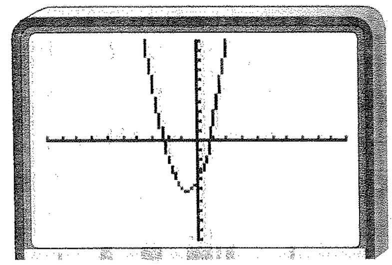
Sometimes you will want to be able to trace the graphs drawn or use other graphing features of the calculator. This often occurs when solving inequalities in one variable. In this case, equations should be entered in the Y = list.

Example 3 **Solve $2x^2 + 3x - 4 < 0$ to the nearest hundredth using a graphing calculator.**

Begin by obtaining a graph of $y = 2x^2 + 3x - 4$. We are interested in determining values for x so that $2x^2 + 3x - 4$ is less than 0. So, we look for points where the y value is less than 0, or where the graph falls below the x-axis.

Graphing calculators cannot show dashed lines for inequalities involving less than or greater than signs.

Enter: [Y=] 2 [X,T,θ,n] [x²] [+] 3 [X,T,θ,n] [−] 4 [ZOOM] 6



Zooming in on the x-intercepts, we find that $2x^2 + 3x - 4 < 0$ when $-2.35 < x < 0.85$.

EXERCISES

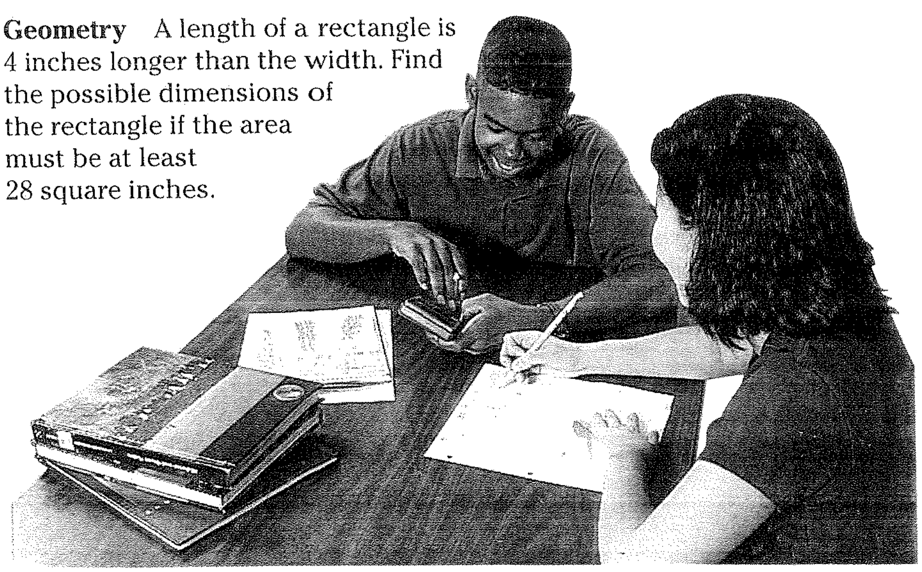
Use a graphing calculator to graph each inequality.

1. $y \geq x^2 + 11x - 3$
2. $y \leq -0.5x^2 + 9$
3. $y \leq 1.2x^2 + 15x$
4. $y \geq 6x^2 - 15x + 7$
5. $y > -x^2 + 6x + 8$
6. $y > -4x^2 - 3x - 6$

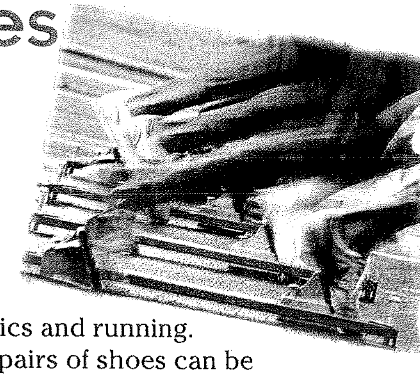
Use a graphing calculator to solve each inequality to the nearest hundredth.

7. $x^2 + 4x - 21 > 0$
8. $2x^2 - 4x + 1 \leq 0$
9. $x(2x + 1) \geq 0$
10. $x^2 - 3 > 0$
11. $x^2 - 9x < 4$
12. $0.5x^2 > 1.8x$

13. **Geometry** A length of a rectangle is 4 inches longer than the width. Find the possible dimensions of the rectangle if the area must be at least 28 square inches.



6-7 Graphing and Solving Quadratic Inequalities



What YOU'LL LEARN
- To graph quadratic inequalities, and
- to solve quadratic inequalities in one variable.

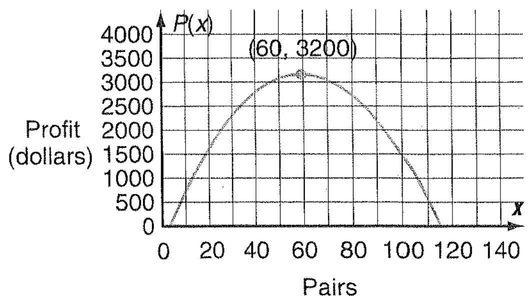
Why IT'S IMPORTANT
You can use quadratic inequalities to solve problems involving sports and forensic science.

LOOK BACK
You can refer to Lesson 2-7 to review linear inequalities.

Real World APPLICATION

Business

Athletic Advantage, Inc. makes athletic shoes for aerobics and running. According to recent sales figures, their profit $P(x)$ on x pairs of shoes can be found by the inequality $P(x) \leq -x^2 + 120x - 400$. The relation is an inequality because the wholesale price is lower for large orders.



The graph of this inequality is the part of the plane enclosed by the parabola whose equation is $P(x) = -x^2 + 120x - 400$. The parabola is the **boundary** of each region. Points on the parabola represent the profit if all the shoes are sold at the list price. Points in the interior of the curve represent profits if some of the shoes are sold at a discount.

You can graph **quadratic inequalities** using the same techniques you used to graph linear inequalities.

1. Graph the boundary. Determine if it should be solid or dashed.
2. Test a point in each region.
3. Shade the region whose ordered pair results in a true inequality.

Example 1 **Graph $y \leq x^2 - 6x + 2$.**

The boundary will be the graph of $y = -x^2 - 6x + 2$.

$$\begin{aligned} y &= -x^2 - 6x + 2 \\ &= -(x^2 + 6x) + 2 \\ &= -(x^2 + 6x + 9) + 2 + 9 \quad \textit{Complete the square.} \\ &= -(x + 3)^2 + 11 \end{aligned}$$

The boundary is a parabola that opens downward with its vertex at $(-3, 11)$. The boundary is included in the graph, so it should be solid. Test points not on the parabola to see whether points inside or outside the parabola belong to the graph.

Region inside parabola

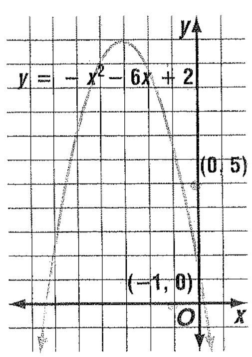
Test $(-1, 0)$:
$$\begin{aligned} y &\leq -x^2 - 6x + 2 \\ 0 &\overset{?}{\leq} -(-1)^2 - 6(-1) + 2 \\ 0 &\leq -1 + 6 + 2 \\ 0 &\leq 7 \quad \text{true} \end{aligned}$$

The point at $(-1, 0)$ does belong.

Region outside parabola

Test $(0, 5)$:
$$\begin{aligned} y &\leq -x^2 - 6x + 2 \\ 5 &\overset{?}{\leq} -(0)^2 - 6(0) + 2 \\ 5 &\leq 0 - 0 + 2 \\ 5 &\leq 2 \quad \text{false} \end{aligned}$$

The point at $(0, 5)$ does not belong.

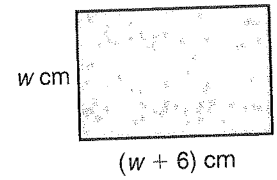


Since $(-1, 0)$ is part of the solution and $(0, 5)$ is not, shade the region inside the parabola.

Example 2 A rectangle is 6 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 216 square centimeters.

INTEGRATION
Geometry

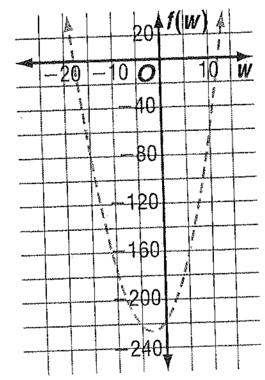
Draw a diagram of the rectangle. Let w represent the width of the rectangle. Then $w + 6$ represents the length and $w(w + 6)$ represents the area.



$$w(w + 6) > 216$$
$$w^2 + 6w > 216$$
$$w^2 + 6w - 216 > 0$$
$$(w^2 + 6w + 9) - 216 - 9 > 0 \quad \textit{Complete the square.}$$
$$(w + 3)^2 - 225 > 0$$

The boundary of the graph of this inequality is a parabola that opens upward with its vertex at $(-3, -225)$. Test a point not on the boundary to determine which region should be included in the graph.

Test (0, 0): $w(w + 6) > 216$
$0(0 + 6) > 216$
$0 > 216$ false



Since the graph of $(0, 0)$ is inside the parabola and it does not satisfy the inequality, the region outside the parabola is included in the graph.

The points on the graph outside Quadrant I should be disregarded since length cannot be negative. So the width of the rectangle should be greater than 12 cm and the length should be greater than 18 cm.

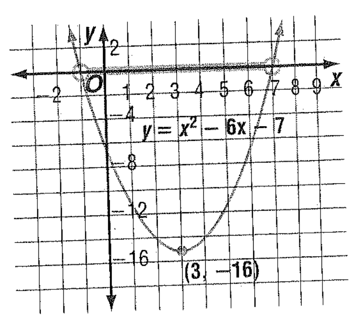
Just as you solve a quadratic equation by graphing its related quadratic function, you can solve a quadratic inequality in one variable by graphing its related quadratic inequality in two variables. For example, to solve $0 > x^2 - 6x - 7$, you can graph $y > x^2 - 6x - 7$. The solutions of the inequality are each point on the x-axis that is included in the graph.

You can also solve quadratic inequalities algebraically.

Example 3 **Solve $0 > x^2 - 6x - 7$.**

Method 1: Graphing

Graph the related inequality $y > x^2 - 6x - 7$. First complete the square and rewrite the inequality as $y > (x - 3)^2 - 16$. The graph of the $y = (x - 3)^2 - 16$ is a parabola that opens upward with its vertex at $(3, -16)$.



The points on the x-axis that satisfy $y > x^2 - 6x - 7$ are solutions to the inequality $0 > x^2 - 6x - 7$. Those solutions are $\{x \mid -1 < x < 7\}$.

(continued on the next page)

Method 2: Factoring

$0 < x^2 - 6x - 7$

$0 < (x - 7)(x + 1)$

Since the product of two numbers is negative if one number is negative and one is positive, we can write the following.

$(x - 7)$ is negative and $(x + 1)$ is positive:

$x - 7 < 0$ and $x + 1 > 0$

$x < 7$ $\quad$ $x > -1$

$-1 < x < 7$

or

$(x - 7)$ is positive and $(x + 1)$ is negative:

$x - 7 > 0$ and $x + 1 < 0$

$x > 7$ $\quad$ $x < -1$

The graphs of $x > 7$ and $x < -1$ never intersect, so $x > 7$ and $x < -1$ can never be true.

You can also solve quadratic inequalities by using three test points.

Example 4 **Solve $x^2 - x - 12 > 0$.**

First solve the equation $x^2 - x - 12 = 0$ by factoring.

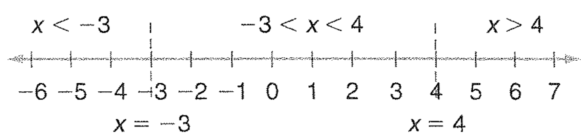
$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x - 4 = 0$ or $x + 3 = 0$

$x = 4$ $\quad$ $x = -3$

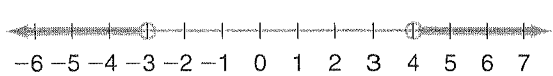
The points at 4 and -3 separate the x-axis into three regions: $x < -3$, $-3 < x < 4$, and $x > 4$.

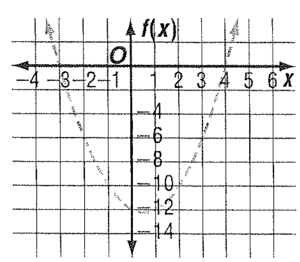


Choose a value from each part and substitute it into $x^2 - x - 12 > 0$. Determine if the result is true or false. Organize your results in a table.

Part of x-axis	Chosen point, x	$x^2 - x - 12$	Is $x^2 - x - 12 > 0$?
$x < -3$	-5	$(-5)^2 - (-5) - 12 = 18$	yes
$-3 < x < 4$	1	$(1)^2 - 1 - 12 = -12$	no
$x > 4$	6	$(6)^2 - 6 - 12 = 18$	yes

The solution set to the inequality $x^2 - x - 12 > 0$ is $\{x \mid x < -3 \text{ or } x > 4\}$, as shown on the number line below.

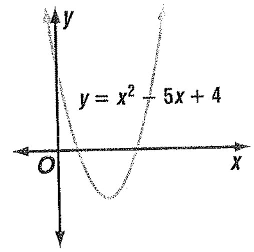




The graph of the related inequality $x^2 - x - 12 > y$, shown above, confirms the solutions.

Communicating Mathematics

Study the lesson. Then complete the following.



1. The equation $y = x^2 - 5x + 4$ is graphed at the right.
   a. If you were to graph the inequality $y \geq x^2 - 5x + 4$, would you include the region inside or outside of the parabola? Explain why.
   b. What are the solutions of $0 \geq x^2 - 5x + 4$?
2. **State** the points you would test to find the solution to $(x - 8)(x + 2) > 0$.
3. **Rewrite** Example 2 so that the region inside the parabola will be included in the graph instead of the region outside the parabola.
4. **Explain** how factoring can help determine the solution set of a quadratic inequality.

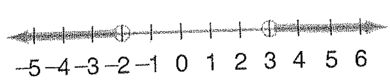
MATH JOURNAL

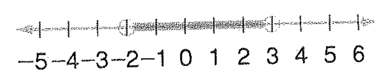
5. **Write** what you have learned about graphing quadratic inequalities. Explain why some boundaries are contained in the region and some are not included. How can you determine if the graph is a region inside or outside of the boundary?

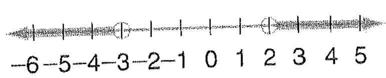
Guided Practice

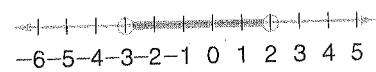
Determine if the ordered pair is a solution of the given inequality.

6. $y < 2x^2 + 4$, (3, 5)
7. $y \geq x^2 - 9$, (0, 5)
8. $y \leq 2x^2 - 3x + 1$, (−1, 4)
9. $y > 5x^2 + 2x - 3$, (−1, −1)
10. Which of the following is the graph of the solution set for $x^2 < x + 6$?

a. 

b. 

c. 

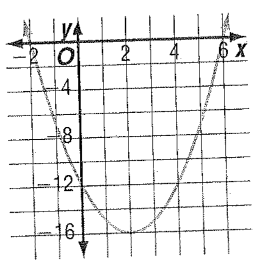
d. 

Graph each inequality.

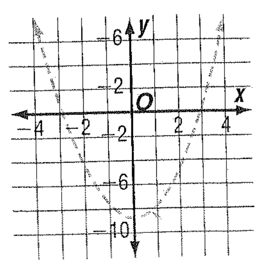
11. $y \leq x^2 + 4x + 4$
12. $y > x^2 - 36$
13. $y \leq -x^2 + 7x + 8$
14. $y \leq -x^2 - 3x + 10$

Use the related graph of each inequality to write its solutions.

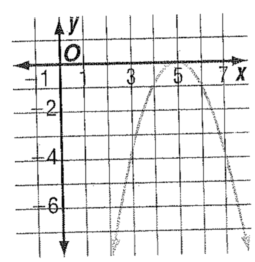
15. $x^2 - 4x - 12 \leq 0$



16. $x^2 - 9 > 0$



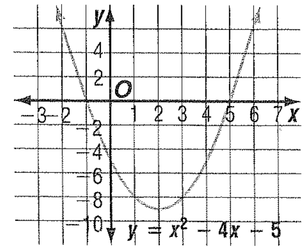
17. $-x^2 + 10x - 25 \geq 0$



Solve each inequality.

18. $(x + 11)(x - 3) > 0$
19. $(n - 2.5)(n + 3.8) \geq 0$
20. $x^2 - 4x \leq 0$
21. $b^2 \geq 10b - 25$
22. $2x^2 > 25$
23. $2b^2 - b < 6$

24. The graph of the quadratic function $y = x^2 - 4x - 5$ is shown at the right.



a. What are the solutions of the equation $0 = x^2 - 4x - 5$?

b. What are the solutions of the inequality $0 \leq x^2 - 4x - 5$?

c. What are the solutions of the inequality $0 \geq x^2 - 4x - 5$?

25. Recreation The YMCA has a 40-foot by 60-foot area in which to build a swimming pool. The pool will be surrounded by a concrete sidewalk of uniform width. What could the width of the sidewalk surrounding the pool be if organizers want the pool to be at least 1500 square feet?

EXERCISES

Practice

Graph each inequality.

26. $y \geq x^2 - 10x + 25$

27. $y < x^2 - 16$

28. $y \leq x^2 - x - 20$

29. $y \geq x^2 + 3x - 18$

30. $y \geq 2x^2 + x - 3$

31. $y \leq -x^2 + 5x + 6$

32. $y > 2x^2 + 3x - 5$

33. $y < -x^2 + 13x - 36$

34. $y \leq -x^2 + 5x + 14$

35. $y \geq -3x^2 + 5x + 2$

36. $y > 4x^2 - 8x + 3$

37. $y \leq -x^2 - 7x + 10$

Solve each inequality.

38. $(x - 4)(x + 7) < 0$

39. $x^2 - 3x - 18 > 0$

40. $m^2 + m - 6 > 0$

41. $q^2 + 2q \geq 24$

42. $p^2 - 4p \leq 5$

43. $2x^2 + 5x - 12 \leq 0$

44. $6s^2 + 5s > 4$

45. $w^2 \geq 2w$

46. $9v^2 - 6v + 1 \leq 0$

47. $2g^2 - 5g - 3 < 0$

48. $f^2 + 12f + 36 < 0$

49. $n^2 \leq 3$

50. $8d + d^2 \geq -16$

51. $4t^2 - 9 \leq -4t$

52. $(x - 1)(x + 4)(x - 3) > 0$

53. $(x + 2)(x + 4)(x - 8) \leq 0$

54. $(x + 5)(x + 1)(x - 4)(x - 6) > 0$

55. $(x - 2)(x + 2)(x - 1)(x + 3) \geq 0$

Critical Thinking

56. Find the intersection of the graphs of $y \geq x^2 - 3$ and $y \leq x^2 + 3$.

Applications and Problem Solving

Real World

57. Sports The instant replay facility at the Superdome in New Orleans, Louisiana, was moved because it was hit by a high punt kicked by Oakland Raider Ray Guy. The original position of the facility was 90 feet above the playing field. Cody, a high school punter, can kick a football with an initial velocity of 65 feet per second. The height of the football t seconds after he kicks it is found by the function $h(t) = -16t^2 + 65t$.

a. If Cody were to have kicked a football in the Superdome before the instant replay facility was moved, would he have been able to hit it? Explain.

b. What was the speed of the ball Ray Guy punted if it hit the facility in 3 seconds?

58. **Geometry** A rectangle is 5 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 104 square centimeters.

59. **Business** Lorena drives a shuttle bus for the National Park Service. The charge is \$1.00 to ride from the parking lot to one of the major attractions in the National Park. During the winter months, about 100 people ride the bus each day. It is estimated that 5 more passengers will ride per day for each \$0.20 decrease in fare. The cost of operating the shuttle bus is \$66 per day. How many \$0.20 decreases in fare could be made and still allow the shuttle bus to make a profit?

60. **Forensic Science** Police are investigating the shooting of a police helicopter. They found a weapon at the scene of the crime that has a suspect's fingerprints on it. Forensic experts have deduced that the weapon is capable of firing with an initial velocity of 980 feet per second. So the height of the bullet t seconds after firing is found by the function $h(t) = -16t^2 + 980t$.

a. Draw a graph to represent this situation.

b. If the helicopter was flying at an altitude of 7000 feet at the time it was shot, is it possible that this weapon shot the helicopter? Explain your answer.

Mixed Review

61. **Architecture** The Gateway Arch of the Jefferson National Expansion Memorial in St. Louis is shaped like a parabola whose equation is $f(x) = \frac{1}{315}(-2x^2 + 1260x)$. (Lesson 6–6)

a. Write the equation in the form $y = a(x - h)^2 + k$.

b. If the bases of the arch are 630 feet apart, how tall is the arch?

c. Graph $f(x)$. Compare your graph to a photo of the Gateway Arch. Describe what you observe.

62. Find a value k such that 1 is a root of $x^2 + kx - 5 = 0$. (Lesson 6–5)

63. Solve $11m^2 - 12m = 10$ by using the quadratic formula. (Lesson 6–4)

64. Solve $x^2 + bx + c = 0$ by completing the square. (Lesson 6–3)

65. Simplify $(5 + \sqrt{8})^2$. (Lesson 5–6)

66. Divide $(y^4 + 3y^3 + y - 1) \div (y + 3)$ by using synthetic division. (Lesson 5–3)

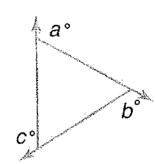
67. Simplify $(5a - 3)(1 - 3a)$. (Lesson 5–2)

68. Find the inverse of $\begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}$. (Lesson 4–4)

69. **SAT Practice** In the figure, $a + b + c =$

A 90 B 180 C 270

D 360 E 540



70. Graph $3x - 2y = 8$. Find the slope, the x-intercept, and the y-intercept. (Lesson 2–3)

71. Solve $4x - (2x + 8) + 3x = 5x - 8$. (Lesson 1–4)

For Extra Practice, see page 891.

6-8 Integration: Statistics
Standard Deviation

What YOU'LL LEARN

- To find the standard deviation for a set of data.

Why IT'S IMPORTANT

Standard deviation can help you describe the spread of a set of data.

Real World APPLICATION

Demographics

As we grow older, we spend our money in different ways. The table shows the money people spend on selected items each year in the United States for several age groups.

Item/Age Group	under 25	35–44	55–64	75 and up
Food at home	$1410	$3324	$2639	$1864
Food away from home	1140	2129	1634	703
Own a home	421	4549	3433	1587
Rent a home	2465	1733	751	1091
Footwear	158	315	206	153
Vehicle purchases	1916	2682	2215	528
Public transportation	181	340	378	156
Health insurance	144	583	749	1152
Electronic items	393	503	404	163
Pets and toys	139	577	442	172
Education	871	507	465	38

It appears that expenses for people in the under 25 category vary less than those in the 35–44 category. But how can you tell for sure?

Sometimes you need information about the spread, or variation, of data. A measure of variation called the **standard deviation** measures how much each value in a set of data differs from the mean. The symbol commonly used for standard deviation is SD or the lowercase Greek letter sigma, σ. The mean is usually labeled with the symbol $\overline{x}$ read "x bar."

To find the standard deviation of a set of data, follow these steps.

1. Find the mean, $\overline{x}$.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.
5. Take the principal square root of this mean.

Standard Deviation

From a set of data with n values, where x_1 represents the first term and x_n represents the nth term, if $\overline{x}$ represents the mean, then the standard deviation can be found as follows.

$$\text{SD or } \sigma_{\overline{x}} = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}}$$

To find the standard deviation of the data given for people under 25 and people aged 35-44, first find the mean of the expenses for both groups.

under 25:

$$\bar{x} = \frac{1410 + 1140 + 421 + 2465 + 158 + 1916 + 181 + 144 + 393 + 139 + 871}{11}$$

$$\approx 839.82$$

35–44:

$$\bar{x} = \frac{3324 + 2129 + 4549 + 1733 + 315 + 2682 + 340 + 583 + 503 + 577 + 507}{11}$$

$$\approx 1567.45$$

Then substitute each mean into the formula for standard deviation.

under 25:

$$\text{SD} = \sqrt{\frac{(1410 - 839.82)^2 + (1140 - 839.82)^2 + \ldots + (139 - 839.82)^2 + (871 - 839.82)^2}{11}}$$

$$= \sqrt{\frac{(570.18)^2 + (300.18)^2 + \ldots + (-700.82)^2 + (31.18)^2}{11}}$$

$$\approx 766.63$$

35–44:

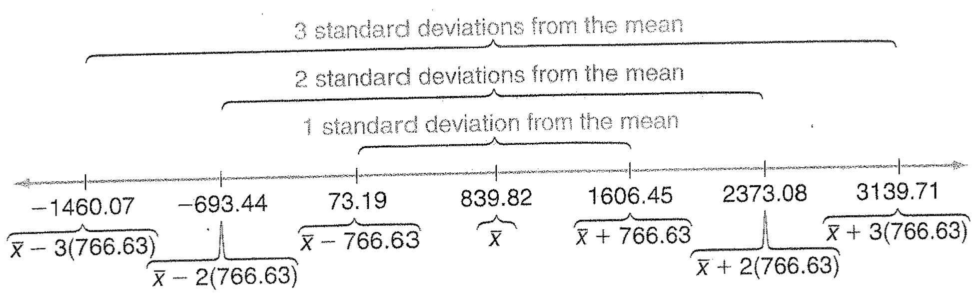
$$\text{SD} = \sqrt{\frac{(3324 - 1567.45)^2 + (2129 - 1567.45)^2 + \ldots + (577 - 1567.45)^2 + (507 - 1567.45)^2}{11}}$$

$$= \sqrt{\frac{(1756.55)^2 + (561.55)^2 + \ldots + (990.45)^2 + (1060.45)^2}{11}}$$

$$\approx 1376.53$$

The standard deviation for the under–25 age group is about \$766.63. The standard deviation for the 35–44 age group is about \$1376.53. So the expenses for the under–25 age group are closer to their mean than those in the 35–44 age group, and they vary less.

Most of the members of a set of data are within 1 standard deviation from the mean. The under 25 age group expenses can be broken down as shown in the diagram below.

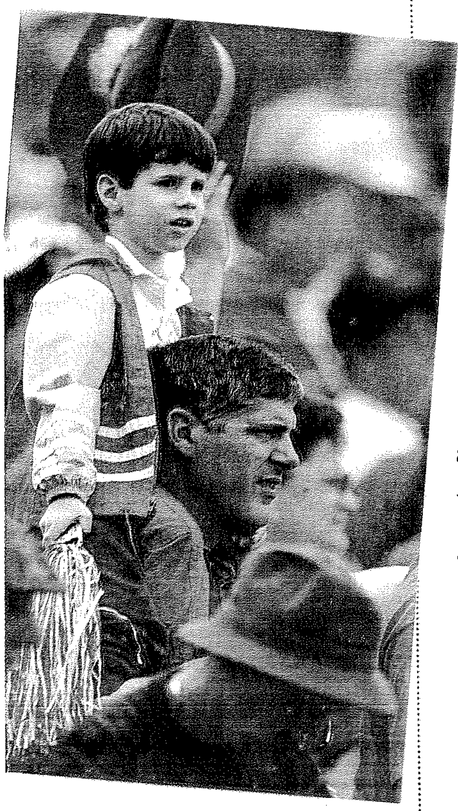


Standard deviation is often used to classify large sets of data.

Example 1

Real World APPLICATION

Sports



A day at a National Football League game for a family of four represents the most expensive outing in professional sports, averaging $184.19 in a recent year. The average ticket price for each NFL team for the year is listed in the table.

Team	Average	Team	Average
Arizona	$27.69	LA Rams	$29.13
Atlanta	27.00	Miami	29.65
Buffalo	33.73	Minnesota	29.79
Chicago	32.23	New England	34.34
Cincinnati	28.43	New Orleans	26.71
Cleveland	27.27	N.Y. Giants	35.59
Dallas	32.85	N.Y. Jets	25.00
Denver	32.34	Philadelphia	40.00
Detroit	30.04	Pittsburgh	30.99
Green Bay	26.13	San Diego	33.86
Houston	31.88	San Francisco	39.75
Indianapolis	26.48	Seattle	28.00
Kansas City	29.16	Tampa Bay	29.57
LA Raiders	31.32	Washington	35.70

a. Estimate the mean ticket price.

b. Calculate the mean ticket price.

c. Find the standard deviation of the data.

a. A quick glance at the data shows that about half of the data are above $30 and about half are below $30. So the mean will probably be about $30.

b. You can use a scientific calculator to perform the arithmetic necessary to calculate the mean and the standard deviation. However, your calculator may also have a statistics mode that simplifies this calculation. Press [MODE] and [STAT] to put your calculator in statistics mode. *The statistics functions are often second key functions.*

Enter the data by entering each number and then pressing the [Σ+] key. This provides a cumulative sum.

Enter: 27.69 [Σ+] 27 [Σ+] 33.73 [Σ+] . . . 35.70 [Σ+]

Wrong entries can be deleted by using [Σ−].

The display will show how many numbers were entered.

To find the mean, press [2nd] [$\bar{x}$]. 30.87964286

The mean is $30.88.

c. Find the standard deviation.

Enter: [2nd] [σn] 3.793188898

The standard deviation is $3.79.

When studying the standard deviation of a set of data, it is very important to keep the mean in mind. For example, suppose a company that sells video equipment found that the standard deviation of monthly prices of their equipment over the last two years was $50. If the mean of the prices over those two years was $200, then the standard deviation indicates a significant variation or change. However, if the mean was $900, then the standard deviation of $50 indicates a much smaller variation.

You can use a graphing calculator to calculate the mean and standard deviation of a set of data.

EXPLORATION — GRAPHING CALCULATORS

First, enter the elements of the data set into the calculator as follows. Press the [STAT] key and choose 1: Edit. Now you may enter the data into List 1 (L1). After each entry, press [ENTER]. After the data entry is complete, press [STAT] and choose [CALC]. Then choose 1: 1-Var Stats. Enter the name of the list that you are using. Press [2nd] [L1]. Then press [ENTER]. $\bar{x}$ and σ_x will be displayed on the screen along with other information.

Your Turn

a. Solve Example 1 by using a graphing calculator.

b. Practice the above process until you can explain it to a friend in your own words. Then write out your explanation.

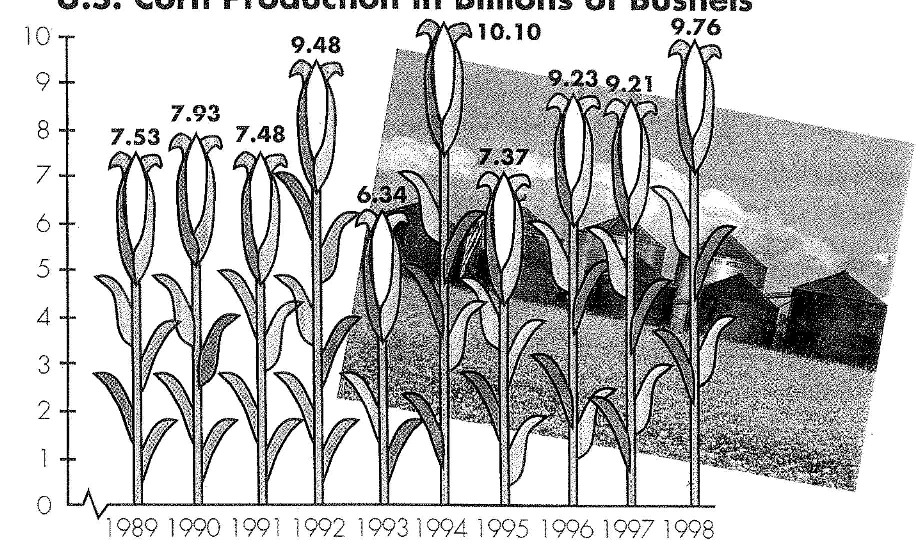
c. Find the mean and standard deviation of the set of data below by using a graphing calculator.

16, 42, 21, 19, 18.6, 41, 37, 24, 29.2, 26, 35

Example 2 — Real World APPLICATION — **Agriculture**

Corn production (in billions of bushels) in the United States from 1989 to 1998 is shown in the graph below. Find the mean corn production and the standard deviation of the data.

U.S. Corn Production in Billions of Bushels

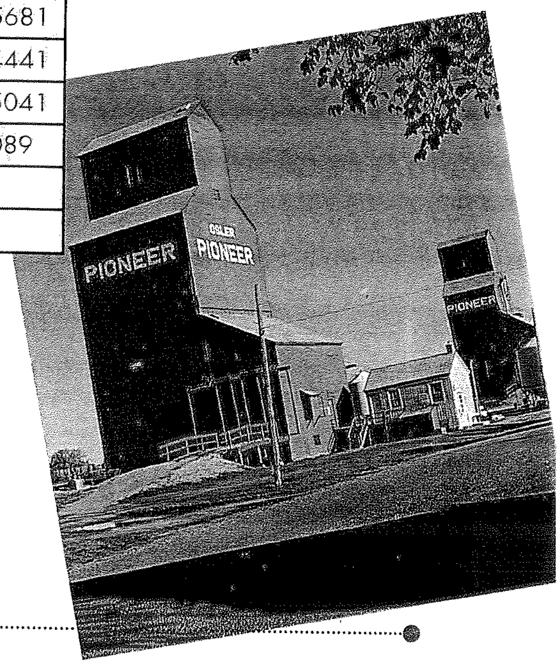


Source: National Agriculture Statistics Service

(continued on the next page)

We can use a spreadsheet to do the calculations as follows.

Year	Billions of Bushels	Difference from Mean	Square of Difference
1989	7.53	−0.909	0.826281
1990	7.93	−0.509	0.259081
1991	7.48	−0.959	0.919681
1992	9.48	1.041	1.083681
1993	6.30	−2.139	4.575321
1994	10.10	1.661	2.758921
1995	7.37	−1.069	1.142761
1996	9.23	0.791	0.625681
1997	9.21	0.771	0.594441
1998	9.76	1.321	1.745041
Total	84.39	Total	14.53089
Mean = B12/10		8.439	
SD = $\sqrt{(D12/10)}$		1.2054414	



The mean is 8.439, and the standard deviation is 1.205.

So the average corn production in the 10-year period 1989–1998 was 8.439 billion bushels. In most of the years, the production was between 8.439 − 1.205 or 7.234 billion and 8.439 + 1.205 or 9.644 billion bushels.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

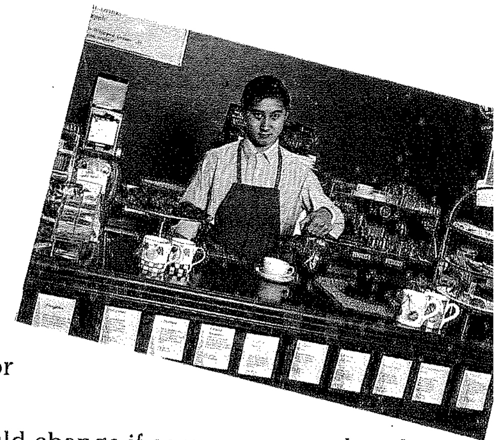
1. **Explain** the meaning of standard deviation in your own words.
2. Refer to Example 1. Find the number of ticket prices that were within one standard deviation of the mean. What percent of the total number of ticket prices is this?
3. **Research** Write a paragraph describing the events that occurred in the summer of 1993 that resulted in lower than usual corn production in the United States.
4. Over the last 12 months, the mean number of full-time employees in a city government was 2783 with a standard deviation of 43.
   a. What does this say about the variation of the number of employees in the government?
   b. If the standard deviation were 430, what would that say about the variation of the number of employees?

Guided Practice

Find the mean and standard deviation to the nearest hundredth for each set of data.

5. {110, 70, 20, 40, 10}
6. {48, 36, 40, 29, 45, 51, 38, 47, 39, 37}
7. {43, 56, 78, 81, 47, 42, 34, 22, 78, 98, 38, 46, 54, 67, 58, 92, 55}
8.

Stem	Leaf
4	3 5 6 8
5	2 4 5 6
6	1 2 4 5 5 6 7 7 7 $\quad 5\|2 = 5.2$



9. **Quality Control** A coffee machine is designed to dispense 250 mL for each cup. The actual measures in milliliters for a sample of 10 cups were 251, 246, 252, 249, 250, 248, 246, 253, 250, and 251.
   a. Find the standard deviation of the amounts.
   b. Do you think the variation is large or small?
   c. How do you think the variation would change if someone poured each cup of coffee, rather than used a machine?

EXERCISES

Practice

Find the mean and standard deviation to the nearest hundredth for each set of data.

10. {45, 65, 145, 85, 25, 25}
11. {400, 300, 325, 275, 425, 375, 350}
12. {5, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 9}
13. {234, 345, 123, 368, 279, 876, 456, 235, 333, 444}
14. {13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 67, 56, 34, 99, 44, 55}

15.

Stem	Leaf
4	4 5 6 7 7
5	3 5 6 7 8 9
6	7 7 8 9 9 9 $\quad 4\|5 = 45$

16.

Stem	Leaf
5	7 7 7 8 9
6	3 4 5 5 6 7
7	2 3 4 5 6 $\quad 6\|3 = 6.3$

17.

Stem	Leaf
3	0 0 1 2 4
•	5 6 6 6 8 9
4	1 1 3 4 4
•	5 5 6 7 $\quad 3\|4 = 3.4$

18.

Stem	Leaf
4	1 3 9
5	2 3 6 9
6	4 4 5 7 8
7	2 4 7 $\quad 5\|2 = 52$

19. {76, 78, 89, 90, 34, 56, 50}
20. {321, 322, 323, 324, 325, 326, 327, 328, 329, 330}

Critical Thinking

21. Suppose you have a set of data in which all elements are the same. What is the standard deviation of this data? Explain your answer.

Applications and Problem Solving

Real World

22. Sports The weights, in pounds, of the starting players for the basketball teams of three high schools are given below.

Jonesboro: 150, 145, 120, 168, 175
Hillview: 124, 157, 195, 205, 177
Murrayville: 146, 155, 176, 186, 199

a. Find the standard deviation for the weights of the players on the Jonesboro basketball team.
b. Find the standard deviation for the weights of the players on the Hillview basketball team.
c. Find the standard deviation for the weights of the players on the Murrayville basketball team.
d. Which team has the most variation in weight? How do you think this variation will impact their play?

23. Airlines The on-time performance of the nation's nine largest airlines has declined recently, although complaints have also dropped.

Airline	Percent of flights arriving on time	Complaints per 100,000 fliers
A	88.7%	0.29
B	84.8	0.68
C	83.0	1.09
D	80.5	0.64
E	80.0	0.38
F	77.9	1.93
G	75.4	0.45
H	73.5	0.83
J	71.9	0.74

a. What are the mean and standard deviation of percent of flights arriving on time?
b. What are the mean and standard deviation of the complaints?
c. Write a paragraph that explains the relationship between on-time flights and passenger complaints.

24. Sales A consumer magazine recently tested jeans for durability and quality in their laboratory. The following table gives the top five lab test winners for men's and women's jeans along with the price of each pair.

Men's	Price	Women's	Price
ProRodeo Cowboy Cut	$20	Jeans That Fit	$19
American Hero	$17	ProRodeo Cowboy Cut	$26
509	$31	Heavenly Blues	$48
Rustler	$15	Full Fit	$21
Long Haul	$23	Straight Leg	$30

a. Find the mean and standard deviation of the prices of the men's jeans.
b. Find the mean and standard deviation of the prices of the women's jeans.
c. Which had the greatest variation in price?
d. Survey 10 students in your school lunchroom and determine the brand of jeans that they wear. Compare your results with the tests above.
e. Suppose during end-of-year clearance sales, the stores lower the prices of all women's jeans by $5. How will the new standard deviation compare with the original standard deviation? Why do you think this occurs?

25. **Food** Professionally-trained food testers working for a consumer magazine dipped into several brands of potato chips three times a day, four days a week, for five weeks in a recent year. The result of their work is listed in the table.

Price per ounce (¢) for Chips Rated "Very Good"	Price per ounce (¢) for Chips Rated "Good"
14	25
20	12
18	17
20	21
23	21
12	21
21	24
23	17
13	10
	28
	21
	22
	20
	24
	19
	25
	21

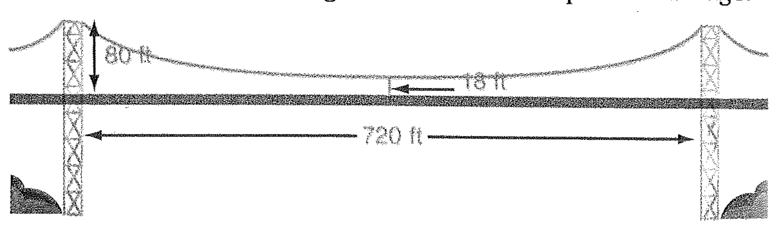
a. Find the mean and standard deviation of the prices of the chips rated "Very Good."

b. Find the mean and standard deviation of the prices of the chips rated "Good."

Mixed Review

26. Solve $(3x - 9)(x + 12) < 0$. (Lesson 6–7)

27. **Architecture** Look at the diagram below of a suspension bridge.



The equation $y = 0.00048x^2 + 18$ can be used to describe the cable hanging between the two upright supports, where x represents the horizontal distance along the roadbed from the lowest point of the cable and y represents the distance up from the roadbed. If a vertical cable is to be installed from the roadbed to the hanging cable and is 48 feet long, how far from the lowest point of the hanging cable will it be? (Lesson 6–2)

28. **SAT Practice** **Grid-in** If $x^2 = 16$ and $y^2 = 4$, what is the greatest possible value of $(x - y)^2$?

29. Write an augmented matrix for the system of equations. Then solve. (Lesson 4–7)
$3x + 4y = 22$
$7x - y = 10$

For Extra Practice, see page 891.

30. Given the function $f(x, y) = 6x - 3y$, find $f(-5, 2)$. (Lesson 3–5)

6–9 Integration: Statistics The Normal Distribution

What YOU'LL LEARN

- To solve problems involving normally-distributed data.

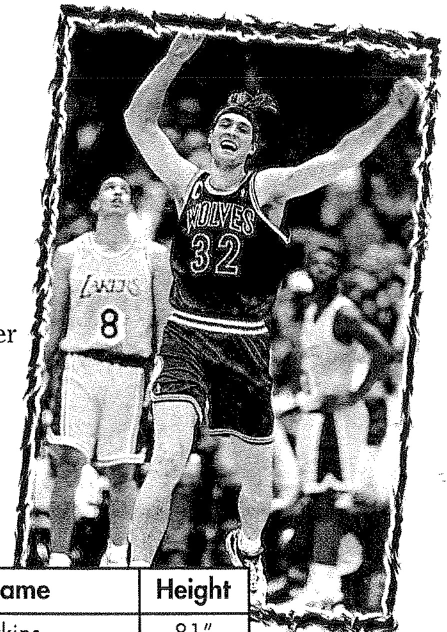
Why IT'S IMPORTANT

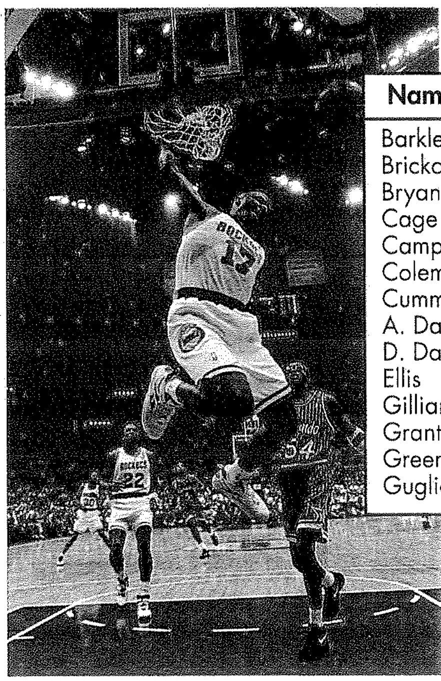
The normal distribution is used to help analyze and describe data.

Real World APPLICATION

Basketball

In professional basketball, a power forward is responsible for rebounding and scoring close to the basket. So, it makes sense that most power forwards are very tall. Forty NBA power forwards and their heights are listed in the table below.

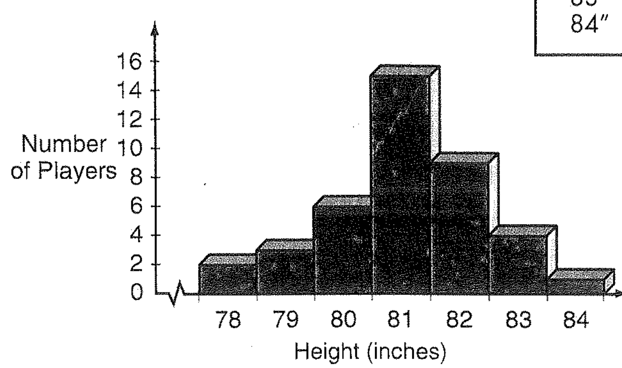




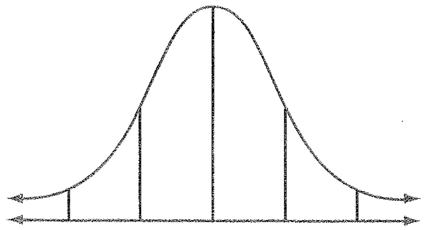
Name	Height	Name	Height	Name	Height
Barkley	78″	Hill	81″	Perkins	81″
Brickowski	82″	Johnson	79″	Pinckney	81″
Bryant	81″	Jones	80″	Reid	81″
Cage	81″	Laettner	83″	Rodman	80″
Campbell	83″	Long	80″	Smith	82″
Coleman	82″	Lynch	80″	Thorpe	82″
Cummings	81″	Malone	81″	Tisdale	81″
A. Davis	81″	Mason	79″	Vaught	81″
D. Davis	83″	McDaniel	79″	Weatherspoon	78″
Ellis	80″	Mills	82″	Br. Williams	83″
Gilliam	81″	Nance	82″	Bu. Williams	80″
Grant	82″	Oakley	81″	J. Williams	82″
Green	81″	Owens	81″	Willis	84″
Gugliotta	82″				

One way of analyzing data is to consider how frequently each value occurs. The table below shows the frequencies of the heights of the forty basketball players. The graph below visually displays the frequencies of the heights in the table.

Height	Frequency
78″	2
79″	3
80″	6
81″	15
82″	9
83″	4
84″	1

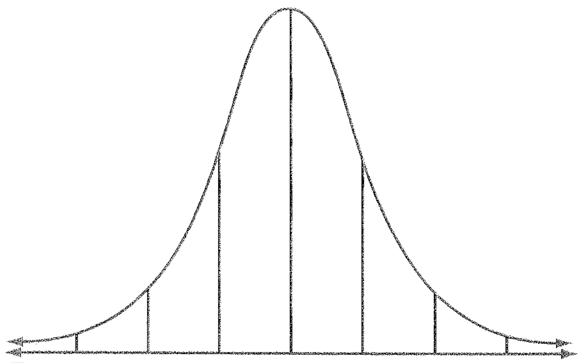


The bar graph shows a **frequency distribution** of the heights. That is, it shows how the heights are spread out over the range from 78 inches to 84 inches. A graph like this one that shows a frequency distribution is called a **histogram.**



Curves are very useful in displaying information. You used parabolas to represent related data. Curves are also used to show frequency distributions, especially when the distribution contains a large number of values. While the curves may be of any shape, many distributions have graphs shaped like the one at the left. Many distributions with this type of graph are **normal distributions.**

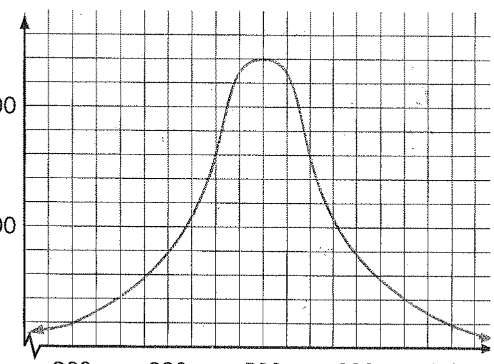
The curve of the graph of a normal distribution is symmetric and is often called a **bell curve.** The shape of the curve indicates that the frequencies in a normal distribution are concentrated around the center portion of the distribution. What does this tell you about the mean?



Normal distributions have these properties.

1. The graph is maximized at the mean.
2. The mean, median, and mode are about equal.
3. The data are symmetrical about the mean.
4. About 68% of the values are within one standard deviation from the mean.
5. About 95% of the values are within two standard deviations from the mean.
6. About 99% of the values are within three standard deviations from the mean.

Suppose the scores on the mathematics component of the Scholastic Assessment Test (SAT) of 1,000,000 students are recorded and the frequency of those scores is normally distributed. If the mean score is 500 and the standard deviation is 90, then the graph at the right approximates the curve for the frequency distribution of the scores.



As shown by the graph, the mean is the most frequent score. Of the 1,000,000 students, the following is true.

- About 680,000 scored between 410 and 590 points.
- About 950,000 students scored between 320 and 680 points.
- About 990,000 students scored between 230 and 770 points.

Normal distributions occur quite frequently in real life. In addition to test scores, the lengths of newborn babies, cholesterol levels, the useful life and size of manufactured items, and production levels can be represented by normal distributions. In all of these cases, the number of data items must be large for the distribution to be normal.

Example 1 **The useful lives of 10,000 batteries are normally distributed. The mean useful life is 20 hours, and the standard deviation is 4 hours.**

a. Sketch a normal curve showing the useful life at one, two, and three standard deviations from the mean.

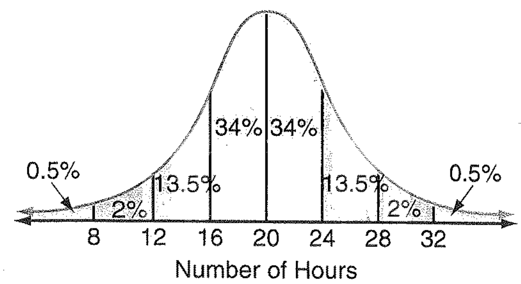
b. How many batteries will last between 16 and 24 hours?

c. How many batteries will last less than 12 hours?

a. Draw a normal curve with 20 hours as the mean.
one standard deviation from the mean: $\bar{x} - 4$ and $\bar{x} + 4$ or 16–24 hours

two standard deviations from the mean: $\bar{x} - 2(4)$ and $\bar{x} + 2(4)$ or 12–28 hours

three standard deviations from the mean: $\bar{x} - 3(4)$ and $\bar{x} + 3(4)$ or 8–32 hours



b. The percentage of batteries lasting between 16 and 24 hours is 34% + 34% or 68% of the batteries.
10,000 × 68% = 6800 batteries

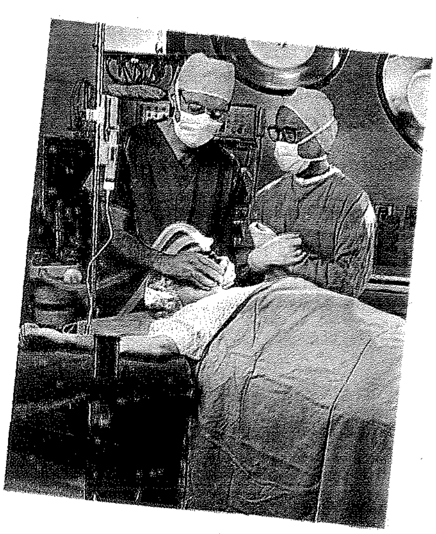
c. The percentage of batteries lasting less than 12 hours is 0.5% + 2% or 2.5%.
10,000 × 2.5% = 250 batteries

You can use normal distributions to estimate the probabilities of certain events occurring.

Example 2

Real World APPLICATION

Medicine



The correct number of milligrams of anesthetic an anesthesiologist must administer to a patient is normally distributed. The mean is 100 milligrams, and the standard deviation is 20 milligrams.

a. Of a sample of 200 patients, about how many people require more than 120 milligrams of anesthetic for a response?

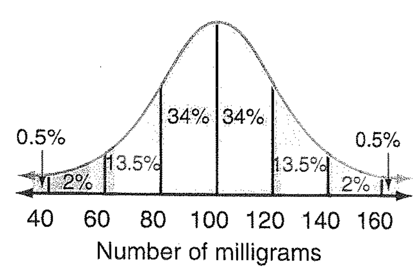
b. What is the probability that a patient chosen at random will require between 80 and 120 milligrams?

a. This frequency distribution is shown by the curve below. The percentages represent the percentages of patients requiring the dosage within the given interval.

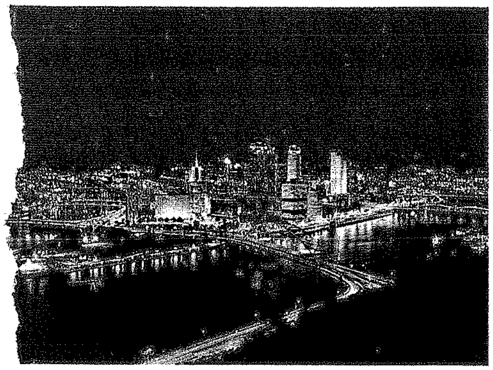
The percentage of people requiring more than 120 milligrams of anesthetic is 13.5% + 2% + 0.5% or 16%.

200 × 16% = 32

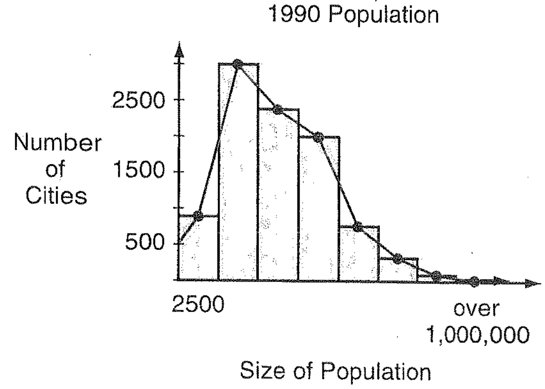
So about 32 of the 200 patients require more than 120 milligrams of anesthetic.



b. The percentage of people requiring between 80 and 120 milligrams of anesthetic is 68%. So the probability that a patient chosen at random will require such amounts of anesthetic is 68%.



The histogram at the right shows the number of urban cities in 1990 in the United States for varying population sizes. You can see that the resulting curve displays a **skewed** distribution. A skewed curve that is high at the left and has a tail at the right like this one is *positively skewed.* A skewed curve that is high at the right and has a tail to the left is *negatively skewed.*

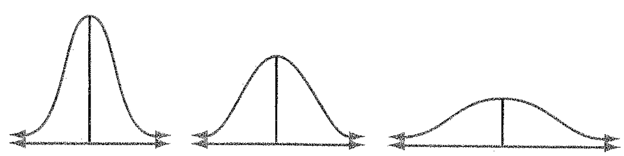


CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Refer** to the application at the beginning of the lesson. Explain why the heights of the power forwards approximate a normal distribution. Explain why the histogram approximates a normal curve.
2. **Sketch** a negatively skewed graph. Describe a situation in which you would expect data to be distributed this way.
3. **Compare and contrast** the means and standard deviations of the graphs.



MATH JOURNAL

4. Explain why you think that the SAT scores of the students in your class would be skewed or why they would be normal.

Guided Practice

interNET CONNECTION

Data Update For more information on the SAT and ACT, visit: **www.algebra2.glencoe.com**

5. The table at the right shows female mathematics SAT scores in 1990.
   a. State whether the tables of data would result in a graph that is positively skewed, is negatively skewed, or appears to be normally distributed.
   b. Sketch a histogram of the data.

Scores	Percent of Females
200–290	5
300–390	18
400–490	26
500–590	26
600–690	18
700–800	7

6. Mrs. Sung gave a test in her trigonometry class. The scores were normally distributed with a mean of 85% and a standard deviation of 3%.
   a. What percent would you expect to score between 82% and 88%?
   b. What percent would you expect to score between 88% and 91%?
   c. What is the probability that a student chosen at random scored between 79% and 91%?
7. **Quality Control** The useful life of a radial tire is normally distributed with a mean of 30,000 miles and a standard deviation of 5000 miles. The company makes 10,000 tires a month.
   a. About how many tires will last between 25,000 and 35,000 miles?
   b. About how many tires will last more than 40,000 miles?
   c. About how many tires will last less than 25,000 miles?
   d. What is the probability that if you buy a radial tire at random, it will last between 20,000 and 35,000 miles?

EXERCISES

Practice

State whether the tables of data would result in a graph that is positively skewed, negatively skewed, or appears to be normally distributed. Then sketch a histogram of the data.

8. the number of hours of TV students watch in a week

Hours	Percent
0–5	16
6–10	24
11–15	5
16–20	8
21–25	4
26+	7

Source: *The Elementary Mathematician*

9. the record low temperatures in the fifty states

Temperature (°F)	Number of States
−80 to −66	2
−65 to −51	10
−50 to −36	17
−35 to −19	16
−18 to −2	4
−1 to 15	1

Source: *The World Almanac,* 1995

10. The shelf life of a particular dairy product is normally distributed with a mean of 12 days and a standard deviation of 3.0 days.

a. About what percent of the products last between 9 and 15 days?

b. About what percent of the products last between 12 and 15 days?

c. About what percent of the products last 3 days or less?

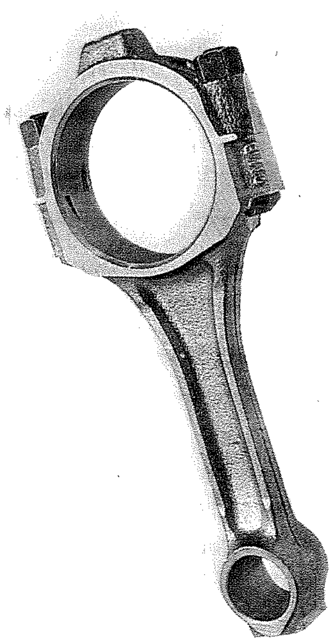
d. About what percent of the products last 15 or more days?

11. The vending machine in the basement of McMicken Hall usually dispenses about 6 oz of soft drink. Lately, it is not working properly and the variability in how much of the soft drink it dispenses has been getting greater. The amounts are normally distributed with a standard deviation of 0.2 oz.

a. What percent of the time will you have more than 6 ounces of soft drink?

b. What percent of the time will you have less than 6 ounces of soft drink?

c. What percent of the time will you have between 5.6 and 6.4 ounces of soft drink?

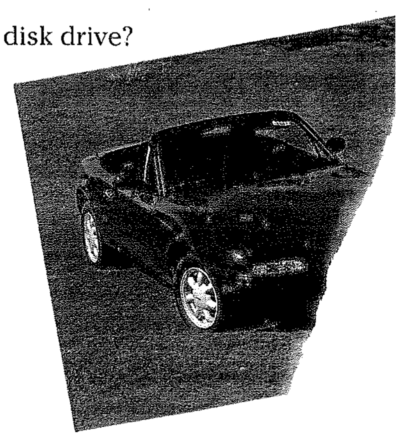


12. The Floppy Disk Company makes 3.5" floppy disks for disk drives that are 3.7" wide. The size of a manufactured disk is normally distributed with a standard deviation of 0.1". The company manufactures 1000 disks every hour.

a. What percent of the disks would you expect to be greater than 3.7"?

b. In one hour, how many disks would you expect to be between 3.4" and 3.7"?

c. About how many disks will be unable to fit in the disk drive?



13. The diameter of the connecting rod in a certain imported sports car must be between 1.480 and 1.500 centimeters, inclusive, to be usable. The diameters of the connecting rods are normally distributed with a mean of 1.495 centimeters and a standard deviation of 0.005 centimeter.

a. Of 1000 connecting rods, how many measure at least 1.495 centimeters?

b. Of 1000 connecting rods, how many measure between 1.485 centimeters and 1.500 centimeters?

c. What percent of rods will have to be discarded?

d. What is the probability that a rod chosen at random will be usable?

Critical Thinking

14. Suppose the weights of the books used in your school were normally distributed.
    a. Where on the normal curve would you find the weight of your Algebra 2 textbook?
    b. Which textbook would be three standard deviations to the left of the mean (extremely light)?
    c. Which textbook would be three standard deviations to the right of the mean (extremely heavy)?

Applications and Problem Solving

Real World

15. **Humor** The following quote appeared in *Chance* magazine in Winter, 1995.

    Someone asked an accountant, a mathematician, an engineer, a statistician, and an actuary how much 2 + 2 was. The accountant said "4." The mathematician said "it depends on your number base." The engineer took out his slide-rule and said "approximately 3.99." The statistician consulted his tables and said "I am 95% confident that it lies between 3.95 and 4.05." The actuary said "What do you want it to add up to?"

    How did each person's perspective affect his or her answer? Describe the information the statistician used to give his answer.

16. **Health** A recent study showed that the systolic blood pressure of high school students ages 14–17 is normally distributed with a mean of 120 mm Hg and a standard deviation of 12 mm Hg. Suppose a high school has 800 students.
    a. About what percent of the students have blood pressure below 108 mm Hg?
    b. About how many students have blood pressure between 108 and 144 mm Hg?

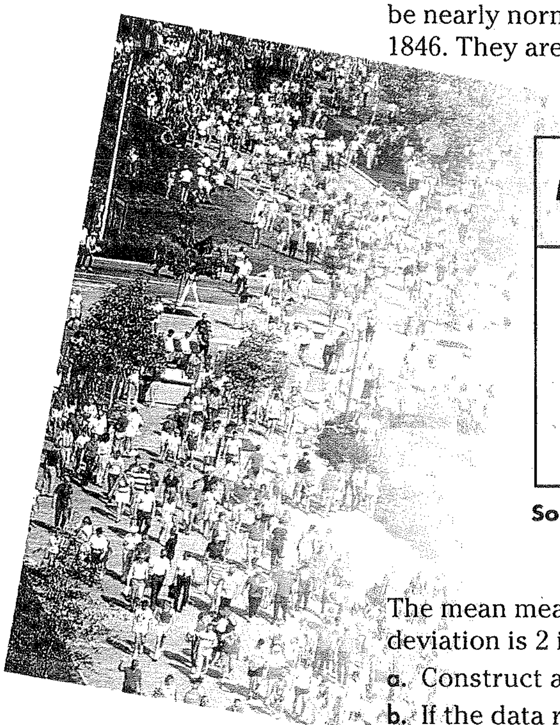
17. **Noise Level** Airplane pilots often suffer from hearing loss as a result of being exposed to high noise levels. A team of researchers measured the cockpit noise levels of 16 commercial aircraft. The results are listed in the table below.

Plane	Decibels of Noise Level	Plane	Decibels of Noise Level
1	80	9	85
2	83	10	80
3	83	11	75
4	86	12	75
5	72	13	74
6	90	14	77
7	87	15	80
8	83	16	82

Source: *Archives of Environmental Health*

    a. Calculate the mean and standard deviation of the data.
    b. Construct a histogram for the noise levels of the planes. Use the intervals 70-73, 74-77, 78-81, 82-85, 86-89, and 90-93.
    c. Do you think the data appear to be normally distributed? Explain.

18. Physiology Nineteenth-century Belgian Lambert Adolphe Jacques Quetelet discovered that if you take a large group of people and measure a physical characteristic such as height, weight, or arm length, the data will be nearly normally distributed. Below is a set of data Quetelet collected in 1846. They are the chest measurements of 5738 Scottish soldiers.



Chest Measures (inches)	Number of men	Chest Measures (inches)	Number of men
33	3	41	934
34	18	42	658
35	81	43	370
36	185	44	92
37	420	45	50
38	749	46	21
39	1073	47	4
40	1079	48	1

Source: *Lettres sur la Theorie des Probabilites, appliquee aux Sciences Morales et Politiques*

The mean measurement to the nearest inch is 40 inches and the standard deviation is 2 inches.

a. Construct a histogram for the chest measurements of Scottish soldiers.

b. If the data represented a perfectly normal distribution, what percentage of Scottish soldiers would have chest measurements that were within two standard deviations of the mean?

c. What percentage of soldiers actually had chest measurements that were within two standard deviations of the mean?

19. Entertainment The 5th Anniversary Issue of *Entertainment Weekly* gave a summary of grades their critics have awarded the movies reviewed since the magazine's premiere issue. The grades were 96 As, 285 Bs, 267 Cs, 136 Ds, and 36 Fs.

a. Construct a histogram for these data and sketch a curve showing the distribution.

b. Do the data appear to be normally distributed? Explain.

Mixed Review

20. Find the mean and the standard deviation for the following set of data. 7, 16, 9, 4, 12, 3, 9, 4 (Lesson 6–8)

21. Football The price of a Super Bowl ticket from Super Bowl I in 1967 to Super Bowl XXIX in 1995 can be described by the function $y = \frac{2}{5}x^2 - 6x + 32$, where y represents the price and x represents the year, with $x = 1$ representing 1967. (Lesson 6–6)

a. Write the equation in the form $y = a(x - h)^2 + k$ and graph the function.

b. What was the approximate price of tickets for Super Bowl XXIX in 1995?

c. What will be the approximate price of tickets for Super Bowl XXXV in 2001?

22. ACT Practice Which of the following is the sum of both solutions of the equation $x^2 - 3x - 18 = 0$?

A −6 B −3 C 3 D 6 E 9

23. Simplify $(-x - 8)(3x + 4)$. (Lesson 5–2)

24. Solve $6(3x - 5y) + (2 + 8x) = -11x$ for x. (Lesson [illegible]–2)

For Extra Practice, see page 891.

INET CONNECTION

Chapter Review For additional lesson-by-lesson review, visit: **www.algebra2.glencoe.com**

VOCABULARY

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Algebra

axis of symmetry (p. 335)
boundary (p. 378)
completing the square (p. 347)
constant term (p. 334)
discriminant (p. 356)
factoring (p. 341)
linear term (p. 334)
parabola (pp. 332, 335)
quadratic equation (p. 336)
quadratic formula (p. 354)
quadratic function (p. 334)
quadratic inequality (p. 378)
quadratic term (p. 334)
roots (pp. 332, 336)
solutions (pp. 332, 336)
vertex (p. 335)
zero product property (p. 341)
zeros (p. 335)

Statistics

bell curve (p. 393)
frequency distribution (p. 392)
histogram (p. 392)
normal distribution (p. 393)
skewed (p. 395)
standard deviation (p. 384)

Problem Solving

guess and check (p. 342)

UNDERSTANDING AND USING THE VOCABULARY

Choose the letter of the term that best matches each statement or phrase.

1. the graph of any quadratic function
2. a process whereby the middle term of a quadratic equation of the form $ax^2 + bx + c = 0$ is altered to help solve the quadratic equation
3. the vertical line passing through the vertex of a parabola and dividing the parabola into two mirror images
4. a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq o$
5. the solutions of an equation
6. a bell-shaped symmetric graph with about 68% of the items within one standard deviation from the mean, about 95% of the items within two standard deviations from the mean, and about 99% of the items within three standard deviations of the mean
7. the process whereby a polynomial of degree 2 or more is simplified into the product of monomials, binomials, or a combination thereof
8. in the quadratic formula, the expression under the radical sign, $b^2 - 4ac$
9. The solution(s) of a quadratic equation of the form $ax^2 + bx + c = 0$ with $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
10. For a set of data with n values, if x_i represents a value such that $1 \leq i \leq n$, and $\bar{x}$ represents the mean, then $\sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$ represents this value.

a. axis of symmetry
b. completing the square
c. discriminant
d. factoring
e. normal distribution
f. parabola
g. quadratic formula
h. quadratic function
i. roots
j. standard deviation

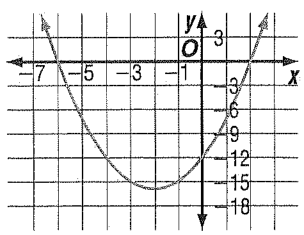
SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Upon completing this chapter, you should be able to:

- solve quadratic equations by graphing (Lesson 6–1)

Solve $x^2 + 4x - 12 = 0$ by graphing.



The graph crosses the x-axis at -6 and 2. Thus, the solutions of the equation are -6 and 2.

REVIEW EXERCISES

Use these exercises to review and prepare for the chapter test.

Solve each equation by graphing.

11. $x^2 + 6x - 40 = 0$
12. $x^2 - 2x - 15 = 0$
13. $a^2 - 8a - 20 = 0$
14. $3m^2 + 9m + 6 = 0$
15. $x^2 + 12x + 35 = 0$
16. $0.5x^2 + 0.5x - 15 = 0$

- solve quadratic equations by factoring (Lesson 6–2)

Solve $x^2 + 9x + 20 = 0$.

$(x + 4)(x + 5) = 0$

$x + 4 = 0$ or $x + 5 = 0$

$x = -4$ $\quad$ $x = -5$

The solutions are -4 and -5.

Solve each equation by factoring.

17. $x^2 - 4x - 32 = 0$
18. $3x^2 + 6x + 3 = 0$
19. $5y^2 = 80$
20. $2c^2 + 18c - 44 = 0$
21. $d^2 + 29d + 100 = 0$
22. $v^3 = 49v$
23. $25x^3 - 25x^2 = 36x$
24. $r^2 - 3r - 70 = 0$

- solve quadratic equations by completing the square (Lesson 6–3)

Solve $x^2 + 10x - 39 = 0$ by completing the square.

$x^2 + 10x + \square = 39 + \square$

$x^2 + 10x + 25 = 39 + 25$

$(x + 5)^2 = 64$

$x + 5 = \pm 8$

$x + 5 = 8$ or $x + 5 = -8$

$x = 3$ $\quad$ $x = -13$

The solutions are -13 and 3.

Solve each equation by completing the square.

25. $-5x^2 - 5x + 9 = 0$
26. $k^2 + 6k - 4 = 0$
27. $b^2 + 4 = 6b$
28. $n^2 - 10n = 23$
29. $h^2 - 4h - 7 = 0$
30. $5x^2 - 15x - 9 = 2$

OBJECTIVES AND EXAMPLES

- solve quadratic equations by using the quadratic formula (Lesson 6–4)

Solve $x^2 - 5x - 66 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)}$$

$$= \frac{5 \pm 17}{2}$$

$x = \frac{5 + 17}{2}$ or 11 and $x = \frac{5 - 17}{2}$ or -6

REVIEW EXERCISES

Solve each equation by using the quadratic formula.

31. $x^2 + 2x + 7 = 0$
32. $x + 2x^2 + 1 = -1 - x$
33. $-x^2 + 5x - 9 = 0$
34. $-2x^2 + 12x - 5 = 0$
35. $3c^2 + 7c - 2 = 0$
36. $8b^2 - b - 15 = 0$

- find a quadratic equation to fit a given condition (Lesson 6–5)

Write an equation that has roots of $-\frac{5}{2}$ and 3.

$s_1 = -\frac{5}{2}$ $\quad s_2 = 3$

$s_1 + s_2 = -\frac{5}{2} + 3 = \frac{1}{2} = -\frac{b}{a}$

$s_1 s_2 = -\frac{5}{2} \cdot 3 = -\frac{15}{2} = \frac{c}{a}$

Therefore $a = 2$, $b = -1$, and $c = -15$.

The equation is $2x^2 - x - 15 = 0$.

Find a quadratic equation that has the given roots.

37. 7, −6
38. 11, 14
39. $-\frac{13}{2}, -4$
40. $\frac{3}{4}, \frac{9}{2}$
41. −2.5, 5.25
42. −0.25, 0.25

- graph quadratic functions of the form $y = a(x - h)^2 + k$ (Lesson 6–6)

Name the vertex, axis of symmetry, and direction of opening for the graph of $f(x) = 3x^2 + 42x + 142$.

$$\begin{aligned} f(x) &= 3x^2 + 42x + 142 \\ &= 3(x^2 + 14x) + 142 \\ &= 3(x^2 + 14x + 49) + 142 - 3(49) \\ &= 3(x + 7)^2 - 5 \end{aligned}$$

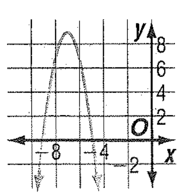
So $a = 3$, $h = -7$, and $k = -5$. The vertex is at $(-7, -5)$, and the axis of symmetry is $x = -7$. Since $a = 3$, the graph opens upward.

Write each equation in the form $f(x) = a(x - h)^2 + k$ if not already in that form. Name the vertex, axis of symmetry, and direction of opening for the graph of each quadratic function. Then graph it.

43. $f(x) = -6(x + 2)^2 + 3$
44. $f(x) = 4(x - 5)^2 - 7$
45. $f(x) = 5x^2 - 35x + 58$
46. $f(x) = -9x^2 + 54x - 8$
47. $f(x) = -\frac{1}{3}x^2 + 8x$
48. $f(x) = 0.25x^2 - 6x - 16$

- graph quadratic inequalities (Lesson 6–7)

Graph $y \leq -2x^2 - 28x - 89$.



Graph each inequality.

49. $y > x^2 - 5x + 15$
50. $y < -3x^2 + 48$
51. $y \leq 4x^2 - 36x + 17$
52. $y \geq -x^2 + 7x - 11$
53. $y < x^2 + 5x + 6$
54. $y \geq 3x^2 - 15x + 22$

OBJECTIVES AND EXAMPLES

- find the standard deviation for a set of data (Lesson 6–8)

To find the standard deviation of this set of data, follow the steps below.

1. Find the mean of the data.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.
5. Take the principal root of this mean.

REVIEW EXERCISES

Find the mean and the standard deviation to the nearest hundredth for each set of data.

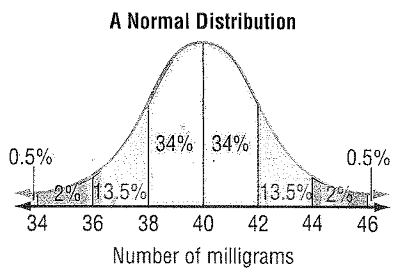
55. {100, 156, 158, 159, 162, 165, 170, 190}

56. {56, 56, 57, 58, 58, 58, 59, 61}

57. {302, 310, 331, 298, 348, 305, 314, 284, 321, 337}

58. {3.4, 4.2, 8.6, 5.1, 3.6, 2.8, 7.1, 4.4, 5.2, 5.6}

- solve problems involving normally-distributed data (Lesson 6–9)



59. Mr. Byrum gave an exam to his 30 Algebra 2 students at the end of the first semester. The scores were normally distributed with a mean of 78% and a standard deviation of 6%.

a. What percentage of the class would you expect to have scored between 72% and 84% on the test?

b. What percentage of the class would you expect to have scored between 90% and 96% on the test?

c. Approximately how many students scored between 84% and 90%?

d. What percentage of the class would you expect to score less than 60%?

e. Approximately how many students scored between 72% and 84%?

APPLICATIONS AND PROBLEM SOLVING

60. Space Exploration The Apollo 11 spacecraft propelled the first men to the moon and contained three stages of rockets. The first stage dropped off 2 min 40 s after takeoff and the second stage ignited. The initial velocity of the second stage was 2760 m/s with a constant acceleration of 200 m/s^2. How long did it take the second stage to travel 7040 m? (Lesson 6–3)

61. Physics The Empire State Building is 1250 feet tall. If an object is thrown upward from the top of the building at an initial velocity of 35 feet per second, its height t seconds after it is thrown is given by the function $h(t) = -16t^2 + 35t + 1250$. How long will it be before the object hits the ground? (Lesson 6–2)

62. Work The monthly incomes of 10,000 workers at the ProComm plant are distributed normally. Suppose the mean monthly income is \$1250 and the standard deviation is \$250. (Lesson 6–9)

a. How many workers earn more than \$1500 per month?

b. How many workers earn less than \$750 per month?

c. What percentage of the workers earn between \$500 and \$1750 per month?

d. What percentage of the workers earn less than \$1750 per month?

A practice test for Chapter 6 is provided on page 917.

ALTERNATIVE ASSESSMENT

COOPERATIVE LEARNING PROJECT

Statistics So far in this text, you have learned several methods of statistical analysis.

- *Standard deviation* is the average measure of how much each value in a set of data differs from the mean.
- A *bar graph* that shows the frequency distribution of data is called a histogram.
- In a *stem-and-leaf plot*, each piece of data is separated into two number columns that are used to form the stem and leaf.
- The *normal distribution* is a symmetric curve of a graph that is often bell-shaped.

In this project, you will have the opportunity to gather data and complete your own statistical analysis using one of the four methods listed above. Your teacher will separate you into groups of four and assign each of you a method of statistical analysis. Once you are assigned to a group and a method, your group should decide which teacher in the school you would like to approach to provide you with a list of his or her latest test scores. Tell the teacher that this is a math project and that only a list of the scores is necessary to complete the project. Be sure to tell the teacher that no names are to be given with any of the test scores. Also, have the teacher provide you with the method by which the test scores were achieved: for example, multiple-choice, true-false, matching, essay, and so on.

Once you have obtained the test scores, begin your analysis of the scores using the method assigned to you by your teacher. Remember, statistics can be used to prove any point an individual wants to make.

When your statistical analysis is complete, share your results with the rest of your group. Are there any similarities and/or differences between the various methods of analysis?

Now share your results with someone in another group who used the same method of analysis as you did. Are there any similarities and/or differences between methods of the same type of analysis? Can any conclusions be drawn about the statistical analysis and the method of testing used by the teacher? Does one method of testing yield better statistical results than another?

THINKING CRITICALLY

Find values of k that meet the following conditions.

a. $x^2 - 7x + k = 0$ has two real roots.

b. $kx^2 - 5x + 7 = 0$ has two imaginary roots.

c. $3x^2 - kx + 15 = 0$ has one root.

PORTFOLIO

Select an item from this chapter that shows your best work and place it in your portfolio. Explain why you believe it to be your best work and how you came to choose this particular piece.

STANDARDIZED TEST PRACTICE

CHAPTERS 1–6

Section One: MULTIPLE CHOICE

There are eight multiple-choice questions in this section. After working each problem, write the letter of the correct answer on your paper.

1. Evaluate $(4.5 \times 10^4)(3.33 \times 10^2)$. Express the answer in scientific notation and in standard notation.

   A. 1.4985×10^6; 1,498,500

   B. 14.985×10^6; 14,985,000,000

   C. 1.4985×10^7; 149,850,000,000

   D. 1.4985×10^7; 14,985,000

2. Which matrix below represents translated $\Delta D'E'F'$ if ΔDEF with $D(7, -2)$, $E(4, 5)$, and $F(-3, 4)$ is moved 5 units left and 1 unit up?

   A. $\begin{bmatrix} 8 & 5 & -2 \\ -7 & 0 & -1 \end{bmatrix}$

   B. $\begin{bmatrix} 2 & -1 & -8 \\ -1 & 6 & 5 \end{bmatrix}$

   C. $\begin{bmatrix} -1 & 6 & 5 \\ 8 & 5 & 2 \end{bmatrix}$

   D. $\begin{bmatrix} 2 & -1 & -8 \\ -7 & 0 & -1 \end{bmatrix}$

3. Choose the equation that represents a parabola that is 1 unit to the right and 8 units below the parabola with equation $f(x) = 5x^2$.

   A. $f(x) = 5(x - 8)^2 + 1$

   B. $f(x) = 5x^2 - 8$

   C. $f(x) = 5(x - 1)^2 - 8$

   D. $f(x) = 5(x - 1)^2 + 8$

4. Find the area of a rectangle with length $3\sqrt{3} - \sqrt{2}$ units and width $\sqrt{3} + 3\sqrt{2}$ units.

   A. $3 + 8\sqrt{6}$

   B. 27

   C. $11\sqrt{6}$

   D. $3 + 8\sqrt{5}$

5. Simplify $4(5x + 2y) + 9(x - 2y)$.

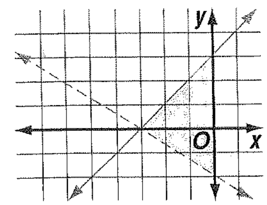
   A. $10xy$

   B. $29x - 10y$

   C. $54x + 13y$

   D. $29x$

6. Choose the system of inequalities whose solution is represented by the graph below.



   A. $y > -\frac{3}{2}x - 2$
   $x < 0$
   $y \leq \frac{1}{2}x - 6$

   B. $-2 < y \leq 3$
   $x > -3$

   C. $y \leq x + 3$
   $y > -\frac{2}{3}x - 2$
   $x \leq 0$

   D. $y \geq -2x$
   $y < -4x + 3$

7. Choose the false statement regarding the value of the discriminant of a quadratic equation.

   A. If it is a perfect square, two real rational roots exist.

   B. If it is zero, infinitely many roots exist.

   C. If it is negative, two complex roots exist.

   D. If it is not a perfect square, two real irrational roots exist.

8. Find the next number in the pattern 2, 8, 18, 32, 50, __?__.

   A. 54

   B. 68

   C. 98

   D. 72