot another point.

**Step 3** Draw the line connecting the two points.

**C** $3x + 2y = 8$

**Step 1** Write the equation in slope-intercept form by solving for $y$.

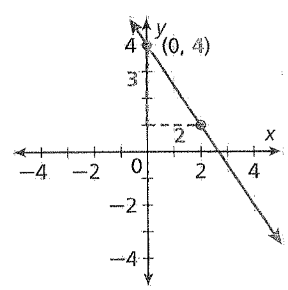
$$\begin{aligned} 3x + 2y &= 8 \\ \underline{-3x \qquad} & \underline{\;-3x} \qquad \textit{Subtract } 3x \textit{ from both sides.} \\ 2y &= 8 - 3x \\ \frac{2y}{2} &= \frac{8 - 3x}{2} \qquad \textit{Since } y \textit{ is multiplied by 2, divide both sides by 2.} \\ y &= 4 - \frac{3}{2}x \qquad \frac{3x}{2} = \frac{3}{2}x \\ y &= -\frac{3}{2}x + 4 \qquad \textit{Write the equation in the form } y = mx + b. \end{aligned}$$

**Step 2** Graph the line.

$y = -\frac{3}{2}x + 4$ is in the form $y = mx + b$.

slope: $m = -\frac{3}{2} = \frac{-3}{2}$

$y$-intercept: $b = 4$




- Plot $(0, 4)$.
- Then count 3 units down and 2 units right and plot another point.
- Draw the line connecting the two points.

**Helpful Hint**

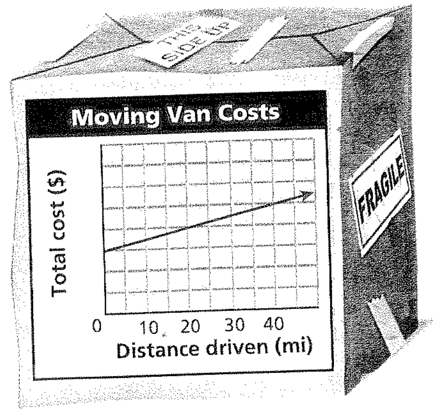
To divide $(8 - 3x)$ by 2, you can multiply by $\frac{1}{2}$ and use the Distributive Property.

$$\begin{aligned} \frac{8 - 3x}{2} &= \frac{1}{2}(8 - 3x) \\ &= \frac{1}{2}(8) + \frac{1}{2}(-3x) \\ &= 4 - \frac{3}{2}x \end{aligned}$$

**CHECK IT OUT!** **Write each equation in slope-intercept form. Then graph the line described by the equation.**

**3a.** $y = \frac{2}{3}x$ **3b.** $6x + 2y = 10$ **3c.** $y = -4$

## EXAMPLE 4 *Consumer Application*




To rent a van, a moving company charges $30.00 plus $0.50 per mile. The cost as a function of the number of miles driven is shown in the graph.

**a. Write an equation that represents the cost as a function of the number of miles.**

| Cost | is | \$0.50 per mile | times | miles | plus | \$30.00 |
|---|---|---|---|---|---|---|
| $y$ | $=$ | $0.5$ | $\cdot$ | $x$ | $+$ | $30$ |

An equation is $y = 0.5x + 30$.

**b. Identify the slope and $y$-intercept and describe their meanings.**

The $y$-intercept is 30. This is the cost for 0 miles, or the initial fee of $30.00.

The slope is 0.5. This is the rate of change of the cost: $0.50 per mile.

**c. Find the cost of the van for 150 miles.**

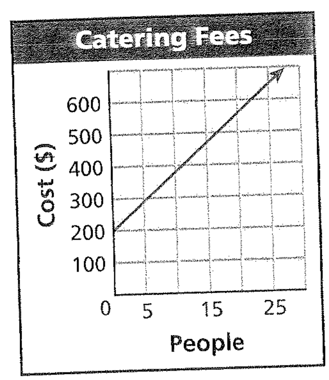
$y = 0.5x + 30$

$= 0.5(150) + 30 = 105$ *Substitute 150 for x in the equation.*

The cost of the van for 150 miles is $105.

**CHECK IT OUT!**

4. A caterer charges a $200 fee plus $18 per person served. The cost as a function of the number of guests is shown in the graph.
   a. Write an equation that represents the cost as a function of the number of guests.
   b. Identify the slope and $y$-intercept and describe their meanings.
   c. Find the cost of catering an event for 200 guests.




## THINK AND DISCUSS

1. If a linear function has a $y$-intercept of $b$, at what point does its graph cross the $y$-axis?
2. Where does the line described by $y = 4.395x - 23.75$ cross the $y$-axis?
3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

Graphing the Line Described by $y = mx + b$

1. Plot the point __?__ . → 2. Find a second point on the line by __?__ . → 3. Draw __?__ .

# 5-7 Exercises

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## GUIDED PRACTICE

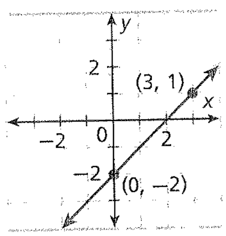
SEE EXAMPLE 1
p. 344

**Graph each line given the slope and $y$-intercept.**

1. slope $= \frac{1}{3}$, $y$-intercept $= -3$
2. slope $= 0.5$, $y$-intercept $= 3.5$
3. slope $= 5$, $y$-intercept $= -1$
4. slope $= -2$, $y$-intercept $= 2$

SEE EXAMPLE 2
p. 345

**Write the equation that describes each line in slope-intercept form.**

5. 


6. slope $= 8$, $y$-intercept $= 2$
7. slope $= 0$, $y$-intercept $= -3$
8. slope $= 5$, $(2, 7)$ is on the line.
9. slope $= -2$, $(1, -3)$ is on the line.

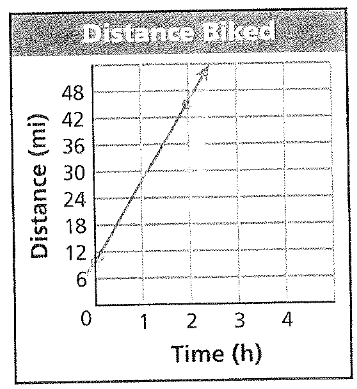
SEE EXAMPLE 3
p. 346

**Write each equation in slope-intercept form. Then graph the line described by the equation.**

10. $y = \frac{2}{5}x - 6$
11. $3x - y = 1$
12. $2x + y = 4$

SEE EXAMPLE 4
p. 347

13. Helen is in a bicycle race. She has already biked 10 miles and is now biking at a rate of 18 miles per hour. Her distance as a function of time is shown in the graph.
    a. Write an equation that represents the distance Helen has biked as a function of time.
    b. Identify the slope and $y$-intercept and describe their meanings.
    c. How far will Helen have biked after 2 hours?




## PRACTICE AND PROBLEM SOLVING

**Independent Practice**

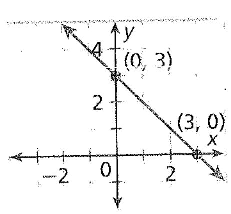
| For Exercises | See Example |
|---|---|
| 14–17 | 1 |
| 19–22 | 2 |
| 23–25 | 3 |
| 26 | 4 |

**Extra Practice**
Skills Practice p. S13
Application Practice p. S32

**Graph each line given the slope and $y$-intercept.**

14. slope $= \frac{1}{4}$, $y$-intercept $= 7$
15. slope $= -6$, $y$-intercept $= -3$
16. slope $= 1$, $y$-intercept $= -4$
17. slope $= -\frac{4}{5}$, $y$-intercept $= 6$

**Write the equation that describes each line in slope-intercept form.**

18. 


19. slope $= 5$, $y$-intercept $= -9$
20. slope $= -\frac{2}{3}$, $y$-intercept $= 2$
21. slope $= -\frac{1}{2}$, $(6, 4)$ is on the line.
22. slope $= 0$, $(6, -8)$ is on the line.

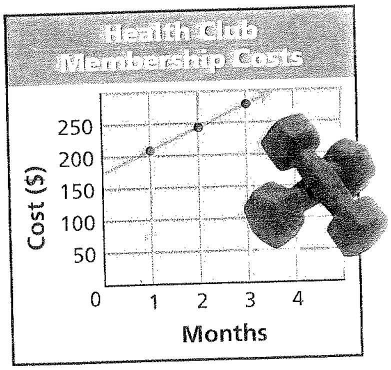
**Write each equation in slope-intercept form. Then graph the line described by the equation.**

**23.** $-\frac{1}{2}x + y = 4$

**24.** $\frac{2}{3}x + y = 2$

**25.** $2x + y = 8$

**26. Fitness** Pauline's health club has an enrollment fee of \$175 and costs \$35 per month. Total cost as a function of number of membership months is shown in the graph.




a. Write an equation that represents the total cost as a function of months.

b. Identify the slope and $y$-intercept and describe their meanings.

c. Find the cost of one year of membership.

**27.** A company rents video games. The table shows the linear relationship between the number of games a customer can rent at one time and the monthly cost of the service.

a. Graph the relationship.

b. Write an equation that represents the monthly cost as a function of games rented at one time.

| Games Rented at One Time | 1 | 2 | 3 |
|---|---|---|---|
| Monthly Cost (\$) | 14 | 18 | 22 |

**Critical Thinking** **Tell whether each situation is possible or impossible. If possible, draw a sketch of the graphs. If impossible, explain.**

**28.** Two different lines have the same slope.

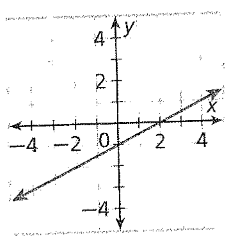
**29.** Two different linear functions have the same $y$-intercept.

**30.** Two intersecting lines have the same slope.

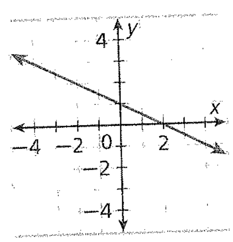
**31.** A linear function does not have a $y$-intercept.

**Match each equation with its corresponding graph.**

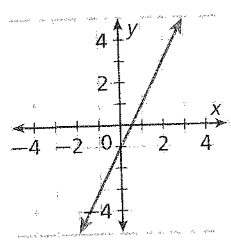
**32.**



**33.**



**34.**



**A.** $y = 2x - 1$

**B.** $y = \frac{1}{2}x - 1$

**C.** $y = -\frac{1}{2}x + 1$

**35. Write About It** Write an equation that describes a vertical line. Can you write this equation in slope-intercept form? Why or why not?

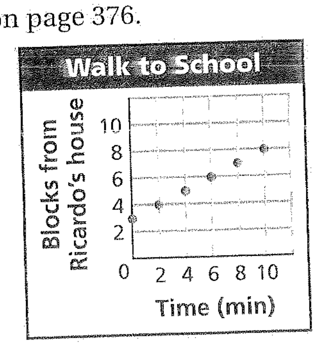
**MULTI-STEP TEST PREP**

**36.** This problem will prepare you for the Multi-Step Test Prep on page 376.

a. Ricardo and Sam walk from Sam's house to school. Sam lives 3 blocks from Ricardo's house. The graph shows their distance from Ricardo's house as they walk to school. Create a table of these values.

b. Find an equation for the distance as a function of time.

c. What are the slope and $y$-intercept? What do they represent in this situation?




## TEST PREP

37. Which function has the same $y$-intercept as $y = \frac{1}{2}x - 2$?

(A) $2x + 3y = 6$ (B) $x + 4y = -8$ (C) $-\frac{1}{2}x + y = 4$ (D) $\frac{1}{2}x - 2y = -2$

38. What is the slope-intercept form of $x - y = -8$?

(F) $y = -x - 8$ (G) $y = x - 8$ (H) $y = -x + 8$ (J) $y = x + 8$

39. Which function has a $y$-intercept of 3?

(A) $2x - y = 3$ (B) $2x + y = 3$ (C) $2x + y = 6$ (D) $y = 3x$

40. **Gridded Response** What is the slope of the line described by $-6x = -2y + 5$?

41. **Short Response** Write a function whose graph has the same slope as the line described by $3x - 9y = 9$ and the same $y$-intercept as $8x - 2y = 6$. Show your work.

## CHALLENGE AND EXTEND

42. The standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. What is the slope? What is the $y$-intercept?

43. What value of $n$ in the equation $nx + 5 = 3y$ would give a line with slope $-2$?

44. If $b$ is the $y$-intercept of a linear function whose graph has slope $m$, then $y = mx + b$ describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

| Statements | Reasons |
|---|---|
| 1. $m = \frac{y_2 - y_1}{x_2 - x_1}$ | 1. Slope formula |
| 2. $m = \frac{y - b}{x - 0}$ | 2. By definition, if $b$ is the $y$-intercept, then $(\quad, b)$ is a point on the line. $(x, y)$ is any other point on the line. |
| 3. $m = \frac{y - b}{x}$ | 3. ___?___ |
| 4. $m \quad = y - b$ | 4. Multiplication Property of Equality (Multiply both sides of the equation by $x$.) |
| 5. $mx + b = y$, or $y = mx + b$ | 5. ___?___ |

## SPIRAL REVIEW

**Define a variable and write an inequality for each situation. Graph the solutions.** *(Lesson 3-1)*

45. Molly has, at most, 2 hours to work out at the gym today.

46. Mishenko is hoping to save at least $300 this month.

**Solve each inequality.** *(Lesson 3-5)*

47. $3n \leq 2n + 8$

48. $4x + 4 > 2(x + 5)$

49. $2(2t + 1) > 6t + 8$

50. The amount of water conditioner needed for an aquarium varies directly with the capacity of the aquarium. For every 10 gallons of water, you need 1 teaspoon of conditioner. Write a direct variation equation for the amount of water conditioner $y$ needed for an aquarium that holds $x$ gallons of water. Then graph. *(Lesson 5-6)*

# 5-8 Point-Slope Form


**Objectives**

Graph a line and write a linear equation using point-slope form.

Write a linear equation given two points.

**Why learn this?**

You can use point-slope form to represent a cost function, such as the cost of placing a newspaper ad. (See Example 5.)

If you know the slope and any point on the line, you can write an equation of the line by using the slope formula. For example, suppose a line has a slope of 3 and contains $(2, 1)$. Let $(x, y)$ be any other point on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \longrightarrow 3 = \frac{y - 1}{x - 2}$$ *Substitute into the slope formula.*

*Slope formula*

$$3(x-2) = \left(\frac{y-1}{x-2}\right)(x-2)$$ *Multiplication Property of Equality*

$$3(x-2) = y - 1$$ *Simplify.*

$$y - 1 = 3(x-2)$$

**Know it! Note**

**Point-Slope Form of a Linear Equation**

The line with slope $m$ that contains the point $(x_1, y_1)$ can be described by the equation $y - y_1 = m(x - x_1)$.

**EXAMPLE 1 Writing Linear Equations in Point-Slope Form**

**Write an equation in point-slope form for the line with the given slope that contains the given point.**

**A** slope $= \frac{5}{2}$; $(-3, 0)$

$y - y_1 = m(x - x_1)$ *Write the point-slope form.*

$y - 0 = \frac{5}{2}[x - (-3)]$ *Substitute $\frac{5}{2}$ for $m$, $-3$ for $x_1$, and $0$ for $y_1$.*

$y - 0 = \frac{5}{2}(x + 3)$ *Rewrite subtraction of negative numbers as addition.*

**B** slope $= -7$; $(4, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = -7(x - 4)$

**C** slope $= 0$; $(-2, -3)$

$y - y_1 = m(x - x_1)$

$y - (-3) = 0[x - (-2)]$

$y + 3 = 0(x + 2)$

**CHECK IT OUT!** Write an equation in point-slope form for the line with the given slope that contains the given point.

**1a.** slope $= 2$; $\left(\frac{1}{2}, 1\right)$

**1b.** slope $= 0$; $(3, -4)$

In Lesson 5-7, you graphed a line given its equation in slope-intercept form. You can also graph a line when given its equation in point-slope form. Start by using the equation to identify a point on the line. Then use the slope of the line to identify a second point.

## EXAMPLE 2 Using Point-Slope Form to Graph

**Graph the line described by each equation.**

**A** $y - 1 = 3(x - 1)$

$y - 1 = 3(x - 1)$ is in the form $y - y_1 = m(x - x_1)$.

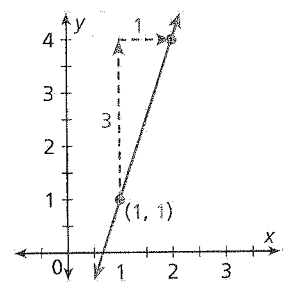
The line contains the point $(1, 1)$.

slope: $m = 3 = \frac{3}{1}$

**Step 1** Plot $(1, 1)$.

**Step 2** Count 3 units up and 1 unit right and plot another point.

**Step 3** Draw the line connecting the two points.




**B** $y + 2 = -\frac{1}{2}(x - 3)$

**Step 1** Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

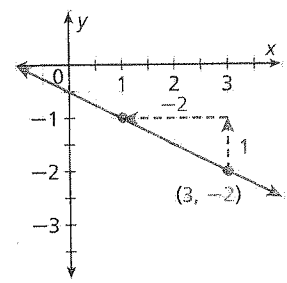
$y - (-2) = -\frac{1}{2}(x - 3)$ *Rewrite addition of 2 as subtraction of −2.*

**Step 2** Graph the line.

The line contains the point $(3, -2)$.

slope: $m = -\frac{1}{2} = \frac{1}{-2}$

- Plot $(3, -2)$.
- Count 1 unit up and 2 units left and plot another point.
- Draw the line connecting the two points.




**Helpful Hint**

For a negative fraction, you can write the negative sign in one of three places.

$$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$$

**CHECK IT OUT!** **Graph the line described by each equation.**

**2a.** $y + 2 = -(x - 2)$ **2b.** $y + 3 = -2(x - 1)$

## EXAMPLE 3 Writing Linear Equations in Slope-Intercept Form

**Write the equation that describes each line in slope-intercept form.**

**A** **slope = −4, (−1, −2) is on the line.**

**Step 1** Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

$y - (-2) = -4[x - (-1)]$

**Step 2** Write the equation in slope-intercept form by solving for $y$.

$y - (-2) = -4[x - (-1)]$

$$\begin{aligned} y + 2 &= -4(x + 1) && \textit{Rewrite subtraction of negative numbers as addition. Distribute −4 on the right side.} \\ y + 2 &= -4x - 4 \\ \underline{-2} &\quad \underline{-2} && \textit{Subtract 2 from both sides.} \\ y &= -4x - 6 \end{aligned}$$

**B** **$(1, -4)$ and $(3, 2)$ are on the line.**

**Step 1** Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3$$

**Step 2** Substitute the slope and one of the points into the point-slope form. Then write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$ *Use (3, 2).*

$y - 2 = 3x - 9$ *Distribute 3 on the right side.*

$y = 3x - 7$ *Add 2 to both sides.*

**C** **$x$-intercept $= -2$, $y$-intercept $= 4$**

**Step 1** Use the intercepts to find two points: $(-2, 0)$ and $(0, 4)$.

**Step 2** Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$$

**Step 3** Write the equation in slope-intercept form.

$y = mx + b$ *Write the slope-intercept form.*

$y = 2x + 4$ *Substitute 2 for m and 4 for b.*

**CHECK IT OUT!** **Write the equation that describes each line in slope-intercept form.**

**3a.** slope $= \frac{1}{3}$, $(-3, 1)$ is on the line.

**3b.** $(1, -2)$ and $(3, 10)$ are on the line.

## EXAMPLE 4 Using Two Points to Find Intercepts

**The points (4, 8) and (−1, −12) are on a line. Find the intercepts.**

**Step 1** Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 8}{-1 - 4} = \frac{-20}{-5} = 4$$

**Step 2** Write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ *Write the point-slope form.*

$y - 8 = 4(x - 4)$ *Substitute (4, 8) for $(x_1, y_1)$ and 4 for m.*

$y - 8 = 4x - 16$ *Distribute 4 on the right side.*

$y = 4x - 8$ *Add 8 to both sides.*

**Step 3** Find the intercepts.

**$x$-intercept:**

$y = 4x - 8$ *Replace y with 0 and solve for x.*

$0 = 4x - 8$

$8 = 4x$

$2 = x$

**$y$-intercept:**

$y = 4x - 8$ *Use the slope-intercept form to identify the y-intercept.*

$b = -8$

The $x$-intercept is 2, and the $y$-intercept is $-8$.

**CHECK IT OUT!** 4. The points (2, 15) and (−4, −3) are on a line. Find the intercepts.

**EXAMPLE 5** PROBLEM SOLVING

## Problem-Solving Application

The cost to place an ad in a newspaper for one week is a linear function of the number of lines in the ad. The costs for 3, 5, and 10 lines are shown. Write an equation in slope-intercept form that represents the function. Then find the cost of an ad that is 18 lines long.

City Gazette

**Newspaper Ad Costs**

| Lines | 3 | 5 | 10 |
|---|---|---|---|
| Cost ($) | 13.50 | 18.50 | 31 |

### 1 Understand the Problem

- The **answer** will have two parts—an equation in slope-intercept form and the cost of an ad that is 18 lines long.
- The ordered pairs given in the table satisfy the equation.

### 2 Make a Plan

First, find the slope. Then use point-slope form to write the equation. Finally, write the equation in slope-intercept form.

### 3 Solve

**Step 1** Choose any two ordered pairs from the table to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18.50 - 13.50}{5 - 3} = \frac{5}{2} = 2.5 \quad \textit{Use (3, 13.50) and (5, 18.50).}$$

**Step 2** Substitute the slope and any ordered pair from the table into the point-slope form.

$y - y_1 = m(x - x_1)$

$y - 31 = 2.5(x - 10)$ *Use (10, 31).*

**Step 3** Write the equation in slope-intercept form by solving for $y$.

$y - 31 = 2.5(x - 10)$

$y - 31 = 2.5x - 25$ *Distribute 2.5.*

$y = 2.5x + 6$ *Add 31 to both sides.*

**Step 4** Find the cost of an ad containing 18 lines by substituting 18 for $x$.

$y = 2.5x + 6$

$y = 2.5(18) + 6 = 51$

The cost of an ad containing 18 lines is $51.

### 4 Look Back

Check the equation by substituting the ordered pairs (3, 13.50) and (5, 18.50).

| $y = 2.5x + 6$ | |
|---|---|
| 13.50 | 2.5(3) + 6 |
| 13.5 | 7.5 + 6 |
| 13.5 | 13.5 ✓ |

| $y = 2.5x + 6$ | |
|---|---|
| 18.50 | 2.5(5) + 6 |
| 18.5 | 12.5 + 6 |
| 18.5 | 18.5 ✓ |

**CHECK IT OUT!**

5. **What if...?** At a different newspaper, the costs to place an ad for one week are shown. Write an equation in slope-intercept form that represents this linear function. Then find the cost of an ad that is 21 lines long.

| Lines | Cost ($) |
|---|---|
| 3 | 12.75 |
| 5 | 17.25 |
| 10 | 28.50 |

## THINK AND DISCUSS

1. How are point-slope form and slope-intercept form alike? different?
2. When is point-slope form useful? When is slope-intercept form useful?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find the equation of a line by using the given method.

Know it! Note

**Writing the Equation of a Line**

- If you know two points on the line
- If you know the slope and *y*-intercept
- If you know the slope and a point on the line

# 5-8 Exercises

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Homework Help Online
KEYWORD: MA11 5-8
Parent Resources Online
KEYWORD: MA7 Parent

## GUIDED PRACTICE

SEE EXAMPLE 1 p. 351

**Write an equation in point-slope form for the line with the given slope that contains the given point.**

1. slope $= \frac{1}{5}$; $(2, -6)$
2. slope $= -4$; $(1, 5)$
3. slope $= 0$; $(3, -7)$

SEE EXAMPLE 2 p. 352

**Graph the line described by each equation.**

4. $y - 1 = -(x - 3)$
5. $y + 2 = -2(x + 4)$
6. $y + 1 = -\frac{1}{2}(x + 4)$

SEE EXAMPLE 3 p. 352

**Write the equation that describes each line in slope-intercept form.**

7. slope $= -\frac{1}{3}$, $(-3, 8)$ is on the line.
8. slope $= 2$; $(1, 1)$ is on the line.
9. $(-2, 2)$ and $(2, -2)$ are on the line.
10. $(1, 1)$ and $(-5, 3)$ are on the line.
11. $x$-intercept $= 8$, $y$-intercept $= 4$
12. $x$-intercept $= -2$, $y$-intercept $= 3$

SEE EXAMPLE 4 p. 353

**Each pair of points is on a line. Find the intercepts.**

13. $(5, 2)$ and $(7, 4)$
14. $(-1, 5)$ and $(-3, -5)$
15. $(2, 9)$ and $(-4, -9)$

SEE EXAMPLE 5 p. 354

16. **Measurement** An oil tank is being filled at a constant rate. The depth of the oil is a function of the number of minutes the tank has been filling, as shown in the table. Write an equation in slope-intercept form that represents this linear function. Then find the depth of the oil after one-half hour.

| Time (min) | Depth (ft) |
|---|---|
| 0 | 3 |
| 10 | 5 |
| 15 | 6 |

## PRACTICE AND PROBLEM SOLVING

**Write an equation in point-slope form for the line with the given slope that contains the given point.**

17. slope $= \frac{2}{9}$; $(-1, 5)$
18. slope $= 0$; $(4, -2)$
19. slope $= 8$; $(1, 8)$

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 17–19 | 1 |
| 20–22 | 2 |
| 23–30 | 3 |
| 31–33 | 4 |
| 34 | 5 |

**Extra Practice**

Skills Practice p. S13
Application Practice p. S32

**Graph the line described by each equation.**

**20.** $y - 4 = -\frac{1}{2}(x + 3)$ **21.** $y + 2 = \frac{3}{5}(x - 1)$ **22.** $y - 0 = 4(x - 1)$

**Write the equation that describes each line in slope-intercept form.**

**23.** slope $= -\frac{2}{7}$, $(14, -3)$ is on the line.

**24.** slope $= \frac{4}{5}$, $(-15, 1)$ is on the line.

**25.** slope $= -6$, $(9, 3)$ is on the line.

**26.** $(7, 8)$ and $(-7, 6)$ are on the line.

**27.** $(2, 7)$ and $(4, -4)$ are on the line.

**28.** $(-1, 2)$ and $(4, -23)$ are on the line.

**29.** $x$-intercept $= 3$, $y$-intercept $= -6$

**30.** $x$-intercept $= 4$, $y$-intercept $= -1$

**Each pair of points is on a line. Find the intercepts.**

**31.** $(-1, -4)$ and $(6, 10)$ **32.** $(3, 4)$ and $(-6, 16)$ **33.** $(4, 15)$ and $(-2, 6)$

**34. History** The amount of fresh water left in the tanks of a 19th-century clipper ship is a linear function of the time since the ship left port, as shown in the table. Write an equation in slope-intercept form that represents the function. Then find the amount of water that will be left in the ship's tanks 50 days after leaving port.

**Fresh Water Aboard Ship**

| Time (days) | Amount (gal) |
|---|---|
| 1 | 3555 |
| 8 | 3240 |
| 15 | 2925 |

**LINK**

**Science**

As altitude increases, the amount of breathable oxygen decreases. At elevations above 8000 feet, this can cause altitude sickness. To prevent this, mountain climbers often use tanks containing a mixture of air and pure oxygen.

**35. Science** At higher altitudes, water boils at lower temperatures. This relationship between altitude and boiling point is linear. At an altitude of 1000 feet, water boils at 210 °F. At an altitude of 3000 feet, water boils at 206 °F. Write an equation in slope-intercept form that represents this linear function. Then find the boiling point at 6000 feet.

**36. Consumer Economics** Lora has a gift card from an online music store where all downloads cost the same amount. After downloading 2 songs, the balance on her card was $18.10. After downloading a total of 5 songs, the balance was $15.25.

a. Write an equation in slope-intercept form that represents the amount in dollars remaining on the card as a function of songs downloaded.

b. Identify the slope of the line and tell what the slope represents.

c. Identify the $y$-intercept of the line and tell what it represents.

d. How many additional songs can Lora download when there is $15.25 left on the card?

**Graph the line with the given slope that contains the given point.**

**37.** slope $= -3$; $(2, 4)$ **38.** slope $= -\frac{1}{4}$; $(0, 0)$ **39.** slope $= \frac{1}{2}$; $(-2, -1)$

**Tell whether each statement is sometimes, always, or never true.**

**40.** A line described by the equation $y = mx + b$ contains the point $(0, b)$.

**41.** The slope of the line that contains the points $(0, 0)$ and $(c, d)$ is negative if both $c$ and $d$ are negative.

**42.** The $y$-intercept of the graph of $y - y_1 = m(x - x_1)$ is negative if $y_1$ is negative.

**43. Meteorology** Snowfall accumulates at an average rate of 2.5 inches per hour during a snowstorm. Two hours after the snowstorm begins, the average depth of snow on the ground is 11 inches.

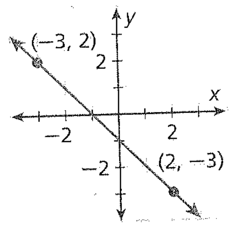
a. Write an equation in point-slope form that represents the depth of the snow in inches as a function of hours since the snowstorm began.

b. How much snow is on the ground when the snowstorm starts?

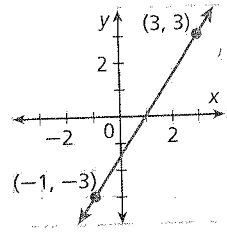
c. The snowstorm begins at 2:15 P.M. and continues until 6:30 P.M. How much snow is on the ground at the end of the storm?

Write an equation in point-slope form that describes each graph.

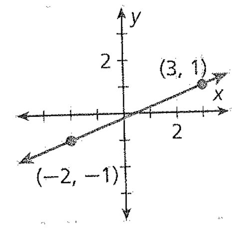
44.




45.




46.




The tables show linear relationships between $x$ and $y$. Copy and complete the tables.

47.

| $x$ | −2 | 0 | | 7 |
|---|---|---|---|---|
| $y$ | −18 | | 12 | 27 |

48.

| $x$ | −4 | 1 | 0 | |
|---|---|---|---|---|
| $y$ | 14 | 4 | | −6 |

49. **ERROR ANALYSIS** Two students used point-slope form to find an equation that describes the line with slope −3 through $(-5, 2)$. Who is incorrect? Explain the error.

A

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - 5)$$

B

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3[x - (-5)]$$
$$y - 2 = -3(x + 5)$$

50. **Critical Thinking** Compare the methods for finding the equation that describes a line when you know

- a point on the line and the slope of the line.
- two points on the line.

How are the methods alike? How are they different?

51. **Write About It** Explain why the first statement is false but the second is true.

- All linear equations can be written in point-slope form.
- All linear equations that describe functions can be written in point-slope form.

52. **Multi-Step** The table shows the mean scores on a standardized test for several different years.

| Years Since 1985 | 0 | 5 | 10 | 17 | 21 |
|---|---|---|---|---|---|
| Mean Combined Score | 994 | 1009 | 1001 | 1016 | 1020 |

a. Make a scatter plot of the data and add a trend line to your graph.

b. Use your trend line to estimate the slope and $y$-intercept, and write an equation in slope-intercept form.

c. What do the slope and $y$-intercept represent in this situation?

**MULTI-STEP TEST PREP**


53. This problem will prepare you for the Multi-Step Test Prep on page 376.

a. Stephen is walking from his house to his friend Sharon's house. When he is 12 blocks away, he looks at his watch. He looks again when he is 8 blocks away and finds that 6 minutes have passed. Write two ordered pairs for these data in the form (time, blocks).

b. Write a linear equation for these two points.

c. What is the total amount of time it takes Stephen to reach Sharon's house? Explain how you found your answer.

## TEST PREP

54. Which equation describes the line through $(-5, 1)$ with slope of 1?
    - (A) $y + 1 = x - 5$
    - (B) $y + 5 = x - 1$
    - (C) $y - 1 = -5(x - 1)$
    - (D) $y - 1 = x + 5$

55. A line contains (4, 4) and (5, 2). What are the slope and $y$-intercept?
    - (F) slope $= -2$; $y$-intercept $= 2$
    - (G) slope $= 1.2$; $y$-intercept $= -2$
    - (H) slope $= -2$; $y$-intercept $= 12$
    - (J) slope $= 12$; $y$-intercept $= 1.2$

## CHALLENGE AND EXTEND

56. A linear function has the same $y$-intercept as $x + 4y = 8$ and its graph contains the point $(2, 7)$. Find the slope and $y$-intercept.

57. Write the equation of a line in slope-intercept form that contains $\left(\frac{3}{4}, \frac{1}{2}\right)$ and has the same slope as the line described by $y + 3x = 6$.

58. Write the equation of a line in slope-intercept form that contains $\left(-\frac{1}{2}, -\frac{1}{3}\right)$ and $\left(1\frac{1}{2}, 1\right)$.

## SPIRAL REVIEW

**Solve each compound inequality and graph the solutions.** *(Lesson 3-6)*

59. $-4 \le x + 2 \le 1$

60. $m - 5 > -7$ AND $m + 1 < 2$

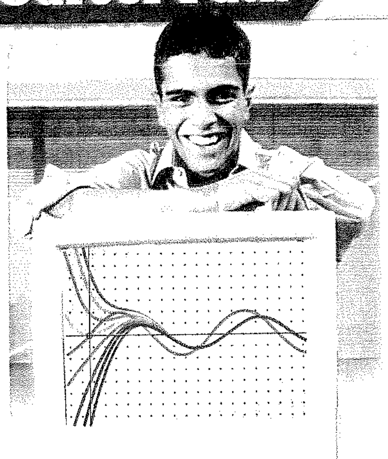
61. A group of hikers is walking the Appalachian Trail. The function $y = 12x$ describes how many miles $y$ the group has traveled in $x$ days. Graph the function. Then use the graph to estimate how many miles the group will hike in 18 days. *(Lesson 4-4)*

**Write the equation that describes each line in slope-intercept form.** *(Lesson 5-7)*

62. slope $= 3$, $y$-intercept $= -5$

63. slope $= -2$, $(2, 4)$ is on the line

## Career Path



**Michael Raynor**
Data mining major

**Q: What math classes did you take in high school?**
A: Algebra 1 and 2, Geometry, and Statistics

**Q: What math classes have you taken in college?**
A: Applied Statistics, Data Mining Methods, Web Mining, and Artificial Intelligence

**Q: How do you use math?**
A: Once for a class, I used software to analyze basketball statistics. What I learned helped me develop strategies for our school team.

**Q: What are your future plans?**
A: There are many options for people with data mining skills. I could work in banking, pharmaceuticals, or even the military. But my dream job is to develop game strategies for an NBA team.

5-8 Technology LAB

# Graph Linear Functions

You can use a graphing calculator to quickly graph lines whose equations are in point-slope form. To enter an equation into your calculator, it must be solved for $y$, but it does not necessarily have to be in slope-intercept form.

*Use with Lesson 5-8*

go.hrw.com
Lab Resources Online
KEYWORD: MA11 LAB5

## Activity

**Graph the line with slope 2 that contains the point $(2, 6.09)$.**

1. Use point-slope form.

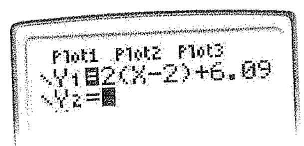
$$y - y_1 = m(x - x_1)$$
$$y - 6.09 = 2(x - 2)$$

2. Solve for $y$ by adding 6.09 to both sides of the equation.

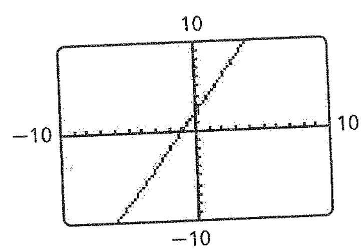
$$\begin{aligned} y - 6.09 &= 2(x - 2) \\ \underline{+6.09} \quad & \quad \underline{+6.09} \\ y \quad &= 2(x - 2) + 6.09 \end{aligned}$$

3. Enter this equation into your calculator.

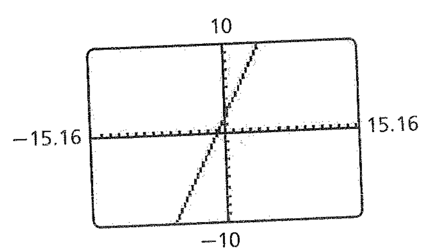
Y= 2 ( X,T,θ,n − 2 ) + 6.09 ENTER




4. Graph in the *standard viewing window* by pressing ZOOM and selecting **6:ZStandard**. In this window, both the $x$- and $y$-axes go from $-10$ to 10.




5. Notice that the scale on the $y$-axis is smaller than the scale on the $x$-axis. This is because the width of the calculator screen is about 50% greater than its height. To see a more accurate graph of this line, use the *square viewing window*. Press ZOOM and select **5:ZSquare**.




## Try This

1. Graph the function represented by the line with slope $-1.5$ that contains the point $(2.25, -3)$. View the graph in the standard viewing window.
2. Now view the graph in the square viewing window. Press WINDOW and write down the minimum and maximum values on the $x$- and $y$-axes.
3. In which graph does the line appear steeper? Why?
4. Explain why it might sometimes be useful to look at a graph in a square window.

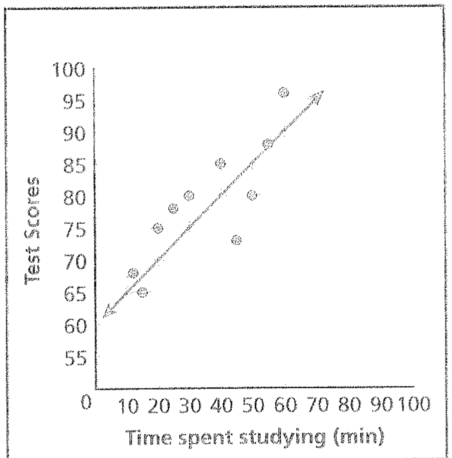
Connecting Algebra to Data Analysis

# Interpreting Trend Lines

In Chapter 4 you learned how to draw trend lines on scatter plots. Now you will learn how to find the equations of trend lines and write them in slope-intercept form.

## Example

**Write an equation for the trend line on the scatter plot.**




Two points on the trend line are (30, 75) and (60, 90).

To find the slope of the line that contains ( , 75) and ( , 90), use the slope formula.

$$m = \frac{y_2 - y_1}{\phantom{x}}$$

$$m = \frac{90 - 75}{\phantom{x}}$$

$$m = \frac{15}{30}$$

$$m = \frac{1}{2}$$

Use the slope and the point (30, 75) to find the $y$-intercept of the line.

$$y = mx + b$$

$$75 = \frac{1}{2}(30) + b$$

$$75 = 15 + b$$

$$=$$

Write the equation.

$$y = \frac{1}{2}x +$$

## Try This

1. In the example above, what is the meaning of the slope?
2. What does the $y$-intercept represent?
3. Use the equation to predict the test score of a student who spent 25 minutes studying.
4. Use the table to create a scatter plot. Draw a trend line and find the equation of your trend line. Tell the meaning of the slope and $y$-intercept. Then use your equation to predict the race time of a runner who ran 40 miles in training.

| Distance Run in Training (mi) | 12 | 15 | 16 | 18 | 21 | 23 | 24 | 25 | 33 |
|---|---|---|---|---|---|---|---|---|---|
| Race Time (min) | 65 | 64 | 55 | 58 | 55 | 50 | 50 | 47 | 36 |