5-9 Slopes of Parallel and Perpendicular Lines

Objectives
Identify and graph parallel and perpendicular lines.

Write equations to describe lines parallel or perpendicular to a given line.

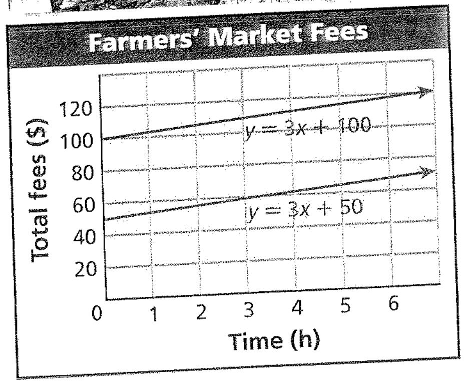
Vocabulary
parallel lines
perpendicular lines

Why learn this?
Parallel lines and their equations can be used to model costs, such as the cost of a booth at a farmers' market.

To sell at a particular farmers' market for a year, there is a \$100 membership fee. Then you pay \$3 for each hour that you sell at the market. However, if you were a member the previous year, the membership fee is reduced to \$50.

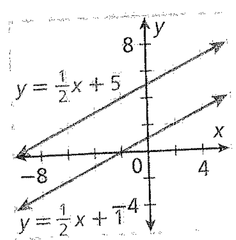
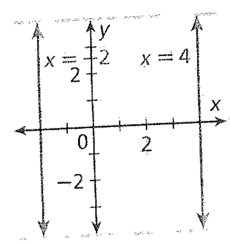
- The **red** line shows the total cost if you are a new member.
- The **blue** line shows the total cost if you are a returning member.

These two lines are *parallel.* **Parallel lines** are lines in the same plane that have no points in common. In other words, they do not intersect.



Know it! Note

Parallel Lines

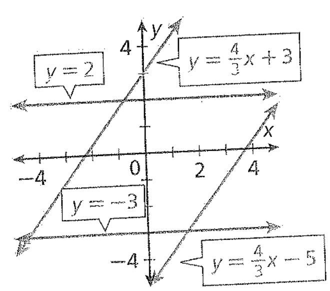
WORDS	Two different nonvertical lines are parallel if and only if they have the same slope.	All different vertical lines are parallel.
GRAPH		

EXAMPLE 1 **Identifying Parallel Lines**

Identify which lines are parallel.

A $y = \frac{4}{3}x + 3;\ y = 2;\ y = \frac{4}{3}x - 5;\ y = -3$

The lines described by $y = \frac{4}{3}x + 3$ and $y = \frac{4}{3}x - 5$ both have slope $\frac{4}{3}$. These lines are parallel. The lines described by $y = 2$ and $y = -3$ both have slope 0. These lines are parallel.



Identify which lines are parallel.

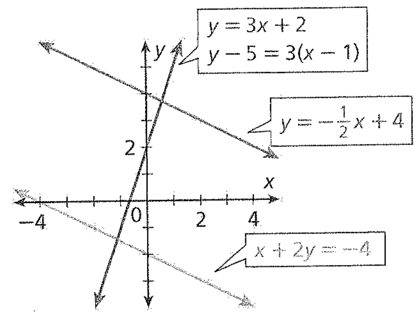
B $y = 3x + 2;\ y = -\frac{1}{2}x + 4;\ x + 2y = -4;\ y - 5 = 3(x - 1)$

Write all equations in slope-intercept form to determine the slopes.

$y = 3x + 2$ slope-intercept form ✓	$y = -\frac{1}{2}x + 4$ slope-intercept form ✓
$x + 2y = -4$ $\underline{-x \qquad -x}$ $2y = -x - 4$ $\frac{2y}{2} = \frac{-x - 4}{2}$ $y = -\frac{1}{2}x - 2$	$y - 5 = 3(x - 1)$ $y - 5 = 3x - 3$ $\underline{+5 \qquad +5}$ $y \quad = 3x + 2$

The lines described by $y = 3x + 2$ and $y - 5 = 3(x - 1)$ have the same slope, but they are not parallel lines. They are the same line.

The lines described by $y = -\frac{1}{2}x + 4$ and $x + 2y = -4$ represent parallel lines. They each have slope $-\frac{1}{2}$.



CHECK IT OUT! **Identify which lines are parallel.**

1a. $y = 2x + 2;\ y = 2x + 1;\ y = -4;\ x = 1$

1b. $y = \frac{3}{4}x + 8;\ -3x + 4y = 32;\ y = 3x;\ y - 1 = 3(x + 2)$

EXAMPLE 2 Geometry Application

Geometry

Remember!

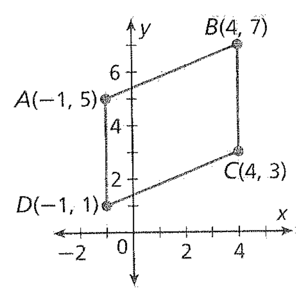
In a parallelogram, opposite sides are parallel.

Show that *ABCD* is a parallelogram.

Use the ordered pairs and the slope formula to find the slopes of $\overline{AB}$ and $\overline{CD}$.

$$\text{slope of } \overline{AB} = \frac{7 - 5}{4 - (-1)} = \frac{2}{5}$$

$$\text{slope of } \overline{CD} = \frac{3 - 1}{4 - (-1)} = \frac{2}{5}$$



$\overline{AB}$ is parallel to $\overline{CD}$ because they have the same slope.

$\overline{AD}$ is parallel to $\overline{BC}$ because they are both vertical.

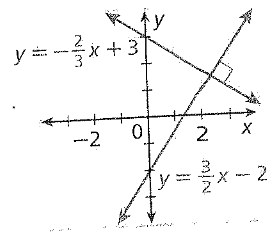
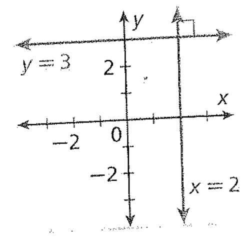
Therefore, *ABCD* is a parallelogram because both pairs of opposite sides are parallel.

CHECK IT OUT! **2.** Show that the points $A(0, 2)$, $B(4, 2)$, $C(1, -3)$, and $D(-3, -3)$ are the vertices of a parallelogram.

Perpendicular lines are lines that intersect to form right angles (90°).

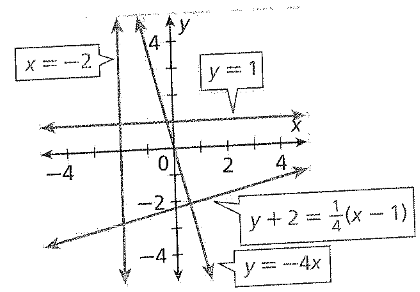
Know it! Note

Perpendicular Lines

WORDS	Two nonvertical lines are perpendicular if and only if the product of their slopes is −1.	Vertical lines are perpendicular to horizontal lines.
GRAPH		

EXAMPLE 3 **Identifying Perpendicular Lines**

Identify which lines are perpendicular: $x = -2$; $y = 1$; $y = -4x$; $y + 2 = \frac{1}{4}(x + 1)$.



The graph described by $x = -2$ is a vertical line, and the graph described by $y = 1$ is a horizontal line. These lines are perpendicular.

The slope of the line described by $y = -4x$ is -4. The slope of the line described by $y + 2 = \frac{1}{4}(x - 1)$ is $\frac{1}{4}$.

$$(-4)\left(\frac{1}{4}\right) = -1$$

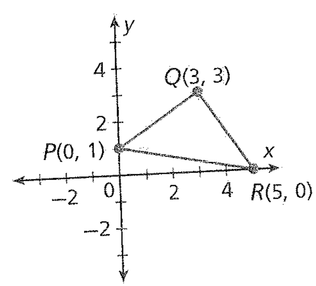
These lines are perpendicular because the product of their slopes is −1.

CHECK IT OUT! **3.** Identify which lines are perpendicular: $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$.

EXAMPLE 4 **Geometry Application**

Geometry

Show that PQR is a right triangle.



If PQR is a right triangle, $\overline{PQ}$ will be perpendicular to $\overline{QR}$.

$$\text{slope of } \overline{PQ} = \frac{3 - 1}{3 - 0} = \frac{2}{3}$$

$$\text{slope of } \overline{QR} = \frac{3 - 0}{3 - 5} = \frac{3}{-2} = -\frac{3}{2}$$

$\overline{PQ}$ is perpendicular to $\overline{QR}$ because $\frac{2}{3}\left(-\frac{3}{2}\right) = -1$.

Therefore, PQR is a right triangle because it contains a right angle.

Helpful Hint

A right triangle contains one right angle. In Example 4, $\angle P$ and $\angle R$ are clearly not right angles, so the only possibility is $\angle Q$.

CHECK IT OUT! **4.** Show that $P(1, 4)$, $Q(2, 6)$, and $R(7, 1)$ are the vertices of a right triangle.

EXAMPLE 5 Writing Equations of Parallel and Perpendicular Lines

A **Write an equation in slope-intercept form for the line that passes through $(4, 5)$ and is parallel to the line described by $y = 5x + 10$.**

Step 1 Find the slope of the line.

$y = 5x + 10$ *The slope is 5.*

The parallel line also has a slope of 5.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

$y - 5 = 5(x - 4)$ *Substitute 5 for m, 4 for x_1, and 5 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 5 = 5(x - 4)$

$y - 5 = 5x - 20$ *Distributive Property*

$y = 5x - 15$ *Addition Property of Equality*

B **Write an equation in slope-intercept form for the line that passes through $(3, 2)$ and is perpendicular to the line described by $y = 3x - 1$.**

Step 1 Find the slope of the line.

$y = 3x - 1$ *The slope is 3.*

The perpendicular line has a slope of $-\frac{1}{3}$, because $3\left(-\frac{1}{3}\right) = -1$.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

$y - 2 = -\frac{1}{3}(x - 3)$ *Substitute $-\frac{1}{3}$ for m, 3 for x_1, and 2 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 2 = -\frac{1}{3}(x - 3)$

$y - 2 = -\frac{1}{3}x + 1$ *Distributive Property*

$y = -\frac{1}{3}x + 3$ *Addition Property of Equality*

Helpful Hint

If you know the slope of a line, the slope of a perpendicular line will be the "opposite reciprocal."

$\frac{2}{3} \rightarrow -\frac{3}{2}$

$\frac{1}{5} \rightarrow -5$

$-7 \rightarrow \frac{1}{7}$

CHECK IT OUT!

5a. Write an equation in slope-intercept form for the line that passes through $(5, 7)$ and is parallel to the line described by $y = \frac{4}{5}x - 6$.

5b. Write an equation in slope-intercept form for the line that passes through $(-5, 3)$ and is perpendicular to the line described by $y = 5x$.

THINK AND DISCUSS

1. Are the lines described by $y = \frac{1}{2}x$ and $y = 2x$ perpendicular? Explain.
2. Describe the slopes and y-intercepts when two nonvertical lines are parallel.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch an example and describe the slopes.

Know it! Note

Parallel lines	Perpendicular lines

5-9 Exercises

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Homework Help Online
KEYWORD: MA11 5-9
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** ___?___ lines have the same slope. (*Parallel* or *Perpendicular*)

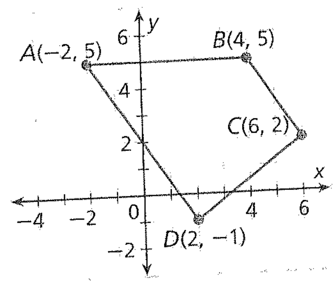
SEE EXAMPLE 1 p. 361

Identify which lines are parallel.

2. $y = 6;\ y = 6x + 5;\ y = 6x - 7;\ y = -8$
3. $y = \frac{3}{4}x - 1;\ y = -2x;\ y - 3 = \frac{3}{4}(x - 5);\ y - 4 = -2(x + 2)$

SEE EXAMPLE 2 p. 362

4. **Geometry** Show that *ABCD* is a trapezoid. (*Hint:* In a trapezoid, exactly one pair of opposite sides is parallel.)



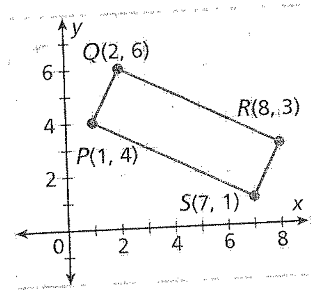
SEE EXAMPLE 3 p. 363

Identify which lines are perpendicular.

5. $y = \frac{2}{3}x - 4;\ y = -\frac{3}{2}x + 2;\ y = -1;\ x = 3$
6. $y = -\frac{3}{7}x - 4;\ y - 4 = -7(x + 2);$
$y - 1 = \frac{1}{7}(x - 4);\ y - 7 = \frac{7}{3}(x - 3)$

SEE EXAMPLE 4 p. 363

7. **Geometry** Show that *PQRS* is a rectangle. (*Hint:* In a rectangle, all four angles are right angles.)



SEE EXAMPLE 5 p. 364

8. Write an equation in slope-intercept form for the line that passes through (5, 0) and is perpendicular to the line described by $y = -\frac{5}{2}x + 6$.

PRACTICE AND PROBLEM SOLVING

Independent Practice

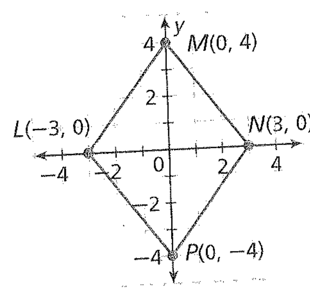
For Exercises	See Example
9–11	1
12	2
13–15	3
16	4
17	5

Extra Practice

Skills Practice p. S13
Application Practice p. S32

Identify which lines are parallel.

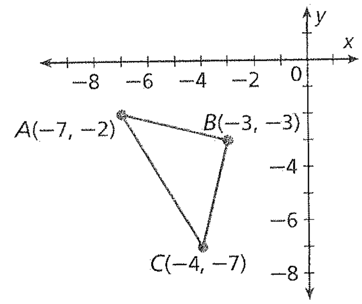
9. $x = 7;\ y = -\frac{5}{6}x + 8;\ y = -\frac{5}{6}x - 4;\ x = -9$
10. $y = -x;\ y - 3 = -1(x + 9);\ y - 6 = \frac{1}{2}(x - 14);\ y + 1 = \frac{1}{2}x$
11. $y = -3x + 2;\ y = \frac{1}{2}x - 1;\ -x + 2y = 17;\ 3x + y = 27$
12. **Geometry** Show that *LMNP* is a parallelogram.



Identify which lines are perpendicular.

13. $y = 6x;\ y = \frac{1}{6}x;\ y = -\frac{1}{6}x;\ y = -6x$
14. $y - 9 = 3(x + 1);\ y = -\frac{1}{3}x + 5;\ y = 0;\ x = 6$
15. $x - 6y = 15;\ y = 3x - 2;\ y = -3x - 3;\ y = -6x - 8;\ 3y = -x - 11$

16. **Geometry** Show that ABC is a right triangle.



17. Write an equation in slope-intercept form for the line that passes through $(0, 0)$ and is parallel to the line described by $y = -\frac{6}{7}x + 1$.

Without graphing, tell whether each pair of lines is parallel, perpendicular, or neither.

18. $x = 2$ and $y = -5$

19. $y = 7x$ and $y - 28 = 7(x - 4)$

20. $y = 2x - 1$ and $y = \frac{1}{2}x + 2$

21. $y - 3 = \frac{1}{4}(x - 3)$ and $y + 13 = \frac{1}{4}(x + 1)$

Write an equation in slope-intercept form for the line that is parallel to the given line and that passes through the given point.

22. $y = 3x + 7$; $(0, 4)$

23. $y = \frac{1}{2}x + 5$; $(4, -3)$
y= 2.5x +5

24. $4y = x$; $(4, 0)$

25. $y = 2x + 3$; $(1, 7)$

26. $\cancel{5x - 2y = 10}$; $(3, -5)$

27. $y = 3x - 4$; $(-2, 7)$

28. $y = 7$; $(2, 4)$

29. $x + y = 1$; $(2, 3)$

30. $2x + 3y = 7$; $(4, 5)$

31. $y = 4x + 2$; $(5, -3)$

32. $y = \frac{1}{2}x - 1$; $(0, -4)$

33. $3x + 4y = 8$; $(4, -3)$

Write an equation in slope-intercept form for the line that is perpendicular to the given line and that passes through the given point.

34. $y = -3x + 4$; $(6, -2)$

35. $y = x - 6$; $(-1, 2)$

36. $3x - 4y = 8$; $(-6, 5)$

37. $5x + 2y = 10$; $(3, -5)$

38. $y = 5 - 3x$; $(2, -4)$

39. $-10x + 2y = 8$; $(4, -3)$

40. $2x + 3y = 7$; $(4, 5)$

41. $4x - 2y = -6$; $(3, -2)$

42. $-2x + 8y = 16$; $(4, 5)$

43. $y = -2x + 4$; $(-2, 5)$

44. $y = x - 5$; $(0, 5)$

45. $x + y = 2$; $(8, 5)$

46. Write an equation describing the line that is parallel to the y-axis and that is 6 units to the right of the y-axis.

47. Write an equation describing the line that is perpendicular to the y-axis and that is 4 units below the x-axis.

48. **Critical Thinking** Is it possible for two linear functions whose graphs are parallel lines to have the same y-intercept? Explain.

49. **Estimation** Estimate the slope of a line that is perpendicular to the line through $(2.07, 8.95)$ and $(-1.9, 25.07)$.

50. **Write About It** Explain in words how to write an equation in slope-intercept form that describes a line parallel to $y - 3 = -6(x - 3)$.

MULTI-STEP TEST PREP

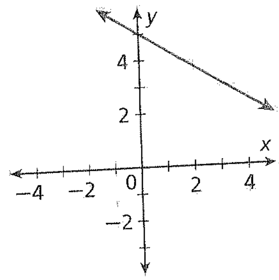
51. This problem will prepare you for the Multi-Step Test Prep on page 376.
    a. Flora walks from her home to the bus stop at a rate of 50 steps per minute. Write a rule that gives her distance from home (in steps) as a function of time.
    b. Flora's neighbor Dan lives 30 steps closer to the bus stop. He begins walking at the same time and at the same pace as Flora. Write a rule that gives Dan's distance from *Flora's* house as a function of time.
    c. Will Flora meet Dan along the walk? Use a graph to help explain your answer.

TEST PREP

52. Which describes a line parallel to the line described by $y = -3x + 2$?

Ⓐ $y = -3x$ Ⓑ $y = \frac{1}{3}x$ Ⓒ $y = 2 - 3x$ Ⓓ $y = \frac{1}{3}x + 2$

53. Which describes a line passing through (3, 3) that is perpendicular to the line described by $y = \frac{3}{5}x + 2$?

Ⓕ


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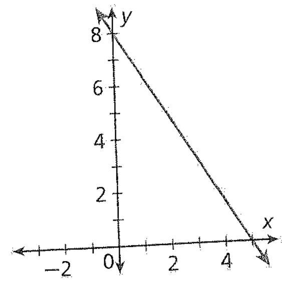


Ⓖ $y = \frac{5}{3}x - 2$ Ⓙ $y = \frac{3}{5}x + \frac{6}{5}$

54. **Gridded Response** The graph of a linear function $f(x)$ is parallel to the line described by $2x + y = 5$ and contains the point $(6, -2)$. What is the y-intercept of $f(x)$?

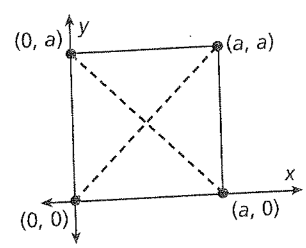
CHALLENGE AND EXTEND

55. Three or more points that lie on the same line are called *collinear points*. Explain why the points A, B, and C must be collinear if the line containing A and B has the same slope as the line containing B and C.

56. The lines described by $y = (a + 12)x + 3$ and $y = 4ax$ are parallel. What is the value of a?

57. The lines described by $y = (5a + 3)x$ and $y = -\frac{1}{2}x$ are perpendicular. What is the value of a?

58. **Geometry** The diagram shows a square in the coordinate plane. Use the diagram to show that the diagonals of a square are perpendicular.



SPIRAL REVIEW

59. The record high temperature for a given city is 112 °F. The morning temperature today was 94 °F and the temperature will increase t degrees. Write and solve an inequality to find all values of t that would break the record for the high temperature. *(Lesson 3-2)*

Graph each function. *(Lesson 4-4)*

60. $y = -3x + 5$ 61. $y = x + 1$ 62. $y = x^2 + 3$

Write an equation in slope-intercept form for the line with the given slope that contains the given point. *(Lesson 5-8)*

63. slope $= \frac{2}{3}$; $(6, -1)$ 64. slope $= -5$; $(2, 4)$ 65. slope $= -\frac{1}{2}$; $(-1, 0)$

66. slope $= -\frac{1}{3}$; $(2, 7)$ 67. slope $= 0$; $(-3, 3)$ 68. slope $= \frac{1}{5}$; $(-4, -2)$

5-10

Technology LAB

The Family of Linear Functions

Use with Lesson 5-10

A *family of functions* is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family. You can use a graphing calculator to explore families of functions.

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KEYWORD: MA11 LAB5

Activity

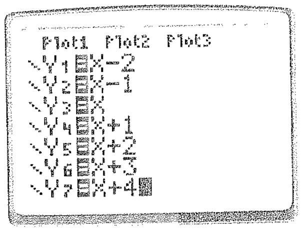
Graph the lines described by $y = x - 2$, $y = x - 1$, $y = x$, $y = x + 1$, $y = x + 2$, $y = x + 3$, and $y = x + 4$. How does the value of b affect the graph described by $y = x + b$?

1. All of the functions are in the form $y = x + b$. Enter them into the **Y=** editor.

   Y= X,T,θ,n − 2 ENTER

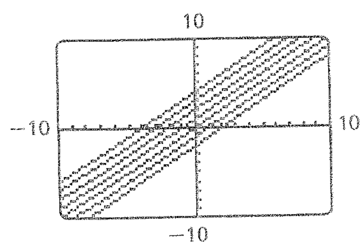
   X,T,θ,n − 1 ENTER

   and so on.



2. Press ZOOM and select **6:Zstandard**. Think about the different values of b as you watch the graphs being drawn. Notice that the lines are all parallel.

3. It appears that the value of b in $y = x + b$ shifts the graph up or down—up if b is positive and down if b is negative.



Try This

1. Make a prediction about the lines described by $y = 2x - 3$, $y = 2x - 2$, $y = 2x - 1$, $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, and $y = 2x + 3$. Then graph. Was your prediction correct?
2. Now use your calculator to explore what happens to the graph of $y = mx$ when you change the value of m.
   a. **Make a Prediction** How do you think the lines described by $y = -2x$, $y = -x$, $y = x$, and $y = 2x$ will be related? How will they be alike? How will they be different?
   b. Graph the functions given in part **a**. Was your prediction correct?
   c. How is the effect of m different when m is positive from when m is negative?

5-10 Transforming Linear Functions

Objective
Describe how changing slope and y-intercept affect the graph of a linear function.

Vocabulary
family of functions
parent function
transformation
translation
rotation
reflection

Who uses this?

Business owners can use transformations to show the effects of price changes, such as the price of trophy engraving. (See Example 5.)

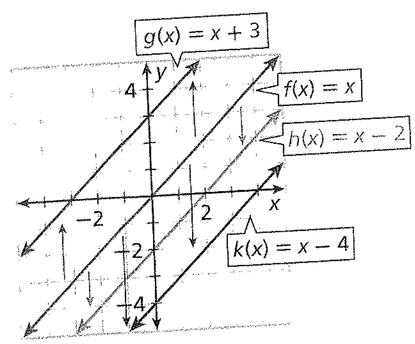
A **family of functions** is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family because all of their graphs are the same basic shape.

A **parent function** is the most basic function in a family. For linear functions, the parent function is $f(x) = x$.

The graphs of all other linear functions are *transformations* of the graph of the parent function, $f(x) = x$. A **transformation** is a change in position or size of a figure.

There are three types of transformations—*translations, rotations,* and *reflections*.

Look at the four functions and their graphs below.



Remember!
Function notation—$f(x)$, $g(x)$, and so on—can be used in place of y.
$y = f(x)$

Notice that all of the lines above are parallel. The slopes are the same but the y-intercepts are different.

The graphs of $g(x) = x + 3$, $h(x) = x - 2$, and $k(x) = x - 4$ are vertical *translations* of the graph of the parent function, $f(x) = x$. A **translation** is a type of transformation that moves every point the same distance in the same direction. You can think of a translation as a "slide."

Know it! Note

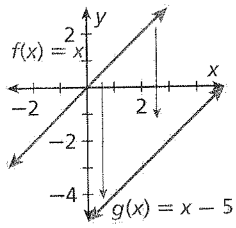
Vertical Translation of a Linear Function

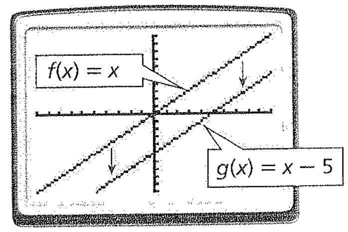
When the y-intercept b is changed in the function $f(x) = mx + b$, the graph is translated vertically.

- If b increases, the graph is translated up.
- If b decreases, the graph is translated down.

EXAMPLE 1 Translating Linear Functions

Graph $f(x) = x$ and $g(x) = x - 5$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

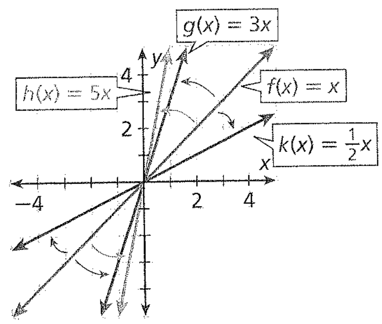




The graph of $g(x) = x - 5$ is the result of translating the graph of $f(x) = x$ 5 units down.

CHECK IT OUT! 1. Graph $f(x) = x + 4$ and $g(x) = x - 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

The graphs of $g(x) = 3x$, $h(x) = 5x$, and $k(x) = \frac{1}{2}x$ are *rotations* of the graph of $f(x) = x$. A **rotation** is a transformation about a point. You can think of a rotation as a "turn." The y-intercepts are the same, but the slopes are different.



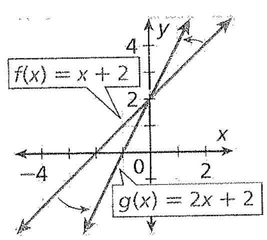
Know it! Note

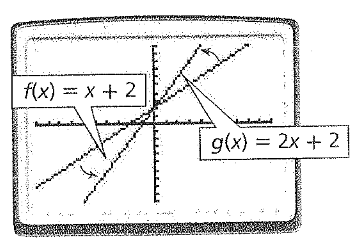
Rotation of a Linear Function

When the slope m is changed in the function $f(x) = mx + b$ it causes a rotation of the graph about the point $(0, b)$, which changes the line's steepness.

EXAMPLE 2 Rotating Linear Functions

Graph $f(x) = x + 2$ and $g(x) = 2x + 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

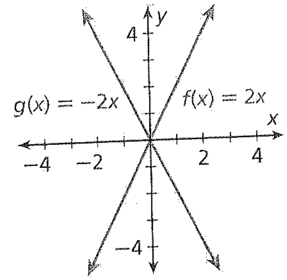




The graph of $g(x) = 2x + 2$ is the result of rotating the graph of $f(x) = x + 2$ about $(0, 2)$. The graph of $g(x)$ is steeper than the graph of $f(x)$.

CHECK IT OUT! 2. Graph $f(x) = 3x - 1$ and $g(x) = \frac{1}{2}x - 1$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

The diagram shows the *reflection* of the graph of $f(x) = 2x$ across the y-axis, producing the graph of $g(x) = -2x$. A **reflection** is a transformation across a line that produces a mirror image. You can think of a reflection as a "flip" over a line.



Know it! Note

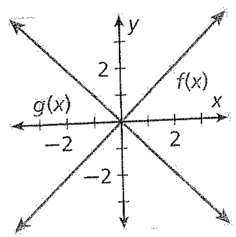
Reflection of a Linear Function

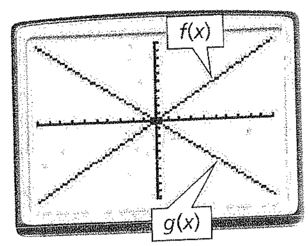
When the slope m is multiplied by -1 in $f(x) = mx + b$, the graph is reflected across the y-axis.

EXAMPLE 3 Reflecting Linear Functions

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

A $f(x) = x$





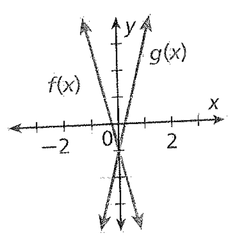
To find $g(x)$, multiply the value of m by -1.

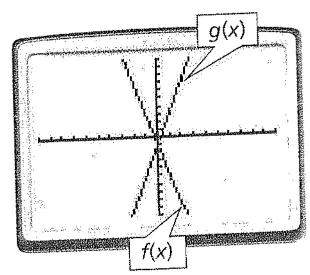
In $f(x) = x$, $m = 1$.

$1(-1) = -1$ *This is the value of m for g(x).*

$g(x) = -x$

B $f(x) = -4x - 1$





To find $g(x)$, multiply the value of m by -1.

In $f(x) = -4x - 1$, $m = -4$.

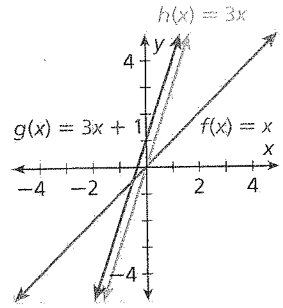
$-4(-1) = 4$ *This is the value of m for g(x).*

$g(x) = 4x - 1$

CHECK IT OUT! 3. Graph $f(x) = \frac{2}{3}x + 2$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

EXAMPLE 4 Multiple Transformations of Linear Functions

Graph $f(x) = x$ and $g(x) = 3x + 1$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.



Find transformations of $f(x) = x$ that will result in $g(x) = 3x + 1$:

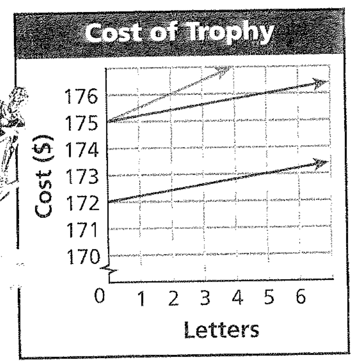
- Multiply $f(x)$ by 3 to get $h(x) = 3x$. This rotates the graph about (0, 0) and makes it steeper.
- Then add 1 to $h(x)$ to get $g(x) = 3x + 1$. This translates the graph 1 unit up.

The transformations are a rotation and a translation.

CHECK IT OUT! **4.** Graph $f(x) = x$ and $g(x) = -x + 2$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

EXAMPLE 5 *Business Application*

A trophy company charges $175 for a trophy plus $0.20 per letter for the engraving. The total charge for a trophy with x letters is given by the function $f(x) = 0.20x + 175$. How will the graph change if the trophy's cost is lowered to $172? if the charge per letter is raised to $0.50?



$f(x) = 0.20x + 175$ is graphed in blue.

If the trophy's cost is lowered to $172, the new function is $g(x) = 0.20x + 172$. The original graph will be translated 3 units down.

If the charge per letter is raised to $0.50, the new function is $h(x) = 0.50x + 175$. The original graph will be rotated about (0, 175) and become steeper.

CHECK IT OUT! **5. What if...?** How will the graph change if the charge per letter is lowered to $0.15? if the trophy's cost is raised to $180?

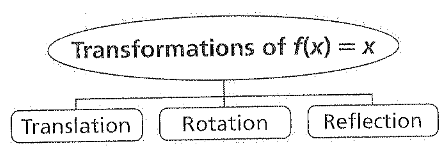
THINK AND DISCUSS

1. Describe the graph of $f(x) = x + 3.45$

2. Look at the graphs in Example 5. For each line, is every point on the line a solution in this situation? Explain.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch a graph of the given transformation of $f(x) = x$, and label it with a possible equation.



5-10 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 5-10
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Changing the value of b in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *reflection*)

2. Changing the value of m in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *rotation*)

Graph $f(x)$ and $g(x)$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE 1 p. 370

3. $f(x) = x,\ g(x) = x - 4$
4. $f(x) = x,\ g(x) = x + 1$
5. $f(x) = x,\ g(x) = x + 2$
6. $f(x) = x,\ g(x) = x - 6.5$

SEE EXAMPLE 2 p. 370

7. $f(x) = x,\ g(x) = \frac{1}{4}x$
8. $f(x) = \frac{1}{5}x + 3,\ g(x) = x + 3$
9. $f(x) = 2x - 2,\ g(x) = 4x - 2$
10. $f(x) = x + 1,\ g(x) = \frac{1}{2}x + 1$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

SEE EXAMPLE 3 p. 371

11. $f(x) = -\frac{1}{5}x$
12. $f(x) = 2x + 4$
13. $f(x) = \frac{1}{3}x - 6$
14. $f(x) = 5x - 1$

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE 4 p. 372

15. $f(x) = x,\ g(x) = 2x - 2$
16. $f(x) = x,\ g(x) = \frac{1}{3}x + 1$
17. $f(x) = -x - 1,\ g(x) = -4x$
18. $f(x) = -x,\ g(x) = -\frac{1}{2}x - 3$

SEE EXAMPLE 5 p. 372

19. **Entertainment** For large parties, a restaurant charges a reservation fee of \$25, plus \$15 per person. The total charge for a party of x people is $f(x) = 15x + 25$. How will the graph of this function change if the reservation fee is raised to \$50? if the per-person charge is lowered to \$12?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–25	3
26–27	4
28	5

Extra Practice

Skills Practice p. S13
Application Practice p. S32

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

20. $f(x) = x,\ g(x) = x + \frac{1}{2}$
21. $f(x) = x,\ g(x) = x - 4$
22. $f(x) = \frac{1}{5}x - 1,\ g(x) = \frac{1}{10}x - 1$
23. $f(x) = x + 2,\ g(x) = \frac{2}{3}x + 2$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

24. $f(x) = 6x$
25. $f(x) = -3x - 2$

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

26. $f(x) = 2x,\ g(x) = 4x - 1$
27. $f(x) = -7x + 5,\ g(x) = -14x$

28. **School** The number of chaperones on a field trip must include 1 teacher for every 4 students, plus 2 parents total. The function describing the number of chaperones for a trip of x students is $f(x) = \frac{1}{4}x + 2$. How will the graph change if the number of parents is reduced to 0? if the number of teachers is raised to 1 for every 3 students?

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$. Graph $f(x)$ and $g(x)$, and compare the slopes and intercepts.

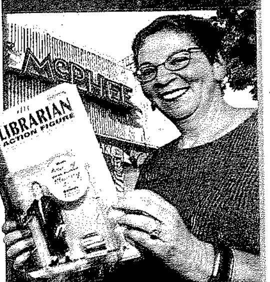
29. $g(x) = -x$
30. $g(x) = x + 8$
31. $g(x) = 3x$
32. $g(x) = -\frac{2}{7}x$
33. $g(x) = 6x - 3$
34. $g(x) = -2x + 1$

Sketch the transformed graph. Then write a function to describe your graph.

35. Rotate the graph of $f(x) = -x + 2$ until it has the same steepness in the opposite direction.
36. Reflect the graph of $f(x) = x - 1$ across the y-axis, and then translate it 4 units down.
37. Translate the graph of $f(x) = \frac{1}{6}x - 10$ six units up.
38. **Hobbies** A book club charges a membership fee of \$20 and then \$12 for each book purchased.
    a. Write and graph a function to represent the cost y of membership in the club based on the number of books purchased x.
    b. **What if...?** Write and graph a second function to represent the cost of membership if the club raises its membership fee to \$30.
    c. Describe the relationship between your graphs from parts **a** and **b**.

LINK

Hobbies



Seattle librarian Nancy Pearl started the first citywide reading program, "If All Seattle Reads the Same Book," in 1998. Many cities have since emulated this program, attempting to unite communities by having everyone read the same book at the same time.

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$.

39. $g(x) = x - 9$
40. $g(x) = -x$
41. $g(x) = 5x$
42. $g(x) = -\frac{2}{3}x + 1$
43. $g(x) = -2x$
44. $g(x) = \frac{1}{5}x$

45. **Careers** Kelly works as a salesperson. She earns a weekly base salary plus a commission that is a percent of her total sales. Her total weekly pay is described by $f(x) = 0.20x + 300$, where x is total sales in dollars.
    a. What is Kelly's weekly base salary?
    b. What percent of total sales does Kelly receive as commission?
    c. **What if...?** What is the change in Kelly's salary plan if the weekly pay function changes to $g(x) = 0.25x + 300$? to $h(x) = 0.2x + 400$?

46. **Critical Thinking** To transform the graph of $f(x) = x$ into the graph of $g(x) = -x$, you can reflect the graph of $f(x)$ across the y-axis. Find another transformation that will have the same result.

47. **Write About It** Describe how a reflection across the y-axis affects each point on a graph. Give an example to illustrate your answer.

MULTI-STEP TEST PREP

48. This problem will prepare you for the Multi-Step Test Prep on page 376.
    a. Maria is walking from school to the softball field at a rate of 3 feet per second. Write a rule that gives her distance from school (in feet) as a function of time (in seconds). Then graph.
    b. Give a real-world situation that could be described by a line parallel to the one in part **a**.
    c. What does the y-intercept represent in each of these situations?

TEST PREP

49. Which best describes the effect on $f(x) = 2x - 5$ if the slope changes to 10?

(A) Its graph becomes less steep.
(B) Its graph moves 15 units up.
(C) Its graph makes 10 complete rotations.
(D) The x-intercept becomes $\frac{1}{2}$.

50. Given $f(x) = 22x - 182$, which does NOT describe the effect of increasing the y-intercept by 182?

(F) The new line passes through the origin.
(G) The new x-intercept is 0.
(H) The new line is parallel to the original.
(J) The new line is steeper than the original.

CHALLENGE AND EXTEND

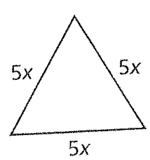
51. You have seen that the graph of $g(x) = x + 3$ is the result of translating the graph of $f(x) = x$ three units up. However, you can also think of this as a *horizontal* translation—that is, a translation left or right. Graph $g(x) = x + 3$. Describe the horizontal translation of the graph of $f(x) = x$ to get the graph of $g(x) = x + 3$.

52. If $c > 0$, how can you describe the translation that transforms the graph of $f(x) = x$ into the graph of $g(x) = x + c$ as a horizontal translation? $g(x) = x - c$ as a horizontal translation?

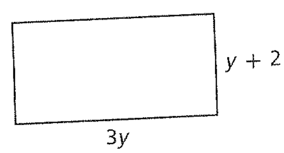
SPIRAL REVIEW

Give an expression in simplest form for the perimeter of each figure. *(Lesson 1-7)*

53.



54.



Identify the correlation you would expect to see between each pair of data sets. Explain. *(Lesson 4-5)*

55. the temperature and the number of people at the local ice cream parlor

56. the amount of electricity used and the total electric bill

57. the number of miles driven after a fill-up and the amount of gasoline in the tank

Identify which lines are parallel. *(Lesson 5-9)*

58. $y = -2x + 3$; $y = 2x$; $y = -2$; $y = -2x - 4$; $y = \frac{1}{2}x$; $y - 1 = -\frac{1}{2}(x + 6)$

59. $y = \frac{3}{5}x + 8$; $y = -\frac{3}{5}x$; $y + 1 = -\frac{3}{5}(x - 2)$; $y = \frac{5}{3}x + 9$; $y = 3x + 5$

Identify which lines are perpendicular. *(Lesson 5-9)*

60. $3x - 5y = 5$; $5y = -2x - 15$; $y = 3x + 5$; $5x + 3y = -21$; $y = \frac{5}{2}x - 2$

61. $x = 4$; $2y + x = 6$; $3x - y = 12$; $y = 2x + 3$; $y = -3$

MULTI-STEP TEST PREP

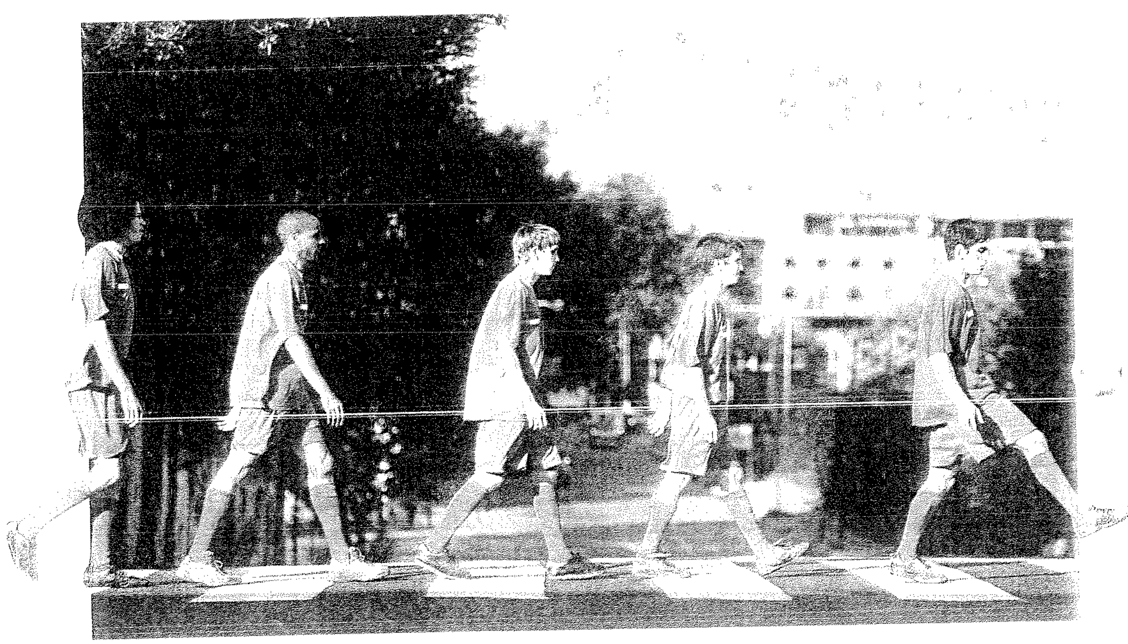
Using Linear Functions

Take a Walk! All intersections in Durango, Colorado, have crossing signals with timers. Once the signal changes to walk, the timer begins at 28 seconds and counts down to show how much time pedestrians have to cross the street.

1. Pauline counted her steps as she crossed the street. She counted 15 steps with 19 seconds remaining. When she reached the opposite side of the street, she had counted a total of 30 steps and had 10 seconds remaining. Copy and complete the table below using these values.

Time Remaining (s)	28		
Steps	0		

2. Find the average rate of change for Pauline's walk.
3. Sketch a graph of the points in the table, or plot them on your graphing calculator.
4. Find an equation that describes the line through the points.
5. How would the graph change if Pauline increased her speed? What if she decreased her speed?



READY TO GO ON?

Quiz for Lessons 5-7 Through 5-10

5-7 Slope-Intercept Form

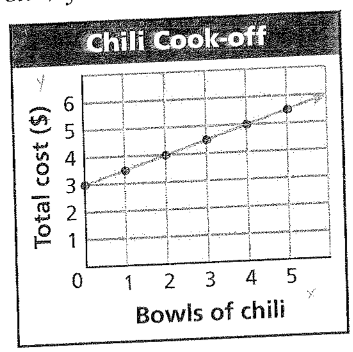
Graph each line given the slope and y-intercept.

1. slope $= \frac{1}{4}$; y-intercept $= 2$
2. slope $= -3$; y-intercept $= 5$
3. slope $= -1$; y-intercept $= -6$

Write each equation in slope-intercept form. Then graph the line described by the equation.

4. $2x + y = 5$
5. $2x - 6y = 6$
6. $3x + y = 3x - 4$

7. **Entertainment** At a chili cook-off, people pay a \$3.00 entrance fee and \$0.50 for each bowl of chili they taste. The graph shows the total cost per person as a function of the number of bowls of chili tasted.
   a. Write an equation that represents the total cost per person as a function of the number of bowls of chili tasted.
   b. Identify the slope and y-intercept and describe their meanings.



5-8 Point-Slope Form

Graph the line with the given slope that contains the given point.

8. slope $= -3$; $(0, 3)$
9. slope $= -\frac{2}{3}$; $(-3, 5)$
10. slope $= 2$; $(-3, -1)$

Write an equation in slope-intercept form for the line through the two points.

11. $(3, 1)$ and $(4, 3)$
12. $(-1, -1)$ and $(1, 7)$
13. $(1, -4)$ and $(-2, 5)$

5-9 Slopes of Parallel and Perpendicular Lines

Identify which lines are parallel.

14. $y = -2x$; $y = 2x + 1$; $y = 2x$; $y = 2(x + 5)$
15. $-3y = x$; $y = -\frac{1}{3}x + 1$; $y = -3x$; $y + 2 = x + 4$

Identify which lines are perpendicular.

16. $y = -4x - 1$; $y = \frac{1}{4}x$; $y = 4x - 6$; $x = -4$
17. $y = -\frac{3}{4}x$; $y = \frac{3}{4}x - 3$; $y = \frac{4}{3}x$; $y = 4$; $x = 3$
18. Write an equation in slope-intercept form for the line that passes through $(5, 2)$ and is parallel to the line described by $3x + 5y = 15$.
19. Write an equation in slope-intercept form for the line that passes through $(3, 5)$ and is perpendicular to the line described by $y = -\frac{3}{2}x - 2$.

5-10 Transforming Linear Functions

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

20. $f(x) = 5x$, $g(x) = -5x$
21. $f(x) = \frac{1}{2}x - 1$, $g(x) = \frac{1}{2}x + 4$
22. An attorney charges an initial fee of \$250 and then \$150 per hour. The total bill after x hours is $f(x) = 150x + 250$. How will the graph of this function change if the initial fee is reduced to \$200? if the hourly rate is increased to \$175?

EXTENSION Absolute-Value Functions

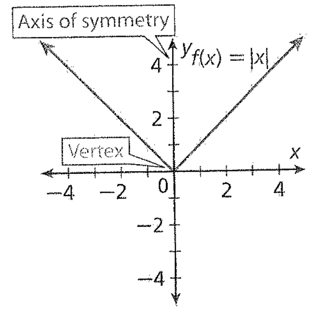
Objectives
Graph absolute-value functions.

Identify characteristics of absolute-value functions and their graphs.

Vocabulary
absolute-value function
axis of symmetry
vertex

An **absolute-value function** is a function whose rule contains an absolute-value expression. To graph an absolute-value function, choose several values of x and generate some ordered pairs.

x	$f(x) = \|x\|$
-2	2
-1	1
0	0
1	1
2	2



Absolute-value graphs are composed of two linear pieces. The **axis of symmetry** is the line that divides the graph into two congruent halves. The **vertex** is the "corner" point on the graph.

From the table and the graph of $f(x) = |x|$, you can tell that
- the axis of symmetry is the y-axis $(x = 0)$.
- the vertex is $(0, 0)$.
- the domain (x-values) is the set of all real numbers.
- the range (y-values) is described by $y \geq 0$.
- the x-intercept and the y-intercept are both 0.
- the slope of the left linear piece is -1 and the slope of the right linear piece is 1.

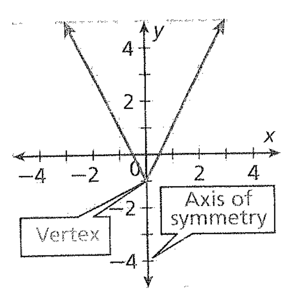
EXAMPLE **1** **Absolute-Value Functions**

Graph $f(x) = |2x| - 1$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

Choose positive, negative, and zero values for x, and find ordered pairs.

x	-2	-1	0	1	2
$f(x) = \|2x\| - 1$	3	1	-1	1	3

Plot the ordered pairs and connect them.



From the table and the graph, you can tell that
- the axis of symmetry is the y-axis $(x = 0)$.
- the vertex is $(0, -1)$.
- the x-intercepts are $\frac{1}{2}$ and $-\frac{1}{2}$.
- the y-intercept is -1.
- the domain is all real numbers.
- the range is described by $y \geq -1$.
- the slope of the left piece is -2 and the slope of the right piece is 2.

CHECK IT OUT!

1. Graph $f(x) = 3|x|$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

The parent function for absolute-value functions is $f(x) = |x|$. The graphs of all other absolute-value functions are transformations of the graph of $f(x) = |x|$.

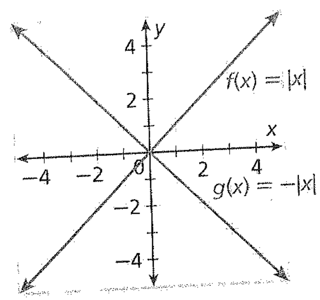
For $f(x) = a|x - b| + c$

WORDS

Opening Direction

- If $a > 0$, the graph opens upward.
- If $a < 0$, the graph opens downward.

EXAMPLES

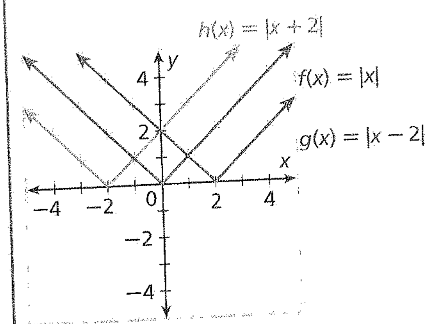


WORDS

Horizontal Translation

- If $b > 0$, the graph is translated b units right from $f(x) = |x|$.
- If $b < 0$, the graph is translated b units left from $f(x) = |x|$.

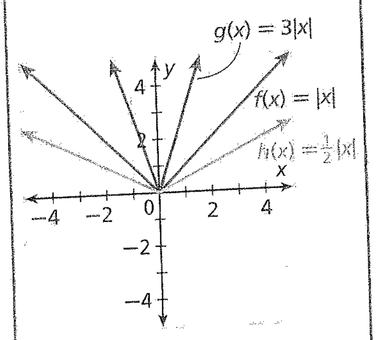
EXAMPLES



Width

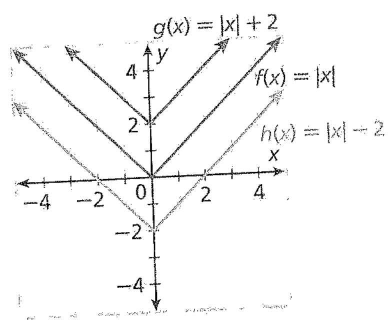
- If $|a| > 1$, the graph is narrower than the graph of $f(x) = |x|$.
- If $|a| < 1$, the graph is wider than the graph of $f(x) = |x|$.

The slopes of the two linear pieces are a and $-a$.



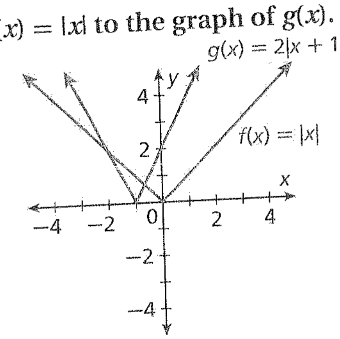
Vertical Translation

- If $c > 0$, the graph is translated c units up from $f(x) = |x|$.
- If $c < 0$, the graph is translated c units down from $f(x) = |x|$.



EXAMPLE 2 Transforming Absolute-Value Functions

Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.



A $g(x) = 2|x + 1|$

Identify a, b, and c.

$g(x) = 2|x + 1| = 2|x - (-1)| + 0$

- $a = 2$: graph is narrower
- $b = -1$: translated 1 unit left
- $c = 0$: no vertical translation

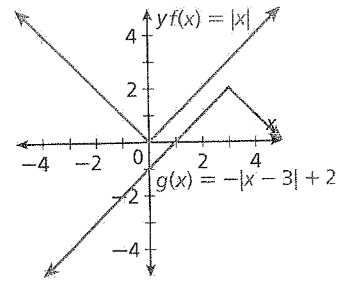
Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

B $g(x) = -|x - 3| + 2$

Identify a, b, and c.

$g(x) = -|x - 3| + 2 = -1|x - 3| + 2$

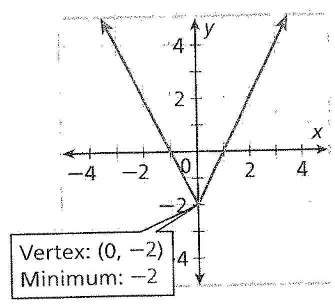
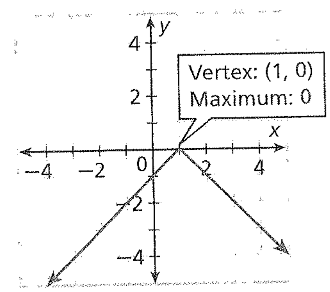
- $a = -1$: graph opens downward and width is unchanged
- $b = 3$: translated 3 units right
- $c = 2$: translated 2 units up



CHECK IT OUT! **Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.**

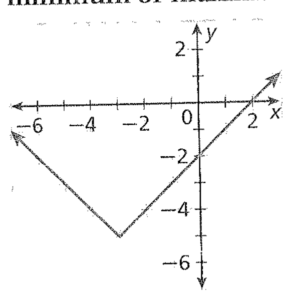
2a. $g(x) = |x - 4| - 2$ 2b. $g(x) = \frac{1}{3}|x| + 1$

Minimum and Maximum Values of Absolute-Value Functions

WORDS	If the graph of an absolute-value function opens upward, the *y*-value of the vertex is the **minimum** value of the function.	If the graph of an absolute-value function opens downward, the *y*-value of the vertex is the **maximum** value of the function.
EXAMPLES	$f(x) = 2\|x\| - 2$ 	$f(x) = -\|x - 1\|$ 

EXAMPLE 3 **Identifying the Minimum or Maximum**

Graph $f(x) = |x + 3| - 5$. Identify the vertex and give the minimum or maximum value of the function.



The vertex is $(-3, -5)$.

The graph opens upward, so the function has a minimum.

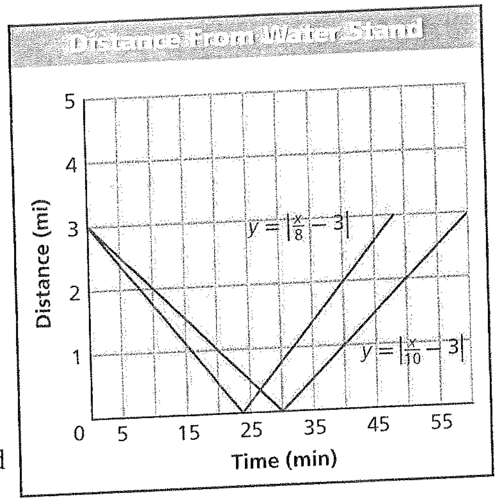
The minimum is -5.

CHECK IT OUT! **Graph each absolute-value function. Identify the vertex and give the minimum or maximum value of the function.**

3a. $f(x) = -|x| + 5$ 3b. $f(x) = |x + 1| - 6$

EXAMPLE 4 ***Sports Application***

In a charity race, a water stand for the runners is halfway between the start and finish lines. The function $y = \left|\frac{x}{8} - 3\right|$ models Riley's distance y in miles from the water stand x minutes into the race.



A How long is the race?

$y = \left|\frac{x}{8} - 3\right|$ is graphed in blue.

At the start of the race ($x = 0$), Riley is 3 mi from the water stand. The water stand is halfway between the start and finish lines, so the race is 6 mi.

B How much time does it take Riley to reach the water stand?

When Riley reaches the water stand, $y = 0$. This happens when $x = 24$. It takes Riley 24 min to reach the water stand.

C The function $y = \left|\frac{x}{10} - 3\right|$ models Dean's distance from the water stand during the same race. Compare Dean's graph to Riley's graph. What can you conclude about Dean's speed?

$y = \left|\frac{x}{10} - 3\right|$ is graphed in red. Both graphs start at the same point, but Dean's graph is translated to the right. It takes him more time to reach the water stand and to finish the race. Therefore, he is running more slowly than Riley.

CHECK IT OUT! 4. How would the graph be different for someone who runs faster than Riley?

EXTENSION Exercises

Graph each absolute-value function and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece. Identify the maximum or minimum.

1. $f(x) = |x| + 3$
2. $f(x) = |x + 3|$
3. $f(x) = \frac{1}{2}|x|$
4. $f(x) = |x - 3|$

Tell whether each statement is sometimes, always, or never true.

5. The absolute value of a number is negative.
6. An absolute-value function has an x-intercept.
7. An absolute-value function has two y-intercepts.

Write a function to describe each of the following.

8. The graph of $f(x) = |x|$ is translated 2 units right.
9. The graph of $f(x) = |x|$ is narrowed and reflected across the x-axis.
10. **Critical Thinking** Suppose that an absolute-value function has no x-intercepts. What can you say about the function rule?

CHAPTER 5

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A(n) ___?___ is a "slide," a(n) ___?___ is a "turn," and a(n) ___?___ is a "flip."
2. The x-coordinate of the point that contains the ___?___ is always 0.
3. In the equation $y = mx + b$, the value of m is the ___?___ , and the value of b is the ___?___ .

5-1 Identifying Linear Functions *(pp. 300–306)*

EXAMPLES

Tell whether each function is linear. If so, graph the function.

■ $y = -3x + 2$

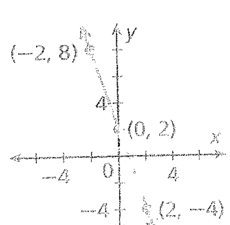
$$\begin{aligned} y &= -3x + 2 \\ +3x & \quad\; +3x \\ 3x + y &= \quad\;\; 2 \end{aligned}$$

Write the equation in standard form.

This is a linear function.

Generate ordered pairs

x	$y = -3x + 2$	(x, y)
-2	$y = -3\quad + 2 = 8$	$(\quad, 8)$
0	$y = -3\quad + 2 = 2$	$(\quad, 2)$
2	$y = -3\quad + 2 = -4$	$(\quad, -4)$



■ $y = 2x^3$

This is not a linear function because x has an exponent other than 1.

Tell whether the given ordered pairs satisfy a linear function. Explain.

4.

x	y
-3	3
-1	1
1	1
3	3

5.

x	y
0	-3
1	-1
2	1
3	3

6. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$
7. $\{(1, 7), (3, 6), (6, 5), (9, 4), (13, 3)\}$

Each equation below is linear. Write each equation in standard form and give the values of A, B, and C.

8. $y = -5x + 1$
9. $\frac{x+2}{2} = -3y$
10. $4y = 7x$
11. $9 = y$
12. Helene is selling cupcakes for \$0.50 each. The function $f(x) = 0.5x$ gives the total amount of money Helene makes after selling x cupcakes. Graph this function and give its domain and range.

5-2 Using Intercepts *(pp. 307–312)*

EXAMPLE

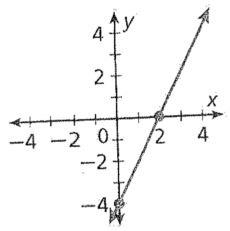
- **Find the x- and y-intercepts of $2x + 5y = 10$.**

Let $y = 0$.	Let $x = 0$.
$2x + 5(0) = 10$	$2(0) + 5y = 10$
$2x + 0 = 10$	$0 + 5y = 10$
$2x = 10$	$5y = 10$
$\frac{2x}{2} = \frac{10}{2}$	$\frac{5y}{5} = \frac{10}{5}$
$x = 5$	$y = 2$
The x-intercept is 5.	The y-intercept is 2.

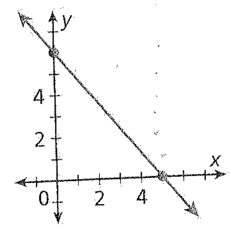
EXERCISES

Find the x- and y-intercepts.

13.



14.



15. $3x - y = 9$

16. $-2x + y = 1$

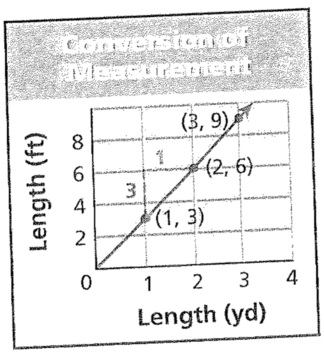
17. $-x + 6y = 18$

18. $3x + 4y = 1$

5-3 Rate of Change and Slope *(pp. 314–321)*

EXAMPLE

- **Find the slope of the line.**



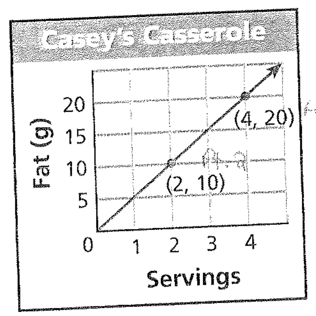
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3}{1} = 3$$

EXERCISES

19. Graph the data and show the rates of change.

Time (s)	Distance (ft)
0	0
1	16
2	64
3	144
4	256

20. Find the slope of the line graphed below.



5-4 The Slope Formula *(pp. 324–329)*

EXAMPLE

- **Find the slope of the line described by $2x - 3y = 6$.**

Step 1 Find the x- and y-intercepts.

Let $y = 0$.	Let $x = 0$.
$2x - 3(0) = 6$	$2(0) - 3y = 6$
$2x = 6$	$-3y = 6$
$x = 3$	$y = -2$

The line contains $(3, 0)$ and $(0, -2)$.

Step 2 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{0 - 3} = \frac{-2}{-3} = \frac{2}{3}$$

EXERCISES

Find the slope of the line described by each equation.

21. $4x + 3y = 24$

22. $y = -3x + 6$

23. $x + 2y = 10$

24. $3x = y + 3$

25. $y + 2 = 7x$

26. $16x = 4y + 1$

Find the slope of the line that contains each pair of points.

27. $(1, 2)$ and $(2, -3)$

28. $(4, -2)$ and $(-5, 7)$

29. $(-3, -6)$ and $(4, 1)$

30. $\left(\frac{1}{2}, 2\right)$ and $\left(\frac{3}{4}, \frac{5}{2}\right)$

31. $(2, 2)$ and $(2, 7)$

32. $(1, -3)$ and $(5, -3)$

5-5 The Midpoint and Distance Formulas (pp. 330–335)

EXAMPLE

■ Find the coordinates of the midpoint of $\overline{AB}$ with endpoints $A(-4, 3)$ and $B(5, -6)$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-4 + 5}{2}, \frac{3 + (-6)}{2}\right) = M\left(\frac{1}{2}, -\frac{3}{2}\right)$$

Use the Distance Formula to find the distance, to the nearest hundredth, from $C(-4, 1)$ to $D(5, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{[5 - (-4)]^2 + (-3 - 1)^2}$$

$$d = \sqrt{9^2 + (-4)^2}$$

$$d = \sqrt{97} \approx 9.85$$

Find the coordinates of the midpoint of each segment.

33. $\overline{EF}$ with endpoints $E(9, 12)$ and $F(21, 18)$
34. $\overline{GH}$ with endpoints $G(-5, -7)$ and $H(4, -11)$

Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.

35. $J(3, 10)$ and $K(-2, 4)$
36. $L(-6, 0)$ and $M(8, -7)$
37. Each unit on a map of a forest represents 1 mile. To the nearest tenth of a mile, what is the distance from a ranger station at (1, 2) on the map to a river crossing at (2, 4)?

5-6 Direct Variation (pp. 336–341)

EXAMPLE

■ **Tell whether $6x = -4y$ represents a direct variation. If so, identify the constant of variation.**

$$6x = -4y$$

$$\frac{6x}{-4} = \frac{-4y}{-4}$$

$$-\frac{6}{4}x = y$$

$$y = -\frac{3}{2}x$$

This equation represents a direct variation because it can be written in the form $y = kx$, where $k = -\frac{3}{2}$.

Tell whether each equation is a direct variation. If so, identify the constant of variation.

38. $y = -6x$
39. $x - y = 0$
40. $y + 4x = 3$
41. $2x = -4y$
42. The value of y varies directly with x, and $y = -8$ when $x = 2$. Find y when $x = 3$.
43. Maleka charges \$8 per hour for baby-sitting. The amount of money she makes varies directly with the number of hours she baby-sits. Write a direct variation equation for the amount y Maleka earns for baby-sitting x hours. Then graph.

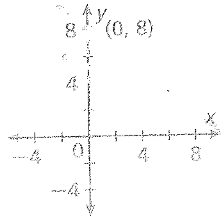
5-7 Slope-Intercept Form (pp. 344–350)

EXAMPLE

■ **Graph the line with slope $= -\frac{4}{5}$ and y-intercept $= 8$.**

Step 1 Plot (0, 8).

Step 2 For a slope of $\frac{-4}{5}$, count 4 down and 5 right from (0, 8). Plot another point.



Step 3 Connect the two points with a line.

Graph each line given the slope and y-intercept.

44. slope $= -\frac{1}{2}$; y-intercept $= 4$
45. slope $= 3$; y-intercept $= -7$

Write the equation that describes each line in slope-intercept form.

46. slope $= \frac{1}{3}$, y-intercept $= 5$
47. slope $= 4$, $(1, -5)$ is on the line

5-8 Point-Slope Form (pp. 351–358)

EXAMPLE

- **Write an equation in slope-intercept form for the line through $(4, -1)$ and $(-2, 8)$.**

  Step 1 Find the slope.

  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{-2 - 4} = \frac{9}{-6} = -\frac{3}{2}$$

  Step 2 Write the point-slope form.

  $y - 8 = -\frac{3}{2}[x - (-2)]$ *Substitute into the point-slope form.*

  $y - 8 = -\frac{3}{2}(x + 2)$

  Step 3 Write the slope-intercept form.

  $y = -\frac{3}{2}x + 5$ *Solve for y.*

EXERCISES

Graph the line described by each equation.

48. $y + 3 = \frac{1}{2}(x + 4)$ **49.** $y - 1 = -(x + 3)$

Write the equation that describes each line in slope-intercept form.

50. slope = 2, (1, 3) is on the line.

51. slope = −5, (−6, 4) is on the line.

52. (1, 4) and (3, 8) are on the line.

53. (−2, 4) and (−1, 6) are on the line.

5-9 Slopes of Parallel and Perpendicular Lines (pp. 361–367)

EXAMPLE

- **Write an equation in slope-intercept form for the line that passes through $(4, -2)$ and is perpendicular to the line described by $y = -4x + 3$.**

  The slope of $y = -4x + 3$ is -4.

  The perpendicular line has a slope of $\frac{1}{4}$ and contains $(4, -2)$.

  $y + 2 = \frac{1}{4}(x - 4)$ *Substitute into the point-slope form.*

  $y = \frac{1}{4}x - 3$ *Solve for y.*

EXERCISES

Identify which lines are parallel.

54. $y = -\frac{1}{3}x$; $y = 3x + 2$; $y = -\frac{1}{3}x - 6$; $y = 3$

55. $y - 2 = -4(x - 1)$; $y = 4x - 4$; $y = \frac{1}{4}x$; $y = -4x - 2$

Identify which lines are perpendicular.

56. $y - 1 = -5(x - 6)$; $y = \frac{1}{5}x + 2$; $y = 5$; $y = 5x + 8$

57. $y = 2x$; $y - 2 = 3(x + 1)$; $y = \frac{2}{3}x - 4$; $y = -\frac{1}{3}x$

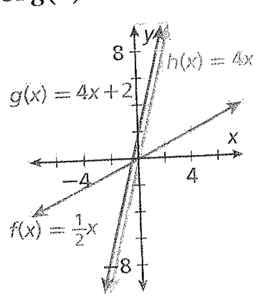
58. Write an equation in slope-intercept form for the line that passes through $(1, -1)$ and is parallel to the line described by $y = 2x - 4$.

5-10 Transforming Linear Functions (pp. 369–375)

EXAMPLE

- **Graph $f(x) = \frac{1}{2}x$ and $g(x) = 4x + 2$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.**

  - Multiply $f(x) = \frac{1}{2}x$ by 8 to get $h(x) = 4x$. This rotates the graph about $(0, 0)$, making it steeper.
  - Then add 2 to $h(x) = 4x$ to get $g(x) = 4x + 2$. This translates the graph 2 units up.



EXERCISES

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

59. $f(x) = x$, $g(x) = x + 4$

60. $f(x) = 4x$, $g(x) = -4x$

61. $f(x) = \frac{1}{3}x - 2$, $g(x) = -\frac{1}{3}x - 2$

62. The entrance fee at a carnival is \$3 and each ride costs \$1. The total cost for x rides is $f(x) = x + 3$. How will the graph of this function change if the entrance fee is increased to \$5? if the cost per ride is increased to \$2?

CHAPTER 5

CHAPTER TEST

Tell whether each function is linear. If so, graph the function.

1. $3y = 2x + 3$

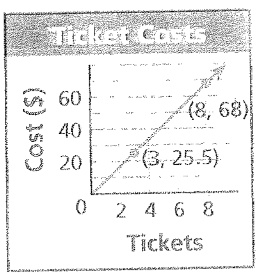
2. $y = x(4 + x)$

3. Lily plans to volunteer at the tutoring center for 45 hours. She can tutor 3 hours per week. The function $f(x) = 45 - 3x$ gives the number of hours she will have left to tutor after x weeks. Graph the function and find its intercepts. What does each intercept represent?

4. The table shows the number of guppies in an aquarium over time. Graph the data and show the rates of change.

Time (mo)	0	1	2	5	8
Guppies	4	4	10	25	31

Find the slope of each line. Then tell what the slope represents.

5.


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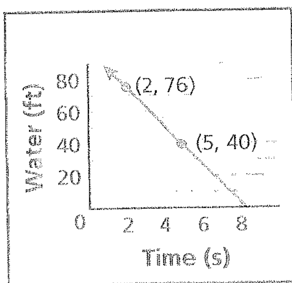

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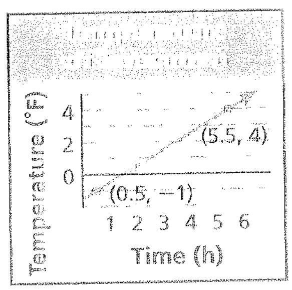


8. Each unit on a map of a bay represents 1 kilometer. Two buoys are located at (1, 5) and (3, 6) on the map. To the nearest tenth of a kilometer, what is the distance between the two buoys?

9. In orbit around Earth, a space shuttle travels at a speed of about 5 miles per second. Write a direct variation equation for the distance y the space shuttle will travel in x seconds. Then graph.

10. Write the equation $2x - 2y = 4$ in slope-intercept form. Then graph the line described by the equation.

Write the equation that describes each line in slope-intercept form.

11. slope $= -2$, y-intercept $= 7$

12. slope $= 4$, $(-3, 3)$ is on the line.

13. $(1, 5)$ and $(-2, 8)$ are on the line.

14. x-intercept $= 3$, y-intercept $= -3$

Write an equation in point-slope form for the line with the given slope that contains the given point.

15. slope $= -1$; $(1, 3)$

16. slope $= 5$; $(-3, 2)$

17. Write an equation in slope-intercept form for the line that passes through $(0, 6)$ and is parallel to the line described by $y = 2x + 3$.

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

18. $f(x) = 8x$, $g(x) = 4x$

19. $f(x) = -x + 2$, $g(x) = -x - 1$

20. $f(x) = 3x$, $g(x) = 6x - 1$

College Entrance Exam Practice

FOCUS ON SAT

SAT scores are based on the total number of items answered correctly minus a fraction of the number of multiple-choice questions answered incorrectly. No points are subtracted for questions unanswered.

HOT TIP! On the SAT, there is a penalty for incorrect answers on multiple-choice items. Guess only when you can eliminate at least one of the answer choices.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

1. The line through $A(1, -3)$ and $B(-2, d)$ has slope -2. What is the value of d?

(A) $-\frac{3}{2}$

(B) -1

(C) $\frac{1}{2}$

(D) 3

(E) 5

2. The ordered pairs $\{(0, -3), (4, -1), (6, 0), (10, 2)\}$ satisfy a pattern. Which is NOT true?

(A) The pattern is linear.

(B) The pattern can be described by $2x - 4y = 12$.

(C) The ordered pairs lie on a line.

(D) $(-4, 1)$ satisfies the same pattern.

(E) The set of ordered pairs is a function.

3. If y varies directly as x, what is the value of x when $y = 72$?

x	7	12	
y	28	48	72

(A) 17

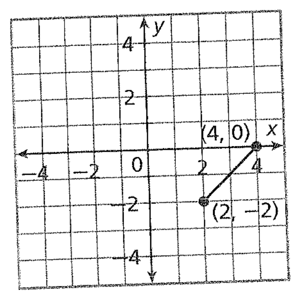
(B) 18

(C) 24

(D) 28

(E) 36

4. The line segment between the points $(4, 0)$ and $(2, -2)$ forms one side of a rectangle. Which of the following coordinates could determine another vertex of that rectangle?



(A) $(-2, 6)$

(B) $(-2, -2)$

(C) $(0, 6)$

(D) $(1, 2)$

(E) $(4, 6)$

5. Which of the following has the same slope as the line described by $2x - 3y = 3$?

(A) $3x - 2y = 2$

(B) $\frac{2}{3}x - y = -2$

(C) $2x - 2y = 3$

(D) $\frac{1}{3}x - 2y = -2$

(E) $-2x - 3y = 2$

TEST TACKLER

Standardized Test Strategies

Multiple Choice: Recognize Distracters

In multiple-choice items, the options that are incorrect are called *distracters*. This is an appropriate name, because these incorrect options can distract you from the correct answer.

Test writers create distracters by using common student errors. Beware! Even if the answer you get when you work the problem is one of the options, it may not be the correct answer.

EXAMPLE 1

What is the y-intercept of $4x + 10 = -2y$?

Ⓐ 10 Ⓒ -2.5

Ⓑ 5 Ⓓ -5

Look at each option carefully.

Ⓐ This is a distracter. The y-intercept would be 10 if the function was $4x + 10 = y$. A common error is to ignore the coefficient of y.

Ⓑ This is a distracter. Another common error is to divide by 2 instead of -2 when solving for y.

Ⓒ This is a distracter. One of the most common errors students make is confusing the x-intercept and the y-intercept. This distracter is actually the *x-intercept* of the given line.

Ⓓ This is the correct answer.

EXAMPLE 2

What is the equation of a line with a slope of -4 that contains $(2, -3)$?

Ⓕ $y - 3 = -4(x - 2)$ Ⓗ $y + 3 = -4(x - 2)$

Ⓖ $y - 2 = -4(x + 3)$ Ⓙ $y + 4 = -3(x - 2)$

Look at each option carefully.

Ⓕ This is a distracter. Students often make errors with positive and negative signs. You would get this answer if you simplified $y - (-3)$ as $y - 3$.

Ⓖ This is a distracter. You would get this answer if you switched the x-coordinate and the y-coordinate.

Ⓗ This is the correct answer.

Ⓙ This is a distracter. You would get this answer if you substituted the given values incorrectly in the point-slope equation.

When you calculate an answer to a multiple-choice test item, try to solve the problem again with a different method to make sure your answer is correct.

Read each test item and answer the questions that follow.

Item A

A line contains $(1, 2)$ and $(-2, 14)$. What are the slope and y-intercept?

Ⓐ Slope $= -4$; y-intercept $= -2$

Ⓑ Slope $= 4$; y-intercept $= 6$

Ⓒ Slope $= -\frac{1}{4}$; y-intercept $= 1$

Ⓓ Slope $= -4$; y-intercept $= 6$

1. What common error does the slope in choice B represent?
2. The slope given in choice A is correct, but the y-intercept is not. What error was made when finding the y-intercept?
3. What formula can you use to find the slope of a line? How was this formula used incorrectly to get the slope in choice C?

Item B

Which of these functions has a graph that is NOT parallel to the line described by $y = \frac{1}{2}x + 4$?

Ⓕ $y = 6 - \frac{1}{2}x$

Ⓖ $y = \frac{1}{2}x + 6$

Ⓗ $-2y = -x + 1$

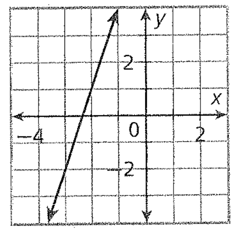
Ⓙ $2y = x$

4. When given two linear functions, describe how to determine whether their graphs are parallel.
5. Which is the correct answer? Describe the errors a student might make to get each of the distracters.

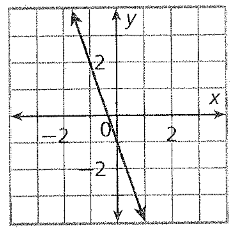
Item C

Which of these lines has a slope of -3?

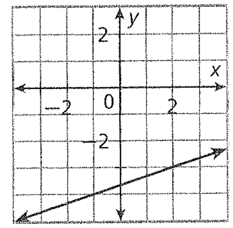
Ⓐ



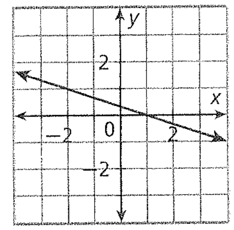
Ⓑ



Ⓒ



Ⓓ



6. Which two answer choices can be eliminated immediately? Why?
7. Decribe how to find the slope of a line from its graph.
8. What common error does choice A represent?
9. What common error does choice D represent?
10. Which is the correct answer?

Item D

Which is NOT a linear function?

Ⓕ $f(x) = 4 + x$

Ⓖ $f(x) = -x - 4$

Ⓗ $f(x) = 4x^2$

Ⓙ $f(x) = \frac{1}{4}x$

11. When given a function rule, how can you tell if the function is linear?
12. What part of the function given in choice G might make someone think it is not linear?
13. What part of the function given in choice J might make someone think it is not linear?
14. What part of the function given in choice H makes it NOT linear?

CHAPTER 5

STANDARDIZED TEST PREP

KEYWORD: MA7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–5

Multiple Choice

1. What is the value of $2 - [1 - (2 - 1)]$?

   (A) -2
   (B) 0
   (C) 2
   (D) 4

2. Frank borrowed \$5000 with an annual simple interest rate. The amount of interest he owed after 6 months was \$300. What is the interest rate of the loan?

   (F) 1%
   (G) 6%
   (H) 10%
   (J) 12%

3. Patty's Pizza charges \$5.50 for a large pizza plus \$0.30 for each topping. Pizza Town charges \$5.00 for a large pizza plus \$0.40 for each topping. Which inequality can you use to find the number of toppings x so that the cost of a pizza at Pizza Town is greater than the cost of a pizza at Patty's Pizza?

   (A) $(5 + 0.4)x > (5.5 + 0.3)x$
   (B) $5.5x + 0.3 > 5x + 0.4$
   (C) $5.5 + 0.3x > 5 + 0.4x$
   (D) $5 + 0.4x > 5.5 + 0.3x$

4. The side length of a square s can be determined by the formula $s = \sqrt{A}$ where A represents the area of the square. What is the side length of a square with area 0.09 square meter?

   (F) 0.0081 meters
   (G) 0.81 meters
   (H) 0.03 meters
   (J) 0.3 meters

5. What is the value of $f(x) = -3 - x$ when $x = -7$?

   (A) -10
   (B) -4
   (C) 4
   (D) 10

6. Which relationship is a direct variation?

   (F)

x	1	2	3	4
y	−1	0	1	2

   (G)

x	1	2	3	4
y	0	−1	−2	−3

   (H)

x	1	2	3	4
y	3	5	7	9

   (J)

x	1	2	3	4
y	3	6	9	12

7. Which function has x-intercept -2 and y-intercept 4?

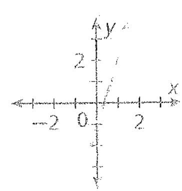
   (A) $2x - y = 4$
   (B) $2y - x = 4$
   (C) $y - 2x = 4$
   (D) $x - 2y = 4$

8. Which equation describes the relationship between x and y in the table below?

x	−8	−4	0	4	8
y	2	1	0	−1	−2

   (F) $y = -4x$
   (G) $y = -\frac{1}{4}x$
   (H) $y = 4x$
   (J) $y = \frac{1}{4}x$

9. Which graph is described by $x - 3y = -3$?

   (A)


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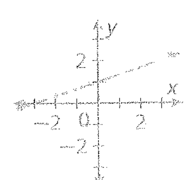
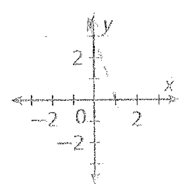


   (C)


   (D)