Focus On
PREREQUISITE SKILLS

To be successful in this chapter, you'll need to understand these concepts and be able to apply them. Refer to the example or to the lesson in parentheses if you need more review before beginning the chapter.

Evaluate expressions. (Lesson 1–1)

Example Evaluate $x^2 + 2x - 7$ if $x = -3$.

$x^2 + 2x - 7$	$= (-3)^2 + 2(-3) - 7$	*Replace x with −3.*
	$= 9 + (-6) - 7$	*Follow the order of operations*
	$= -4$	*Simplify.*

Evaluate each expression if $a = -1$, $b = 3$, $c = -2$, and $d = 0$.

1. $a^2 - 5a + 3$
2. $c + d$
3. $4c - b$
4. $\frac{a-b}{d-c}$

Use the Distributive Property to simplify expressions. (Lesson 1–2)

Use the Distributive Property to rewrite each expression without parentheses.

5. $0.75(y - 5)$
6. $-0.36(z + 10)$
7. $\frac{1}{3}(x - 6)$
8. $-\frac{5}{4}(a + 12)$

Evaluate expressions with absolute values. (Lesson 1–5)

Evaluate each expression if $x = -3$, $y = 4$, and $z = -4.5$.

9. $|5x|$
10. $-|2z|$
11. $5|y + z|$
12. $-3|x + y| - |x + z|$

Focus On
READING SKILLS

In this chapter, you will learn about ***relations*** and ***functions.*** A relation shows how two sets of data are related. A function is a special type of relation. Sometimes a linear equation can be graphed to represent a function. Notice that the word *linear* contains the word "line." When you graph a linear equation, the result is a straight line.

2-1 Relations and Functions

What YOU'LL LEARN

- To graph a relation, state its domain and range, and determine if it is a function, and
- to find values of functions for given elements of the domain.

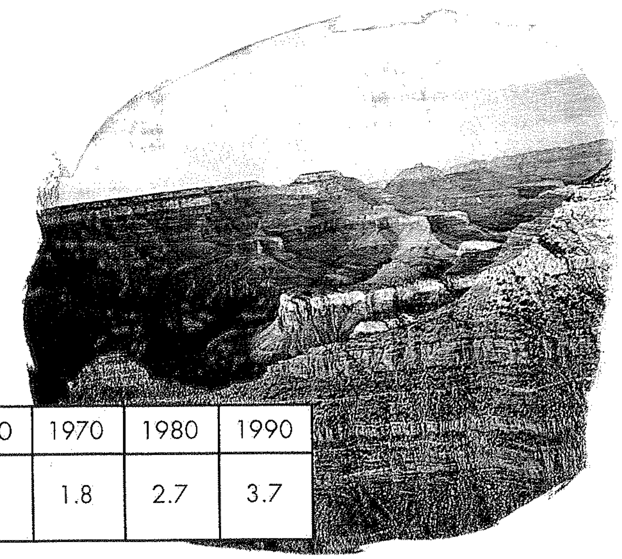
Why IT'S IMPORTANT

You can use relations to solve problems involving geography, forestry, and sports.

CONNECTION

Geography

The growth in the population of the state of Arizona over the last several decades can be shown by using a table.



Year	1930	1940	1950	1960	1970	1980	1990
Population (millions)	0.4	0.5	0.7	1.3	1.8	2.7	3.7

Source: U.S. Census

Another way to represent the population growth is to use **ordered pairs.** The ordered pairs for the data above are: (1930, 0.4), (1940, 0.5), (1950, 0.7), (1960, 1.3), (1970, 1.8), (1980, 2.7), and (1990, 3.7). The first number in the ordered pair is the year, and the second number is the population in millions.

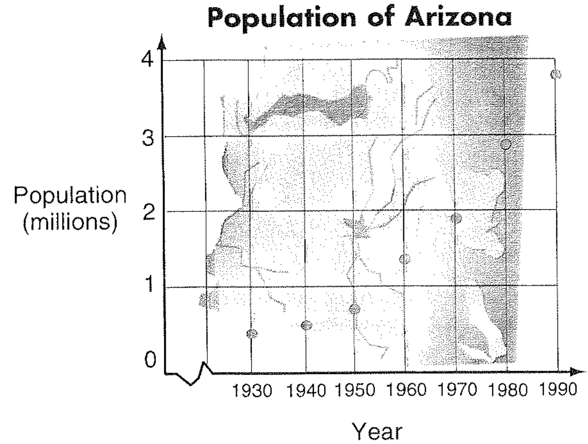
fabulous **FIRSTS**

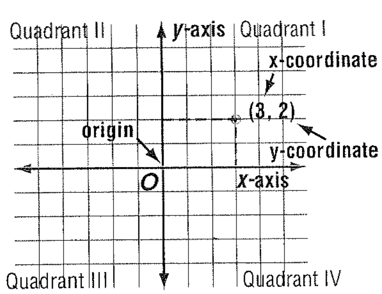
N. Scott Momaday (1934–)

The first American Indian to win the Pulitzer Prize for Literature was Kiowa Indian N. Scott Momaday. His 1969 novel, *House Made of Dawn*, was based on his experiences growing up in the Southwest.

You can *graph* these ordered pairs by creating a **coordinate system** with two axes. The horizontal axis represents the year, and the vertical axis represents the population. Each point represents an ordered pair shown in the table above. Remember that each point in the coordinate plane can be named by exactly one ordered pair and that every ordered pair names exactly one point in the coordinate plane.

Population of Arizona





In this book, assume that each square on a graph represents 1 unit unless otherwise labeled.

In the graph above, only one part of the Cartesian coordinate plane was shown—the one with all positive numbers. The Cartesian coordinate plane is composed of the ***x*-axis** (horizontal) and the ***y*-axis** (vertical), which meet at the **origin** (0, 0) and divide the plane into four **quadrants.**

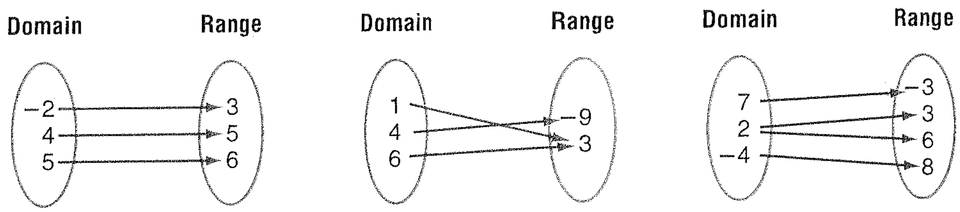
The points on the two axes do <u>not</u> lie in any quadrant.

The ordered pairs graphed on this plane can be represented by (x, y).

A set of ordered pairs, such as the one for the population of Arizona, forms a **relation.** The **domain** of a relation is the set of all first coordinates (x-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (y-coordinates) from the ordered pairs.

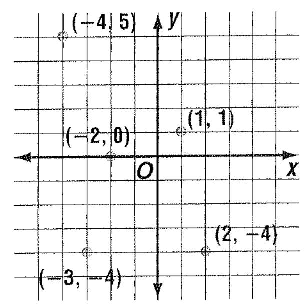
A **mapping** shows how each member of the domain is paired with each member of the range.

$\{(4, 5), (-2, 3), (5, 6)\}$ $\{(1, 3), (4, -9)\ (6, 3)\}$ $\{(2, 3), (-4, 8), (2, 6), (7, -3)\}$



A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element from the range. The first two relations above are functions. The third relation is *not* a function because the 2 in the domain is paired with both 3 and 6 in the range.

Example 1 **State the domain and range of the relation shown in the graph. Is the relation a function?**

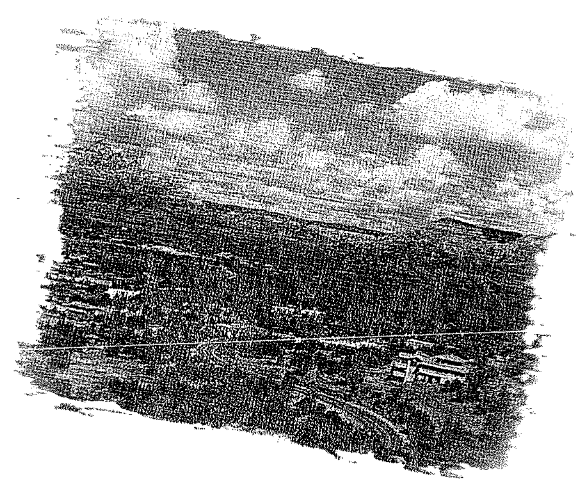


The relation is $\{(-4, 5), (-3, -4), (-2, 0), (1, 1), (2, -4)\}$.
The domain is $\{-4, -3, -2, 1, 2\}$.
The range is $\{-4, 0, 1, 5\}$

Each member of the domain is paired with exactly one member of the range, so this relation is a function.

Since the domain of the function in Example 1 is a set of individual points, it is called a **discrete function**. Notice that its graph consists of points that are not connected.

You can use the **vertical line test** to determine if a relation is a function. Using the graph in Example 1, place your pencil at the left of the graph to represent a vertical line. Slowly move the pencil to the right across the graph. At each point of the domain, the vertical line intersects the graph of the relation at only one point. Therefore, the relation is a function. If the vertical line intersects the graph at more than one point, the relation is *not* a function.

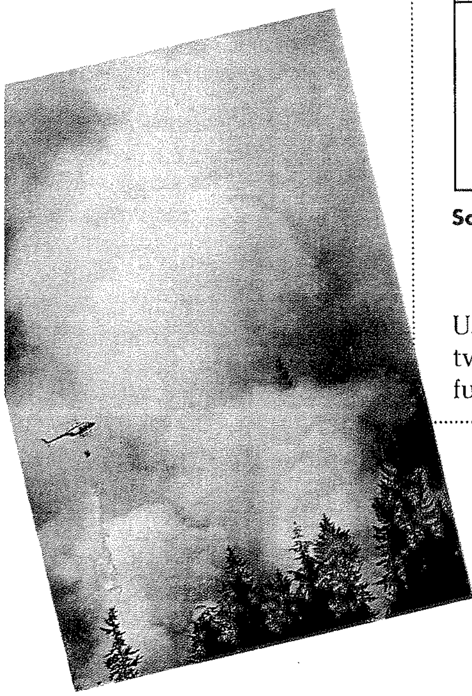


Is the population growth in Arizona a function? Why or why not?

Example 2 **The table below shows the number of fires and the number of acres burned over six years on the lands owned by the Bureau of Indian Affairs in the state of Washington. Graph this information and determine if it is a function.**

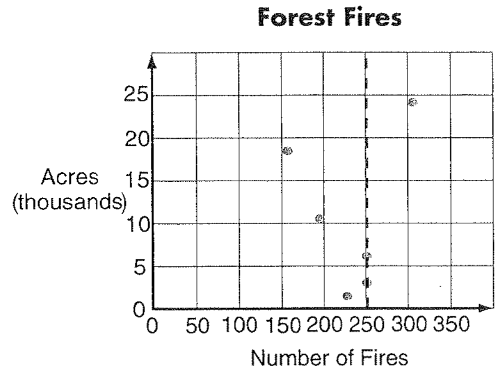
Real World APPLICATION

Forestry



Year	Number of Fires	Acreage
1987	198	10,288
1988	152	17,842
1989	250	3320
1990	227	1726
1991	250	6218
1992	310	24,499

Source: Washington State Department of Natural Resources



Using the vertical line test, you can see that 250 in the domain is mapped to two different range values, 3320 and 6218. Therefore, the relation is *not* a function.

Functions and their graphs can help you discover many relationships in mathematics.

MODELING MATHEMATICS

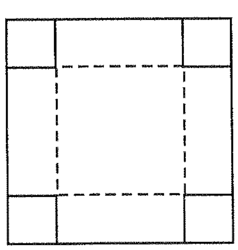
Volume

Materials: centimeter grid paper scissors

In this activity, you will make open boxes from identical square pieces of centimeter paper and investigate their volumes.

Your Turn

a. Cut identical squares from the corners of one square piece of paper and fold along the edges to form an open box.



b. Find the volume of the open box.

c. Repeat steps a and b for other cutout squares with whole-number lengths.

d. Organize the data into ordered pairs (length of the side of cutout square, volume of open box). Is this relation a function?

e. Graph the ordered pairs.

f. Which ordered pair results in the greatest volume?

g. Why might some businesses be interested in data like these?

An equation is another way to represent a relation. The solutions of an equation in x and y are the ordered pairs (x, y) that make the equation true. Consider the equation $y = 5x - 4$. Since x can be any real number, the domain has an infinite number of elements. To determine whether an equation represents a function, it is often simpler to look at the graph of the relation.

One way to graph an equation is to make a table of solutions, graph enough ordered pairs to see a pattern, and then connect the points with a line or smooth curve. You can then use the vertical line test to determine whether the equation represents a function.

When the domain of the function has an infinite number of elements and can be graphed with a line or smooth curve, the function is a **continuous function**.

Example 3 **Determine whether each equation represents a function.**

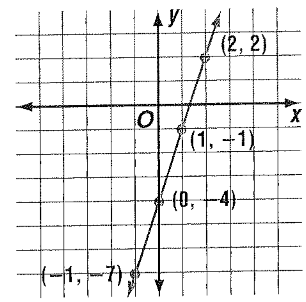
a. $y = 3x - 4$

Prepare a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

x	y
−1	
0	
1	
2	

→

x	y
−1	−7
0	−4
1	−1
2	2



Since x can be any real number, there is an infinite number of ordered pairs that can be graphed. If all of them were graphed, they would form a line. For each x value, there is exactly one y value. Thus, this set of ordered pairs passes the vertical line test and is a function.

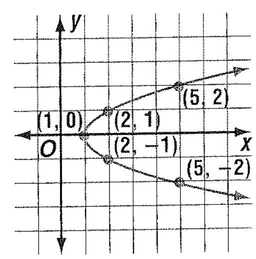
b. $x = y^2 + 1$

Complete the table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a curved line.

x	y
	−2
	−1
	0
	1
	2

→

x	y
5	−2
2	−1
1	0
2	1
5	2



This graph is called a *parabola*. You can see from the table as well as the vertical line test that there are two y values for all but one of the x values. Therefore, this set of ordered pairs is *not* a function.

Letters other than f can be used to represent a function. For example, the equation $y = 4x + 3$ can also be written as $g(x) = 4x + 3$.

Equations that represent functions are often written in *functional notation*. The equation $y = 2x + 3$ can be written as $f(x) = 2x + 3$. The symbol $f(x)$ replaces the y and is read "f of x." The f is just the name of the function, not a variable. Suppose you want to find the value in the range that corresponds to the element 6 in the domain. This is written $f(6)$ and is read "f of 6." The value $f(6)$ is found by substituting 6 for each x in the equation. Therefore, $f(6) = 2(6) + 3$ or 15.

Example 4 **Given the function $g(x) = x^2 - 8$, find each value.**

a. $g(-2)$ **b.** $g(7a)$

$g(x) = x^2 - 8$

$g(-2) = (-2)^2 - 8$ *Substitute.*

$= 4 - 8$ or -4

$g(x) = x^2 - 8$

$g(7a) = (7a)^2 - 8$ *Substitute.*

$= 49a^2 - 8$ $(ab)^2 = a^2b^2$

Example 5 **Use a calculator to find $f(4.6)$ if $f(x) = 0.5x^2 + 4x - 2.5$.**

$f(4.6) = 0.5(4.6)^2 + 4(4.6) - 2.5$

Estimate: *$0.5(5)^2 + 4(5) - 2.5 = 12.5 + 20 - 2.5$ or 30*

Enter: 0.5 $\boxed{\times}$ 4.6 $\boxed{x^2}$ $\boxed{+}$ 4 $\boxed{\times}$ 4.6 $\boxed{-}$ 2.5 $\boxed{=}$ 26.48

Therefore, $f(4.6) = 26.48$. *Compare with the estimate.*

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

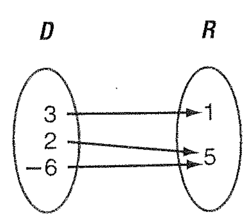
1. **Graph** a function that has a domain of $-1 \le x \le 5$ and a range of $-3 \le y \le 2$. Be sure that your function passes the vertical line test.
2. **Explain** the difference between a discrete function and a continuous function. Give an example of a graph of each type of function.
3. **Find a counterexample** for the statement "Every straight line is a function."
4. **Draw** a Cartesian coordinate plane. Name seven possible locations on the plane where a point might be graphed. Then graph and label a point in each of these locations.
5. **List** four ways to show how the relationship between members of a domain and range can be represented. Give an example of each.

MODELING MATHEMATICS

6. Suppose you are going to construct a graph that shows the relationship between the number of hours worked per week by a student at a fast-food restaurant and the amount of money earned. Identify a reasonable domain and range for the graph. Justify your answer. Then draw a sample graph.

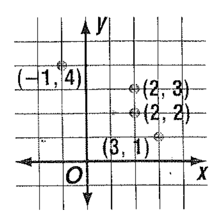
Guided Practice

State whether each relation is a function or not.

7. 

8.

x	y
5	−2
10	−2
15	−2
20	−2

9. 

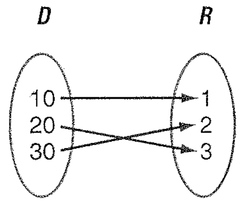
State the domain and range of each relation. Then graph the relation and identify whether it is a function or not. For each function, state whether it is discrete or continuous.

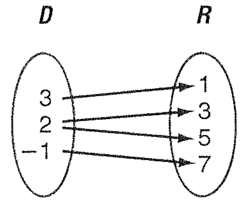
10. $\{(7, 8), (7, 5), (7, 2), (7, -1)\}$
11. $\{(6, 2.5), (3, 2.5), (4, 2.5)\}$
12. $y = -2x + 1$
13. $x = y^2$
14. Find $f(5)$ if $f(x) = x^2 - 3x$.
15. Find $h(-2)$ if $h(x) = x^3 + 1$.

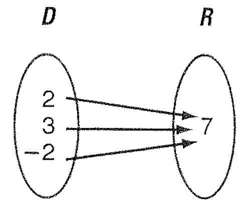
EXERCISES

Practice

State whether each relation is a function or not.

16. 

17. 

18. 

19.

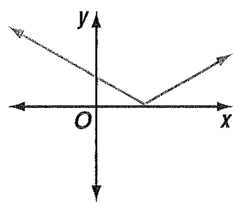
x	y
0.5	−3
2	0.8
0.5	8

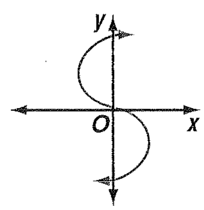
20.

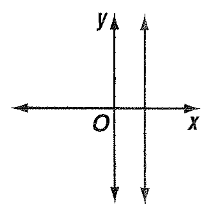
Year	Expenses
1994	$4000
1995	$4300
1996	$4000
1997	$4500

21.

x	y
3	5
3	10
3	15
3	20

22. 

23. 

24. 

State the domain and range of each relation. Then graph the relation and identify whether it is a function or not. ~~For each function, state whether it is discrete or continuous.~~

25. $\{(4, 5), (6, 5), (3, 5)\}$

26. $\{(-2, 5), (3, 7), (-2, 8)\}$

27. $\{(3, 4), (4, 3), (6, 5), (5, 6)\}$

28. $y = 7x - 6$

29. $y = -5x$

30. $x = y^2 + 1$

A function h includes the ordered pairs $(-2, 1)$, $(1, 2)$, and $(3.5, -0.3)$. State whether h will still be a function if each ordered pair given below is also included in h.

31. $(-2, 2)$

32. $(0, 0)$

33. $(2, 1)$

Find each value if $f(x) = 3x - 5$ and $g(x) = x^2 - x$.

34. $f(-3)$

35. $g(3)$

36. $g\left(\frac{1}{3}\right)$

37. $f\left(\frac{2}{3}\right)$

38. $f(a)$

39. $g(5n)$

Find each value if $h(x) = \dfrac{x^2 + 5x - 6}{x + 3}$.

40. $h(3)$

41. $h(-2)$

42. $h(a - 1)$

43. Write an example of a function that has a value of 5 when $x = 2$.

44. The function $f(x) = 3x$ can be represented by the notation $f : x \rightarrow 3x$, where x "maps" to $3x$. If a function g is defined as $g : x \rightarrow x^2 + x$, find the number that -2 maps to in function g.

The graph of each figure described below can be a function or a relation that is *not* a function depending on how it appears on a coordinate plane. Graph each figure as both a function and as a relation that is *not* a function.

45. a set of ordered pairs

46. a wavy line

47. an angle

48. a semicircle

Critical Thinking

49. When a fraction contains a variable in the denominator, there are some values of the variable for which the fraction is undefined. Find the domain of $f(x) = \frac{15}{x^2 - 9}$.

50. If $f(x) = x^2 + 2x + 1$, for what value(s) of x would $f(x) = 0$? On a graph of $f(x)$, describe the graph at which $f(x) = 0$.

Applications and Problem Solving

Real World

51. Finance On January 1, 1984, a large telephone system gave up its local telephone monopolies. Despite its break-up, the stock price nearly tripled over the next ten years.

Year	'83	'84	'85	'86	'87	'88	'89	'90	'91	'92	'93
Stock Price	\$19	\$20	\$25	\$25	\$28	\$29	\$45	\$30	\$39	\$50	\$55

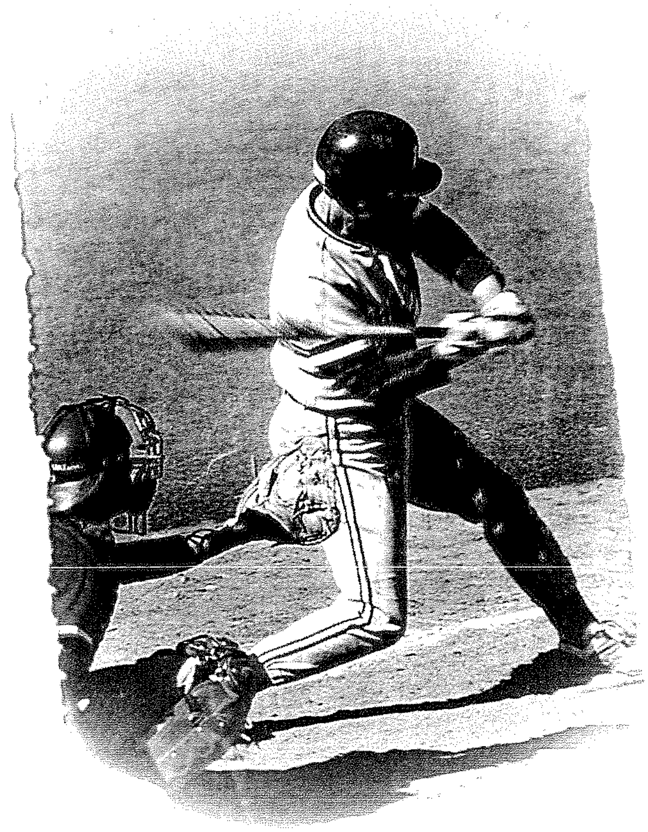
a. Identify the domain and range.

b. Write the ordered pairs and graph.

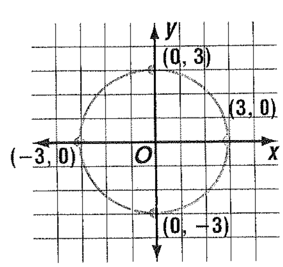
c. Is this a function?

d. Would buying the stock in 1983 have been a sound investment? Explain.

52. Sports Sketch a graph of the flight of a baseball, in which the vertical coordinate at each point measures the height of the baseball above the ground and the horizontal coordinate measures the time since the ball was hit. Does the graph represent a function? Explain.



53. Geometry Identify the domain and range of the circle shown below. Is this relation a function?



Internet Connection

Data Update For more information on the representatives in Congress, visit: **www.algebra2.glencoe.com**

54. **Government** The table below shows the number of Latino representatives in the U.S. Congress for 1981–1995.

Year	1981	1983	1985	1987	1989	1991	1993	1995
Latino Representatives	6	8	10	11	10	11	17	15

Source: National Association of Latino Elected and Appointed Officials

a. Identify the domain and range.

b. Write the ordered pairs and draw the graph.

c. Is this relation a function? If it is a function, determine whether it is discrete or continuous.

d. If the domain and range were switched, would the relation be a function? Explain.

Mixed Review

Solve each equation or inequality.

55. $|y + 1| < 7$ (Lesson 1–7)

56. $|5 - m| < 1$ (Lesson 1–7)

57. $x - 5 < 0.1$ (Lesson 1–6)

58. $3|2x - 5| = -\frac{1}{3}$ (Lesson 1–5)

59. **Consumerism** Ryan had \$25.04 with him when he went to the mall. His friend, Tim, had \$32.67. Ryan wanted to buy a golf shirt for \$27.89. (Lesson 1–4)

a. How much money did he have to borrow from Tim to buy the shirt?

b. How much money did that leave Tim? 48

60. **SAT Practice** In 1998, Kashan had a collection of 30 music CDs. Since then he has given away 2 CDs, purchased 6 new CDs, and traded 3 of his CDs to Marcus for 4 of Marcus' CDs. Since 1998, what has been the net percent increase in Kashan's CDs collection?

A $3\frac{1}{3}\%$ B 10% C $14\frac{2}{7}\%$ D $16\frac{2}{3}\%$ E $23\frac{1}{3}\%$

61. Write an algebraic expression to represent *seven less than the sum of a number and two times its square.* (Lesson 1–4)

62. **Statistics** At a bowling party, the members of the junior class decided to separate into two teams and the team with the lower mean would have to make dinner for the other team. The list at the right shows the two teams and their scores.

Team A	Score	Team B	Score
Ed	281	Laurel	101
Kelley	212	Tammy	236
Paul	72	Smitty	143
Mandi	147	Jeff	154
Maria	110	Renee	111
Bryce	212	Yolanda	88
Ryan	69	Percy	69
Sue	28	Debra	205

a. What are the mean, median, and mode for Team A? for Team B? (Lesson 1–3)

b. Make a back-to-back stem-and-leaf plot of the data. (Lesson 1–3)

c. Which team had to make dinner?

Simplify each expression.

63. $3(5a + 6b) + 8(2a - b)$ (Lesson 1–2)

64. $3^2(2^2 - 1^2) + 4^2$ (Lesson 1–1)

For Extra Practice, see page 878.

2-2A Graphing Technology
Linear Equations

A Preview of Lesson 2-2

Graphing calculators are powerful tools for studying a wide variety of graphs. The examples below show graphs of linear equations.

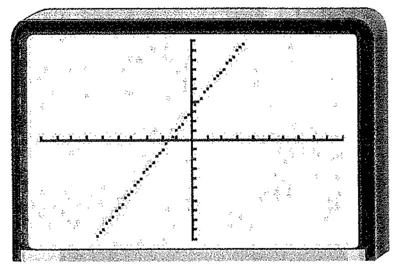
Example

Graph each equation in the standard viewing window.

a. $y - 2x = 3$

First, rewrite the equation so y is isolated on one side. Enter the equation for the line and graph.

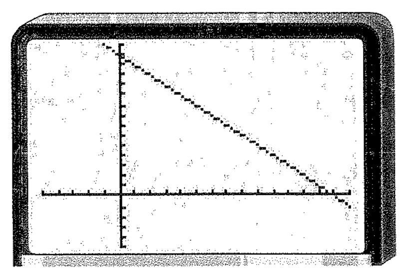
Enter: $\boxed{Y=}$ 2 $\boxed{X,T,\theta,n}$ $\boxed{+}$ 3 $\boxed{ZOOM}$ 6



A graph that appears on the graphics screen showing all important characteristics of the graph is called a **complete graph**. The graph of $y = 2x + 3$ is complete because we can see both the x- and y-intercepts.

b. $y = -x + 14$

Enter: $\boxed{Y=}$ $\boxed{(-)}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ 14 $\boxed{ZOOM}$ 6



What happened? Only a small portion of the graph is shown in the $[-10, 10]$ by $[-10, 10]$ window. This graph is not complete. Adjust the viewing window to include more of the graph. Try $[-5, 15]$ by $[-5, 15]$. This graph is shown at the right.

EXERCISES

Use a graphing calculator to graph each equation. Describe the viewing window that you used to view a complete graph for each equation.

1. $y = 3x - 3$	**2.** $y = -2x + 5$	**3.** $y = 4 - x$
4. $y = 5x - 35$	**5.** $y = -12x$	**6.** $y = 0.1x - 1$
7. $y = -0.3x + 15$	**8.** $y = 0.01x$	**9.** $y = 100x + 5$

2–2 Linear Equations

What YOU'LL LEARN

- To identify equations that are linear and graph them,
- to write linear equations in standard form, and
- to determine the intercepts of a line and use them to graph an equation.

Why IT'S IMPORTANT

You can write equations to represent relations in education, physics, and geology.

CONNECTION

Physics

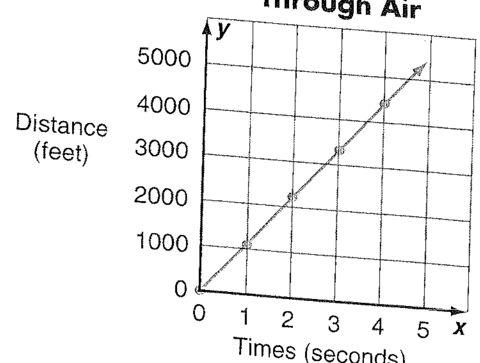
You might guess that sound would travel fastest through air since air is less dense than other mediums like water, glass, or steel. However, just the opposite is true. Sound travels through air most slowly of all. Through air it travels 1129 feet per second, through water about 4760 feet per second, and through glass and steel about 16,000 feet per second. Distances through air for various numbers of seconds are given in the table below.

Time (seconds)	0	1	2	3	4
Distance (feet)	0	1129	2258	3387	4516

The open sentence that describes this relationship is $y = 1129x$, where x represents the number of seconds, and y represents the distance in feet. Since the value of y *depends* on the value of x, y is called the **dependent variable**, and x is called the **independent variable**.

When the relation is graphed on a coordinate plane, the independent variable is graphed on the horizontal axis, and the dependent variable is graphed on the vertical axis. In this case, the points appear to lie on a line. This graph is a function.

Speed of Sound Through Air



Suppose we connect the points with a line. The line would contain an infinite number of points, whose ordered pairs are solutions of the equation $y = 1129x$. An equation whose graph is a line is called a **linear equation**. A linear equation is an equation that can be written in **standard form**, $Ax + By = C$.

Standard Form of a Linear Equation	The standard form of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

Usually A, B, and C are given as integers whose greatest common factor is 1.

When variables other than x are used, assume that the letter coming first in the alphabet represents the domain or horizontal coordinate.

Linear equations contain one or two variables, with no variable having an exponent other than 1.

Linear equations

$5x - 3y = 7$

$x = 9$

$6s = -3t - 15$

Not linear equations

$7a + 4b^2 = -8$

$y = \sqrt{x + 5}$

$x + xy = 1$

Example 1 **Write each equation in standard form where A, B, and C are integers whose greatest common factor is 1. Identify A, B, and C.**

a. $y = -5x + 6$

$$y = -5x + 6$$

$$5x + y = 6 \quad \textit{Add 5x to each side.}$$

So $A = 5$, $B = 1$, and $C = 6$.

b. $\frac{2}{7}x = 2y + 5$

$$\frac{2}{7}x = 2y + 5$$

$$\frac{2}{7}x - 2y = 5 \quad \textit{The equation is in standard form.}$$

$$2x - 14y = 35 \quad \textit{Multiply each side by 7.}$$

So $A = 2$, $B = -14$, and $C = 35$.

c. $5x - 10y = 25$

$$5x - 10y = 25 \quad \textit{The GCF of 5, 10, and 25 is 5.}$$

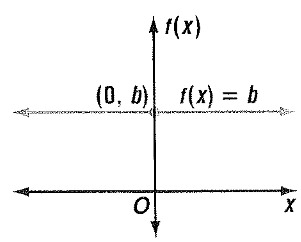
$$x - 2y = 5 \quad \textit{Divide each side by 5.}$$

So $A = 1$, $B = -2$, and $C = 5$.

Any function whose ordered pairs satisfy a linear equation is called a **linear function.**

Definition of Linear Function	**A function is linear if it can be defined by $f(x) = mx + b$, where m and b are real numbers.**

In the definition of a linear function, m or b may be zero. If $m = 0$, then $f(x) = b$. The graph is a horizontal line. This function is called a **constant function.** If $f(x) = 0$, the function is called the *zero function*.



Example 2 **State whether each function is a linear function.**

a. $f(x) = 4x + 5$ This is a linear function because it is in the form $f(x) = mx + b$, with $m = 4$ and $b = 5$.

b. $g(x) = x^3 + 2$ This is *not* a linear function because x has an exponent other than 1.

c. $f(x) = 9 - 6x$ This is a linear function because it can be written as $f(x) = -6x + 9$, with $m = -6$ and $b = 9$.

In Lesson 2–1, you graphed an equation or function by making a table of values, graphing enough ordered pairs to see a pattern, and connecting the points with a line or smooth curve. However, there are quicker ways to graph a linear equation or function. One way is to find the points at which the graph intersects each axis and connect them with a line. The y-coordinate of the point at which a graph crosses the y-axis is called the **y-intercept.** Likewise, the x-coordinate of the point at which it crosses the x-axis is the **x-intercept**.

Example 3 **Graph $5x - 3y = 15$ using the x- and y-intercepts.**

The x-intercept is the value of x when $y = 0$.

$$5x - 3y = 15$$
$$5x - 3(0) = 15 \quad \textit{Substitute 0 for y.}$$
$$x = 3$$

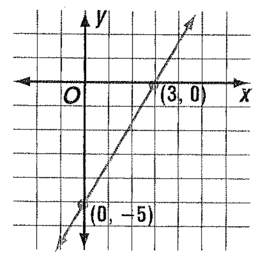
The x-intercept is 3. The graph crosses the x-axis at $(3, 0)$.

Likewise, the y-intercept is the value of y when $x = 0$.

$$5x - 3y = 15$$
$$5(0) - 3y = 15 \quad \textit{Substitute 0 for x.}$$
$$y = -5$$

The y-intercept is -5. The graph crosses the y-axis at $(0, -5)$.

Use these ordered pairs to graph the equation.



Linear equations, functions, and their graphs can be used to model situations that occur in real life.

Example 4

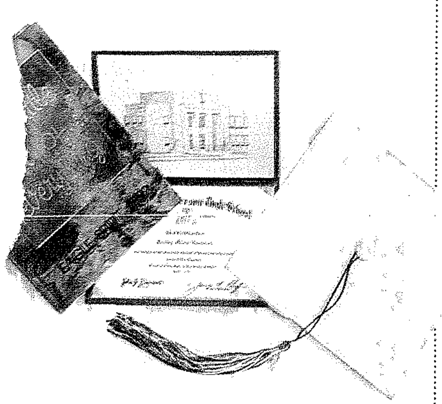
Real World APPLICATION

Education

Each year, more than two million high school juniors and seniors across the nation tackle one or more college admission tests. One of them is the SAT, which is divided into two sections, verbal and mathematical. For each section, the possible scores range from 200 to 800. Suppose your counselor advises you that, as one factor for admission, some colleges expect a combined score of 1300. This situation can be represented by the equation $x + y = 1300$, where x is the verbal score and y is the mathematical score.

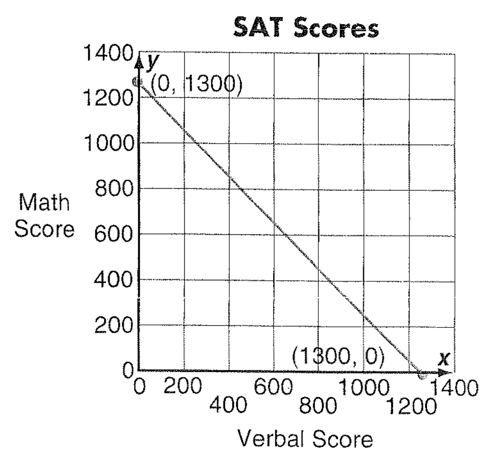
a. Graph the linear equation.

b. Name an ordered pair that satisfies the equation and explain what it represents.



a. Find the x- and y-intercepts.

$$x + y = 1300$$
$$x + 0 = 1300 \quad \textit{Substitute 0 for y.}$$
$$x = 1300$$

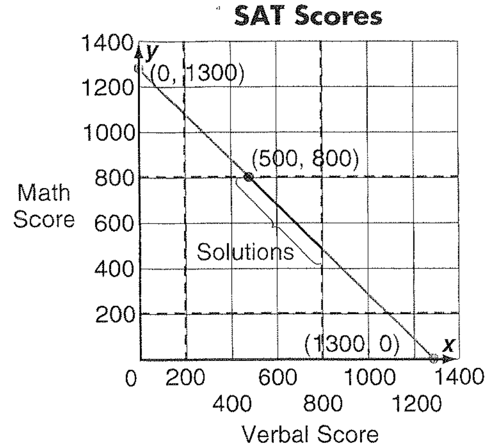


The x-intercept is 1300. Similarly, the y-intercept is 1300.

(continued on the next page)

b. There are many ordered pairs that satisfy the equation. However, not all of them are solutions of the problem. Since the scores on each section range from 200 to 800, it is necessary to restrict the domain to $200 \leq x \leq 800$ and the range to $200 \leq y \leq 800$.

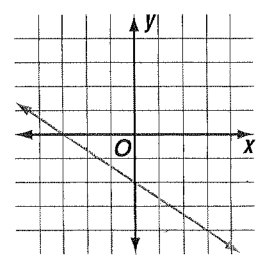
Therefore, the ordered pairs that are solutions to the problem are shown at the right. One ordered pair is (500, 800). It represents having a verbal score of 500 and a math score of 800 for a total of 1300.



CHECK FOR UNDERSTANDING

Communicating Mathematics

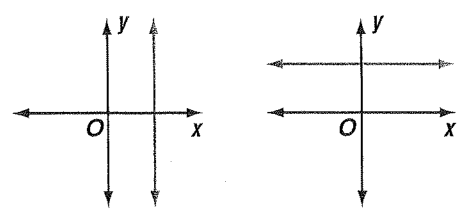
Study the lesson. Then complete the following.



1. **Write** the equation $4y = 3x + 5$ in standard form. Identify A, B, and C.
2. **Name** the x- and y-intercepts of the graph shown at the right.
3. **Explain** how to find the x- and y-intercepts of the graph of $2x + y = 7$.
4. **List** at least two ways to graph a linear equation.

MATH JOURNAL

5. **Write** a paragraph explaining why both graphs at the right are graphs of linear equations, but only one is a linear function. Use drawings with your explanation.



Guided Practice

State whether each equation is linear. Write *yes* or *no* and explain your answer.

6. $x^2 + y^2 = 4$
7. $h(x) = 1.1 - 2x$

Write each equation in standard form where *A*, *B*, and *C* are integers whose greatest common factor is 1. Identify *A*, *B*, and *C*.

8. $y = 3x - 5$
9. $4x = 10y + 6$
10. $y = \frac{2}{3}x + 1$

Find the *x*-intercept and the *y*-intercept of the graph of each equation.

11. $6x + y = 9$
12. $y = -3x - 5$
13. $f(x) = x - 2$

Graph each equation.

14. $3x + 2y = 6$
15. $y = -2x$
16. $4x + 8y = 12$
17. **Economics** On June 20, 1995, the function $f(x) = 0.718x$ was used to convert German marks, x, to U.S. dollars, $f(x)$. For that date, find the value in U.S. dollars of 100 German marks.

EXERCISES

Practice

State whether each equation is linear. Write *yes* or *no* and explain your answer.

18. $x + y = 5$
19. $\frac{1}{x} + 3y = -5$
20. $x + xy = 4$
21. $g(x) = 10$

Write each equation in standard form where *A*, *B*, and *C* are integers whose greatest common factor is 1. Identify *A*, *B*, and *C*.

22. $y = -3x + 4$
23. $y = 12x$
24. $x = 4y - 5$
25. $5y = 10x - 25$
26. $\frac{1}{2}x + \frac{1}{2}y = 6$
27. $0.25y = 10$

Find the *x*-intercept and *y*-intercept of the graph of each equation.

28. $y + 6 = 5x$
29. $3x = y$
30. $y = -2$
31. $x = 8$
32. $g(x) = 4x - 1$
33. $5x + 3y = 15$

Graph each equation.

34. $y = x$
35. $y = 4x + 2$
36. $x + y = 7$
37. $2x - y = 5$
38. $2x + 5y = 10$
39. $y = 0.5x - 3$
40. $b = 2a - 3$
41. $x - y = 6$
42. $2a + 3b = 6$
43. $3 = 3x$
44. $x + 2y = 7$
45. $4x + 3y = 12$
46. $\frac{1}{3}x + \frac{1}{2}y = 1$
47. $\frac{x}{4} - \frac{y}{3} = 2$
48. $\frac{x}{3} + \frac{y}{2} = \frac{15}{2}$

Critical Thinking

49. Graph $x + y = 0$, $x + y = 5$, and $x + y = -5$ on the same coordinate plane.
    a. Compare and contrast the graphs.
    b. Write a linear equation whose graph is between the graphs of $x + y = 0$ and $x + y = 5$.

Applications and Problem Solving

Real World

50. **Geology** Geothermal energy is generated wherever water comes into contact with heated underground rocks. The underground temperature of rocks varies with their depth below the surface. The temperature t in degrees Celsius is given by the function $t(d) = 35d + 20$, where d is the depth in kilometers of the rocks.
    a. Graph the linear equation.
    b. Find the temperature of the rocks at a depth of 3 kilometers.
    c. Is this function discrete or continuous? Explain your reasoning.
    d. Find the depth if the temperature of the rocks is 195°C.

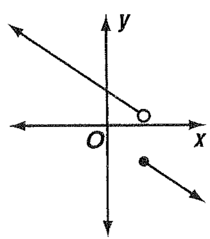
51. **Commercial Fishing** Fishing boats are usually equipped with sonar—a device used to locate schools of fish by the reflection of sound waves.
    a. Refer to the application at the beginning of the lesson. Write a function that is a model for the relationship between the number of seconds it takes the sound signal to return to the boat and the depth of the school of fish.
    b. Suppose the sound signal returned to the boat in 0.05 seconds. Estimate the depth of the school of fish.

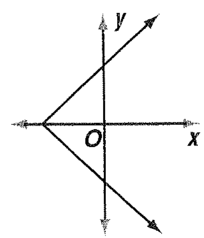
52. **Entertainment** In a recent movie, the crew of the submarine *Alabama* tried desperately to restore power to the ship before it sunk to a depth of 1850 feet. Use the function $f(x) = 1.15x$, where x is the depth in miles and $f(x)$ is the pressure in tons per square inch, to estimate the water pressure on the outside of the hull at that depth.

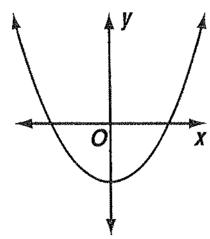
53. **Fundraising** The Central High School Band Boosters have a concessions stand for home football games. They sell beverages for \$1.50 and candy for \$1.25.
Their goal is to sell a total of \$375 for each game.
 a. Write an equation that is a model for the different numbers of beverages and candy that can be sold to meet the goal.
 b. Graph the equation.
 c. Is this equation also a function? If so, is it discrete or continuous? Explain your reasoning.
 d. If they sell 100 beverages and 200 pieces of candy, will the Band Boosters meet their goal?

Mixed Review

54. Which of the following graphs represent functions? (Lesson 2–1)

a.


b.


c.


Solve each equation.

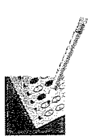
55. $|x + 7| > -2$ (Lesson 1–7)

56. $5(2x - 7) > 10$ (Lesson 1–6)

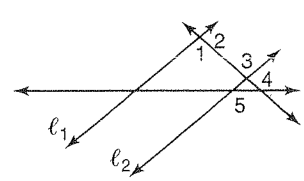
57. $7|3x + 5| = 35$ (Lesson 1–5)

58. $x + 28.3 = 56.0$ (Lesson 1–4)

59. **Statistics** Suppose four tests had been given in your Algebra 2 class this quarter. On the first three, you scored 87, 92, and 81. If you must have at least 350 points to earn an A, what must you score on the fourth test to earn an A? (Lesson 1–6)



60. **ACT Practice** In the figure $\ell_1 \parallel \ell_2$. Which of the labeled angles must be equal to each other?



A 4 and 2
B 5 and 1
C 4 and 3
D 5 and 3
E 2 and 3

For Extra Practice, see page 878.

Simplify each expression.

61. $(9s - 4) - 3(2s - 6)$ (Lesson 1–2)

62. $[19 - (8 - 1)] \div 3$ (Lesson 1–1)

2–2B Graphing Technology
Using Graphs to Estimate Solutions

An Extension of Lesson 2–2

The graphing features of a graphing calculator allow you to approximate solutions to an equation in one variable from a graph. First set each side of the equation equal to y. Then graph the two equations on the same screen. The solution of the original equation is the x-coordinate of the point of intersection of the two graphs.

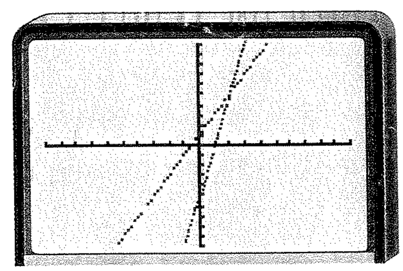
Example **Solve $5x - 5 = 2x + 1$ graphically.**

Set each side of the equation equal to y.
$y = 5x - 5$
$y = 2x + 1$

Then graph both equations in the standard viewing window.

Enter: $\boxed{\text{Y=}}$ 5 $\boxed{\text{X,T,}\theta,n}$ $\boxed{-}$ 5 $\boxed{\text{ENTER}}$
2 $\boxed{\text{X,T,}\theta,n}$ $\boxed{+}$ 1 $\boxed{\text{ZOOM}}$ 6

You are interested in the point at which the values of x are equal—namely, the point of intersection. Use TRACE with the left and right arrow keys to approximate the point of intersection. The display gives an approximation for the coordinates of the point. Or press $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ 5 $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ and read the x-coordinate at the bottom of the screen.



You can see that the point of intersection occurs when $x = 2$.

Therefore, the solution of $5x - 5 = 2x + 1$ is 2.

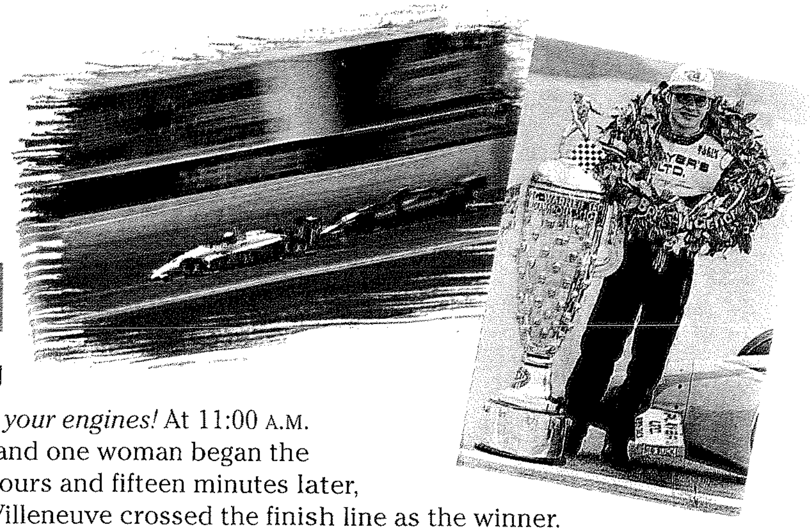
Verify the solution by substituting 2 into the original equation.

EXERCISES

Solve each equation graphically to the nearest tenth. Check your solutions algebraically.

1. $2x - 1 = 2$
2. $3x + 9 = 25$
3. $2x + 1 = 16 - x$
4. $4x + 3 = 5x + 7$
5. $7x + 9 = 3(x + 3)$
6. $5(8 - 2x) = 4x - 2$
7. $-1.5x = -2x + 5.75$
8. $16x - 3.8 = 12x - 3.8$
9. $5.2x + 0.7 = 2.8 + 2.2x$
10. $-2(3x - 5) + 3x = 2 - x$

2-3 Slope



Real World APPLICATION

Auto Racing

Lady and gentlemen, start your engines! At 11:00 A.M. on May 28, 1995, 32 men and one woman began the Indianapolis 500. Three hours and fifteen minutes later, Canadian racer Jacques Villeneuve crossed the finish line as the winner. Meanwhile, the Diaz family left Indianapolis, Indiana, in the family car, heading for Atlanta, Georgia. They took turns driving, stopped only for gasoline, and completed their 500 mile trip in 9.5 hours. Both of these situations can be modeled graphically.

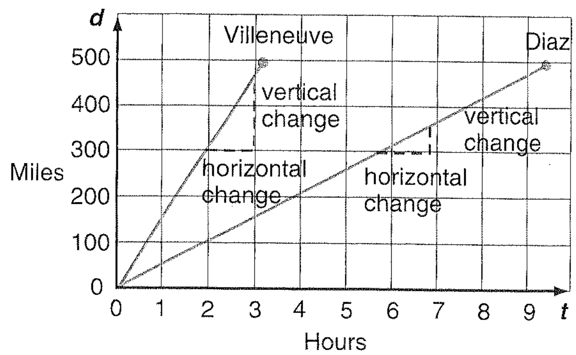
What YOU'LL LEARN

- To determine the slope of a line,
- to use slope and a point to graph an equation,
- to determine if two lines are parallel, perpendicular, or neither, and
- to solve problems by identifying and using a pattern.

Why IT'S IMPORTANT

You can use slope to describe lines and solve problems involving auto racing and entertainment.

The graph representing Villeneuve's race is much steeper than the graph representing the Diaz family's trip because, on average, Villeneuve traveled a greater distance for each unit of time. The **slope** of each line is the ratio of the change in the vertical units (distance) to the change in the horizontal units (time).



$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

The slope of the line representing Villeneuve's race is $\frac{500}{3.25}$ or about 153.8.

The slope of the line representing the Diaz' trip is $\frac{500}{9.5}$ or about 52.6.

The slope of each line indicates its steepness, and in this case, it also indicates the average speed in miles per hour.

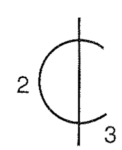
The problem-solving strategy **look for a pattern** is one of the most-used strategies in mathematics. When using this strategy, you will often need to make a table to organize the information.

Example 1

PROBLEM SOLVING

Look for a Pattern

The symbol used for a cent is a lowercase c with a vertical line through it. The line separates the c into 3 parts, as shown at the right. How many parts would there be if the c had 101 lines through it?



Make a table to show the pattern.

Number of Lines	1	2	3	4	5	x
Number of Parts	3	5	7	9	11	$2x + 1$

If the c had x lines through it, there would be $2x + 1$ parts. Thus, if the c had 101 lines through it, there would be $2(101) + 1$ or 203 parts.

ou can look for a pattern to help solve many problems involving slope and
phs.

Examp

Real World APPLICATION

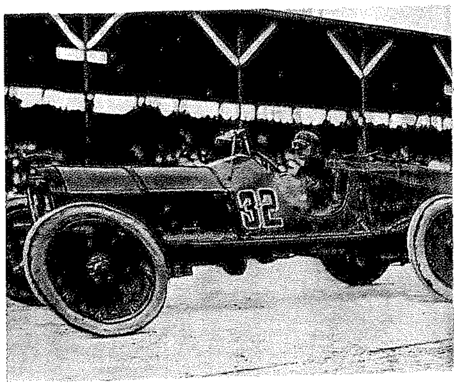
Auto Racing

The winner of the first Indianapolis 500 was Ray Harroun. In 1911, he completed the race in a Marmon Wasp at an average speed of 74.6 miles per hour. Harroun's race can be modeled by the linear function $d(t) = 74.6t$, where t is the time in hours and $d(t)$ is the distance in miles.

a. Make a table of ordered pairs and graph the linear function.

b. Use the graph to predict the value of t when $d(t) = 500$.

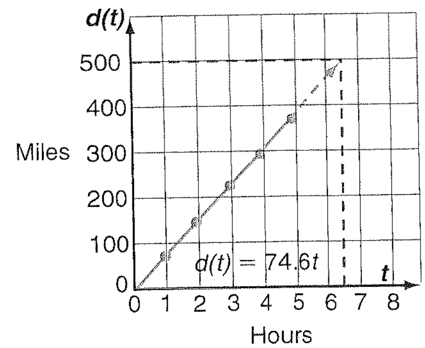
c. The winning speed for the Indianapolis 500 seems to increase each year. Suppose next year's winning speed is 160 mph. Predict where the line representing this race will be graphed.



Ray Harroun winning the Indianapolis 500

a. Choose ordered pairs for $1 \leq t \leq 5$.

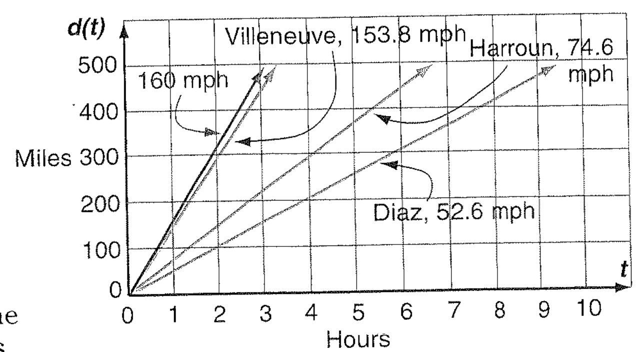
	Time t (hours)	Distance $d(t)$ (miles)	
	1	74.6	
+1	2	149.2	+74.6
+1	3	223.8	+74.6
+1	4	298.4	+74.6
+1	5	373.0	+74.6



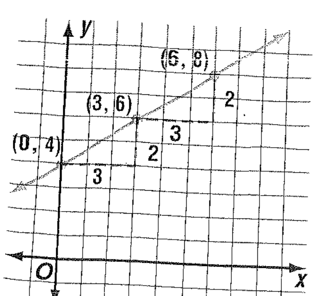
Notice that the ratio of the change in the vertical units to the change in horizontal units is $\frac{74.6}{1}$ or 74.6. Therefore, the slope is 74.6.

b. To predict the value of t when $d(t) = 500$, extend the graph to $d(t) = 500$. The corresponding value of t is between 6 and 7 hours.

c. To predict where the line representing a winning speed of 160 mph will be graphed, look for a pattern in the graphs of Harroun's race and the graphs in the application at the beginning of the lesson. Notice that as the average speed increases, the slope of the line also increases.



The graph representing a winning speed of 160 mph should have the steepest slope and, following the pattern in the graphs, should be to the left of the graph representing Villeneuve's race.



The slopes of linear functions can be defined by looking for a pattern. Consider the equation $y = \frac{2}{3}x + 4$. In the table of values shown at the right and the graph shown at the left, look for a pattern in the relationship of the change in the y-coordinates to the change in the x-coordinates for the points on the graph.

	x	y	
	0	4	
+3	3	6	+2
+3	6	8	+2

Notice that the y-coordinates increase 2 units for each 3-unit increase in the x-coordinates. Thus, the slope of the line whose equation is $y = \frac{2}{3}x + 4$ is $\frac{2}{3}$.

These examples suggest that the slope of a line can be determined from the coordinates of two points on the line.

Definition of Slope	**The slope m of the line passing through points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.**

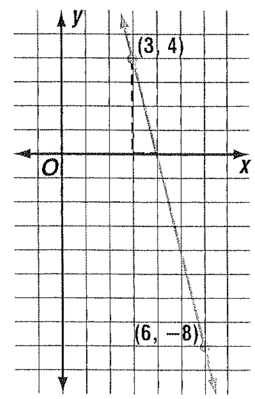
x_2 is read "x sub 2." The 2 is called a subscript.

Example 3 **Determine the slope of the line that passes through the points at (3, 4) and (6, −8). Then graph the line.**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-8 - 4}{6 - 3} \quad (x_1, y_1) = (3, 4), (x_2, y_2) = (6, -8)$$

$$= \frac{-12}{3} \text{ or } -4$$



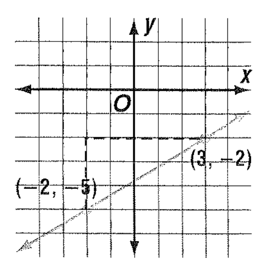
The slope of the line is −4.

Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 4 units and right 1 unit or go up 4 units and left 1 unit. This point should also be on the line.

Example 4 **Graph the line passing through the point (−2, −5) with a slope of $\frac{3}{5}$.**

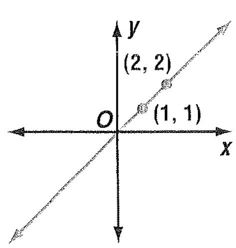
Graph the ordered pair (−2, −5). Then, using the definition of slope, go up 3 units and 5 units to the right. Plot the point. This new point is (3, −2). *You can also go 5 units right and 3 units up to plot the new point.*

Connect the points to draw the line.



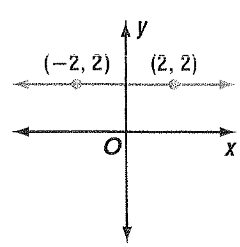
The slope of a line tells the direction in which it rises or falls.

If the line rises to the right, then the slope is positive.



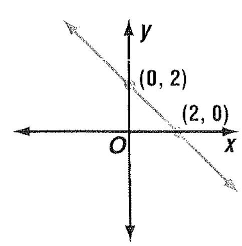
$m = \frac{2 - 1}{2 - 1}$ or 1

If the line is horizontal, then the slope is zero.



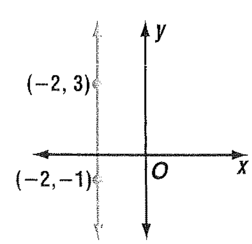
$m = \frac{2 - 2}{2 - (-2)}$ or 0

If the line falls to the right, then the slope is negative.



$m = \frac{0 - 2}{2 - 0}$ or −1

If the line is vertical, then the slope is undefined.



$m = \frac{3 - (-1)}{-2 - (-2)}$ or $\frac{4}{0}$

EXPLORATION — GRAPHING CALCULATORS

A **family of graphs** is a group of graphs that displays one or more similar characteristics. The **parent graph** is the simplest of the graphs in a family. You can graph families on the same screen and observe their common traits.

Your Turn

a. Graph $y = 3x$, $y = 3x + 2$, $y = 3x - 2$, and $y = 3x - 5$ on the same screen.

b. Identify the parent function and describe the family of graphs.

c. Find the slope of each line.

d. Write a function that has the same characteristics as this family of graphs. Check by graphing.

In the Exploration, you saw that lines that have the same slope are parallel.

Definition of Parallel Lines

In a plane, nonvertical lines with the same slope are parallel.

All vertical lines are parallel.

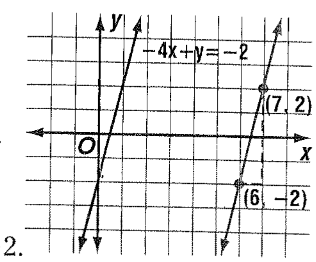
If you know the slope of a line and the coordinates of a point not on the line, you can graph a line through the point that is parallel to the first line.

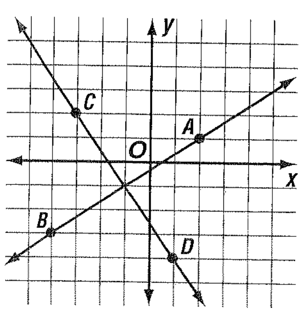
Example 5 **Graph the line that goes through the point at (6, −2) and is parallel to the line whose equation is $-4x + y = -2$.**

The y-intercept is -2, and the x-intercept is $\frac{1}{2}$.

Use the x- and y-intercepts to graph $-4x + y = -2$. The slope of the line is 4.

Now use the slope and the point at $(6, -2)$ to graph the line parallel to the graph of $-4x + y = -2$.





The figure at the left shows the graphs of two lines that are perpendicular. We found that parallel lines have the same slope. Is there a special relationship between the slopes of two perpendicular lines?

slope of line AB

$$\frac{1-(-3)}{2-(-4)} = \frac{4}{6} \text{ or } \frac{2}{3}$$

slope of line CD

$$\frac{2-(-4)}{-3-1} = \frac{6}{-4} \text{ or } -\frac{3}{2}$$

The slopes are negative reciprocals of each other. This and other examples suggest that when you multiply the slopes of two perpendicular lines, the product is always -1.

Definition of Perpendicular Lines

In a plane, two oblique lines are perpendicular if and only if the product of their slopes is −1.

Lines that are not vertical or horizontal are called oblique.
Any vertical line is perpendicular to any horizontal line.

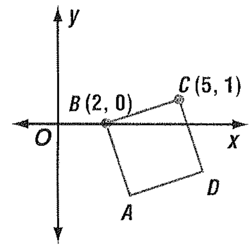
You can use the fact that the slopes of perpendicular lines are negative reciprocals of each other to solve problems involving figures with right angles.

Example 6

INTEGRATION
Geometry

The consecutive sides of a rectangle are perpendicular. In rectangle *ABCD*, the coordinates of point *B* are (2, 0), and the coordinates of point *C* are (5, 1). Find the slope of the line containing side $\overline{CD}$ of the rectangle.

In rectangle $ABCD$, $\overline{BC}$ is perpendicular to $\overline{CD}$. First find the slope of side $\overline{BC}$.



slope of $\overline{BC} = \frac{1-0}{5-2}$ or $\frac{1}{3}$

Let m represent the slope of $\overline{CD}$. Since the product of the slopes must be -1, you can use this equation.

$$m\left(\frac{1}{3}\right) = -1$$

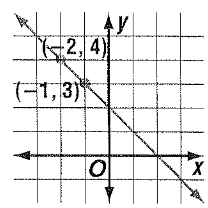
$$m = -1\left(\frac{3}{1}\right) \text{ or } -3$$

The slope of the line containing $\overline{CD}$ is -3.

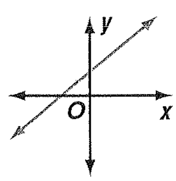
CHECK FOR UNDERSTANDING

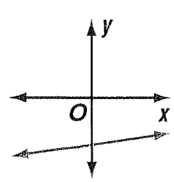
Communicating Mathematics

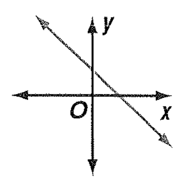
Study the lesson. Then complete the following.

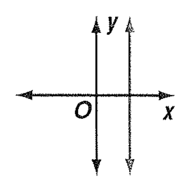


1. **Explain** how to find the slope of the line at the right.
2. **Graph** a line with a slope of 2 and a y-intercept of 3.
3. **Choose** the line that has a negative slope.

a.


b.


c.


d.


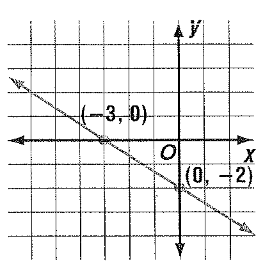
4. **Describe** the relationship between the slopes of two parallel lines.

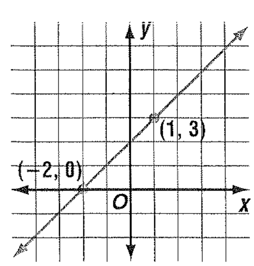
MATH JOURNAL

5. Use a dictionary to find other definitions of slope.

Guided Practice

State the slope of each line.

6.


7.


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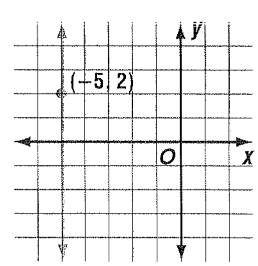


Find the slope of the line that passes through each pair of points. Then determine whether the line rises to the right, falls to the right, is horizontal, or is vertical.

9. (1, 1), (3, 1)   10. (−1, 0), (3, −2)   11. (3, 4), (1, 2)

Find the slope of the graph of each equation.

12. $2x - y = 4$ 13. $x + y = 3$ 14. $2x + 3y = 6$

15. State the slope of a line parallel to the line passing through (−1, −1) and (3, 2). Then state the slope of a line perpendicular to it.

16. Graph a line that passes through (0, 0) and has a slope of 3.

17. Graph the line that passes through (0, 3) and is parallel to the line whose equation is $6y - 10x = 30$.

EXERCISES

Practice

Find the slope of the line that passes through each pair of points. Then determine whether the line rises to the right, falls to the right, is horizontal, or is vertical.

18. (6, 1), (8, −4)   19. (6, 8), (5, −5)   20. (−6, −5), (4, 1)

21. (7, 8), (1, 8)   22. (2.5, 3), (1, −9)   23. (a, 2), (a, −2)

Find the slope of the graph of each equation.

$3y = -2x - 32$
$y = -\frac{2}{3}$

24. $x + y = 5$ $y = -x + 5$ 25. $3x + 9 = 0$ 26. $2x - y = 8$

27. $2x + 3y + 32 = 0$ $2x + 3y = -32$ 28. $3x - 4y = 0$ $-4y = -3x$ 29. $y = 5$

Determine the value of *r* so that a line through the points with the given coordinates has the given slope. Draw a sketch of each situation.

30. (r, 2), (4, −6); slope = $-\frac{8}{3}$ 12   31. (5, r), (2, 3); slope = 2 9 $\frac{-6}{-3}$

32. (r, 6), (8, 4); slope = $\frac{1}{2}$ $\frac{-2}{-4}$ 33. (6, r), (9, 2); slope = $\frac{1}{3}$

34. Graph a line through (2, 6) that has a slope of $\frac{2}{3}$.

35. Graph a line through (−2, 2) that is parallel to a line whose slope is −1.

36. Graph a line through (−4, 1) that is perpendicular to a line whose slope is $-\frac{3}{2}$.

37. Graph a line through (3, 3) that is perpendicular to the graph of $y = 3$.

38. Graph a line through the origin that is parallel to the graph of $x + y = 10$.

39. Graph a line through (−4, −2) that has an undefined slope.

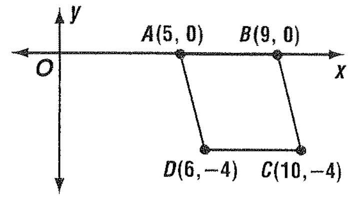
40. One line has a slope of 0 and another line has an undefined slope, but they both pass through (−3, −3). Graph the lines.

41. Graph the line perpendicular to the graph of $3x - 2y = 24$ that intersects it at its x-intercept.

42. **Geometry** In the ordered pairs (3, 0), (4, 2), (5, 5), (6, 9), and (7, ___?___), the first coordinate is the number of sides in a polygon, and the second coordinate is the number of diagonals that can be drawn in each polygon. Find the pattern to complete the last ordered pair. 14

INTEGRATION
Geometry

43. Determine whether the diagonals of parallelogram $ABCD$ at the right are perpendicular. Explain your answer.



44. Are the diagonals of a rectangle perpendicular? Use analytic methods to explain your answer.

Graphing Calculator

45. Use a graphing calculator to investigate each family of graphs. Explain how changing the slope affects the graph of the line.
   a. $y = 2x + 3, y = 4x + 3, y = 8x + 3, y = x + 3$
   b. $y = -3x + 1, y = -x + 1, y = -5x + 1, y = -7x + 1$

Programming

46. Points that lie on the same line are called *collinear* points. The graphing calculator program at the right will help you determine whether three points are collinear.

```
PROGRAM: Slope
: Disp "ENTER X AND Y FOR POINT
  1"
: Input A
: Input B
: Disp "ENTER X AND Y FOR POINT
  2"
: Input C
: Input D
: If (C-A)=0
: Goto 1
: (D-B)/(C-A) → M
: Disp "THE SLOPE IS  "
: Disp M
: Stop
: Lbl 1
: Disp "THE SLOPE IS UNDEFINED"
: End
```

Draw $\overline{AB}$ and $\overline{BC}$ for each set of points on a graphing calculator. Then use the program at the right to find the slopes of $\overline{AB}$ and $\overline{BC}$ and determine which points are collinear.

   a. $A(3, 6), B(5, 7), C(7, 8)$
   b. $A(5, 9), B(7, 12), C(11, 17)$
   c. $A(0, 1.5), B(5, 8.2), C(11, 15.4)$
   d. $A(2.2, -2.1), B(0.6, -1.3), C(-3.8, 0.9)$

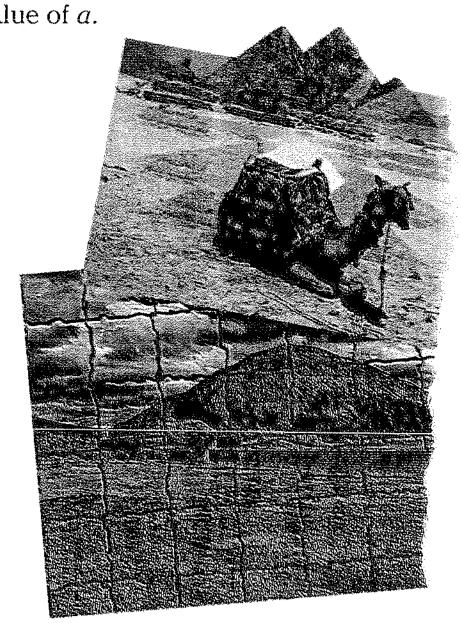
Critical Thinking

47. If the graph of the equation $ax + 2y = 8$ is perpendicular to the graph of the equation $2x + y = -3$, find the value of a.

Applications and Problem Solving

Real World

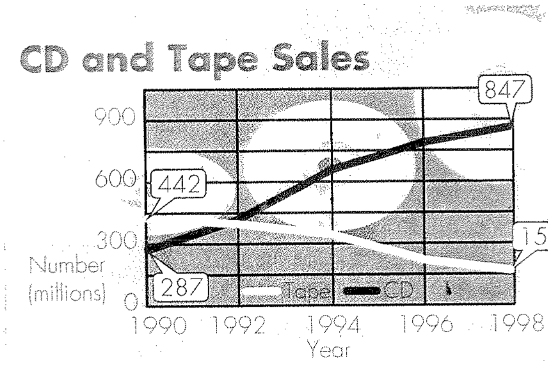
48. **Ancient Cultures** Probably the most famous use of pyramids occurred 4500 years ago as tombs for Egyptian pharaohs and their relatives. But the Western Hemisphere has also had its share of pyramids. Mayan Indians of Mexico and Central America built pyramids that were used as their temples.



   a. The Pyramid of the Sun in Teotihuacán, Mexico, measures about 700 feet on each side of its square base and is about 210 feet high. Estimate the slope that one of its faces makes with the base.
   b. The Great Pyramid in Egypt measures 756 feet on each side of its square base and was originally 481 feet high. Estimate the slope that one of its faces makes with the base.

49. **Look for a Pattern** Find the next number in each sequence.
   a. 6, 10, 15, 21, 28, . . .
   b. 1, 4, 9, 16, 25, . . .
   3 5 7 9

50. **Entertainment** In 1992, CDs passed cassette tapes as the most popular form for pre-recorded music. The graph at the right shows that most of the growth in sales from 1991 to 1998 has been in CDs. If x represents the year and y represents the number of cassettes and CDs sold in millions, find the rate of increase (slope) for both cassette tapes and CDs.



Source: Recording Industry Association of America

Mixed Review

51. Write $y = -2x + 4$ in standard form. (Lesson 2–2)

52. **Aviation** The air pressure in the cabin of a fighter jet decreases as the plane ascends. (Lesson 2–1)

Altitude (feet)	10,000	20,000	30,000	40,000	50,000
Air Pressure (lb/in^2)	10.2	6.4	4.3	2.7	1.6

a. Graph the data above.

b. Predict what you think the air pressure would be at 60,000 feet.

53. Solve $5 < 2x + 7 < 13$. (Lesson 1–7)

54. **SAT Practice** If $\frac{3+x}{7+x} = \frac{3}{7} + \frac{3}{7}$, then $x =$

A $\frac{3}{7}$ B 3 C 7 D 21 E 31

55. Solve $|7 + 3a| = 11 - a$. (Lesson 1–5)

56. Solve $0.75(8a + 20) - 2(a - 1) = 3$. (Lesson 1–4)

57. **Statistics** Find the mean, median, and mode for 100, 45, 105, 98, 97, and 101. (Lesson 1–3)

For Extra Practice, see page 879.

58. Simplify $\frac{1}{3}(15a + 9b) - \frac{1}{7}(28b - 84a)$. (Lesson 1–2)

59. Simplify $3 + (21 \div 7) \times 8 \div 4$. (Lesson 1–1)

SELF TEST

1. **Meteorology** When the temperature is 30° F, the speed of the wind makes the temperature feel colder. This is called the windchill factor. The chart below shows how the wind affects your perception of how cold it is when the temperature is 30°F. (Lesson 2–1)

a. State the domain and range of the relation shown in the table below.

Wind Speed (mph)	0	5	10	15	20	25	30	35	40
Windchill Factor (°F)	30	27	16	9	4	1	−2	−4	−5

b. Graph the relation. Is it a function?

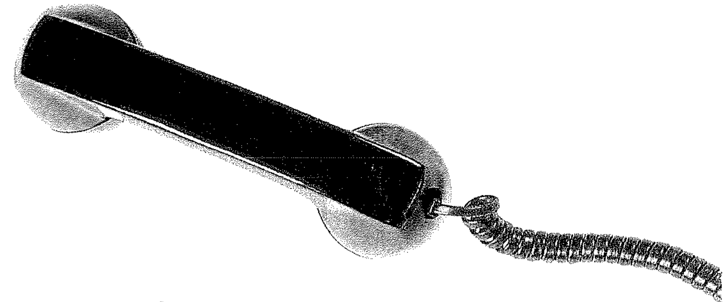
2. Find the value of $f(15)$ if $f(x) = 100x - 5x^2$. (Lesson 2–1)

3. Write $y = -6x + 4$ in standard form. (Lesson 2–2)

4. Graph $3x + 5y = 30$ using the x- and y-intercepts. (Lesson 2–2)

5. Graph the line that goes through $(4, -3)$ and is parallel to the line whose equation is $2x + 3y = 6$. (Lesson 2–3)

2-4 Writing Linear Equations



What YOU'LL LEARN

- To write an equation of a line in slope-intercept form given the slope and one point, and
- to write an equation of a line that is parallel or perpendicular to the graph of a linear equation.

Why IT'S IMPORTANT

You can use equations to explore relations in telecommunications and business.

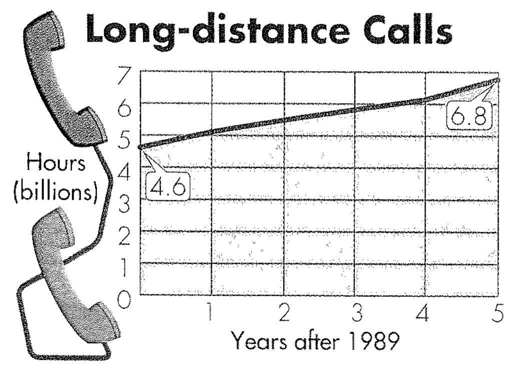
Real World APPLICATION

Telecommunications

Did you ever wonder why there are so many advertisements for long-distance telephone companies on television? Well, the long-distance market is huge! In 1989, there were 4.6 *billion* hours of long-distance calls made in the United States. Since 1989, the number of hours has increased by about 0.4 billion each year.

The equation $y = 0.4x + 4.6$ can be used to find y, the number of hours of long-distance calls (in billions) for any number of years, x, after 1989. This equation is graphed at the right. The y-intercept, 4.6, represents the number of billions of hours in 1989. The slope, 0.4, represents the yearly increase.

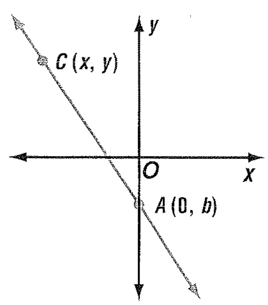
Long-distance Calls



Source: Commonwealth Associates

In Lesson 2–2, you learned if a function can be written in the form $y = mx + b$, then it is a linear function. In the example above, m is 0.4, which is the slope, and b is 4.6, which is the y-intercept. Is it always true that m is the slope and b is the y-intercept?

Consider the graph below. The line passes through points $A(0, b)$ and $C(x, y)$. Notice that b is the y-intercept of $\overleftrightarrow{AC}$. Suppose you need to find the slope of $\overleftrightarrow{AC}$. Substitute the coordinates of points A and C into the slope equation.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y - b}{x - 0} \quad \textit{Substitute } (0, b) \textit{ for } (x_1, y_1) \textit{ and } (x, y) \textit{ for } (x_2, y_2).$$

$$m = \frac{y - b}{x}$$

Now solve the equation for y.

$$mx = y - b \quad \textit{Multiply each side by } x.$$

$$mx + b = y \quad \textit{Add } b \textit{ to each side.}$$

$$y = mx + b \quad \textit{Symmetric property of equality}$$

When an equation is written in this form, it is in **slope-intercept form**.

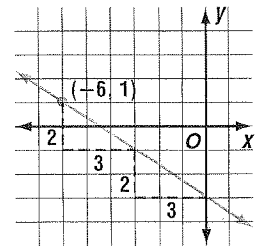
Slope-Intercept Form of a Linear Equation	**The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.**

If you are given the slope and the y-intercept of a line, you can find an equation of the line by substituting the values of m and b into the slope-intercept form. For example, if you know that the slope of a line is -4 and the y-intercept is 5, the equation of the line is $y = -4x + 5$, or, in standard form, $4x + y = 5$.

You can also use the slope-intercept form to find an equation of a line if you know the slope and the coordinates of a point on the line.

Example 1 **Find the slope-intercept form of an equation of the line that has a slope of $-\frac{2}{3}$ and passes through $(-6, 1)$.**

You know the slope and the x and y values of one point on the graph. Substitute for m, x, and y in the slope-intercept form.



$$y = mx + b$$
$$1 = \left(-\frac{2}{3}\right)(-6) + b$$
$$1 = 4 + b$$
$$-3 = b$$

The y-intercept is -3. So, the equation in slope-intercept form is $y = -\frac{2}{3}x - 3$.

TECHNOLOGY Tip

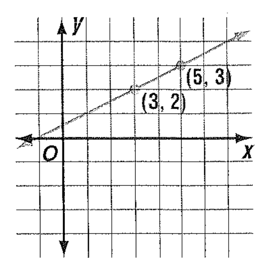
You also use the slope-intercept form when entering an equation into a graphing calculator.

If you are given the coordinates of two points on a line, you can use the **point-slope form** to find the equation of the line that passes through them.

Point-Slope Form of a Linear Equation	**The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.**

Example 2 **Find an equation of the line that passes through (3, 2) and (5, 3).**

First, use the two points given to find the slope of the line.



$$m = \frac{3-2}{5-3} \text{ or } \frac{1}{2}$$

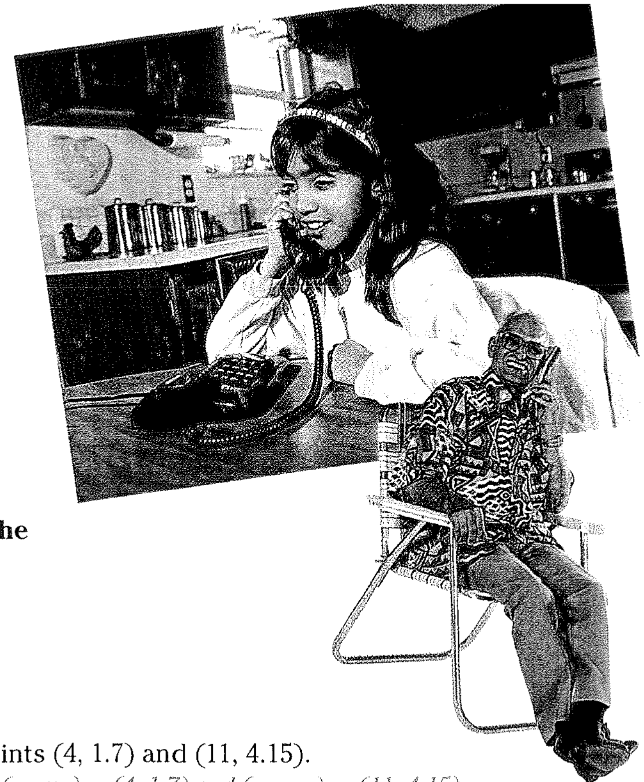
Then use the point-slope form to write the linear equation.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{1}{2}(x - 3) \quad \textit{Replace } m \textit{ with } \tfrac{1}{2} \textit{ and } (x_1, y_1) \textit{ with the coordinates of either point. We chose } (3, 2).$$
$$y - 2 = \frac{1}{2}x - \frac{3}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2}$$

The slope-intercept form of the equation of the line is $y = \frac{1}{2}x + \frac{1}{2}$.

In standard form, the equation is $x - 2y = -1$.

When changes in real-life situations occur at a linear rate, a linear equation can be used as a model for describing the situation.



Example 3

Real World APPLICATION

Business

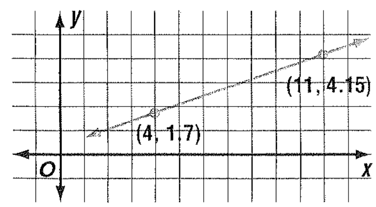
With a certain long-distance company, the price of a 4-minute long-distance call is \$1.70. An 11-minute call with the same company costs \$4.15.

a. Write a linear equation that describes the cost of these telephone calls. Assume that the changes increase linearly.

b. How much would a 20-minute telephone call cost?

a. The line passes through the points (4, 1.7) and (11, 4.15). Find the slope of the line. *Use $(x_1, y_1) = (4, 1.7)$ and $(x_2, y_2) = (11, 4.15)$.*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4.15 - 1.7}{11 - 4}$$
$$= \frac{2.45}{7} \text{ or } 0.35$$



Now use the point-slope form to write the linear equation.

$$y - y_1 = m(x - x_1)$$
$$y - 1.7 = 0.35(x - 4) \quad \textit{Replace } m \textit{ with 0.35 and } (x_1, y_1) \textit{ with (4, 1.7).}$$
$$y - 1.7 = 0.35x - 1.4$$
$$y = 0.35x + 0.3$$

The slope-intercept form of the equation of the line is $y = 0.35x + 0.3$.

b. Use the equation to find the cost of a 20-minute call.

$$y = 0.35x + 0.3$$
$$y = 0.35(20) + 0.3 \quad \textit{Replace } x \textit{ with 20.}$$
$$y = 7 + 0.3 \text{ or } 7.3$$

A 20-minute call would cost \$7.30.

Check this by estimating the cost from the graph.

Top Five List

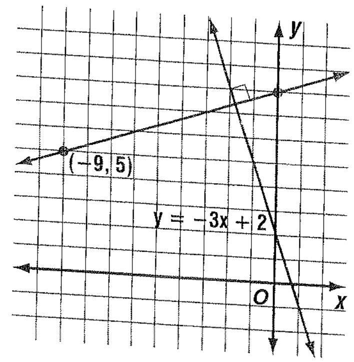
Countries with the most telephones per 100 people

1. Sweden, 68.4
2. Switzerland, 60.8
3. Canada, 59.2
4. Denmark, 58.3
5. United States, 56.1

The slope-intercept form can also be used to find equations of lines that are parallel or perpendicular.

Example **4** Write an equation of the line that passes through $(-9, 5)$ and is perpendicular to the line whose equation is $y = -3x + 2$.

INTEGRATION
Geometry



The slope of the given line is -3. Since the product of this slope and the slope of the perpendicular line is -1, the slope of the perpendicular line is $\frac{1}{3}$.

Use the slope-intercept form and the ordered pair $(-9, 5)$ to write the equation.

$y = mx + b$

$5 = \left(\frac{1}{3}\right)(-9) + b$ *Replace m with $\frac{1}{3}$ and (x, y) with (−9, 5).*

$5 = -3 + b$

$8 = b$

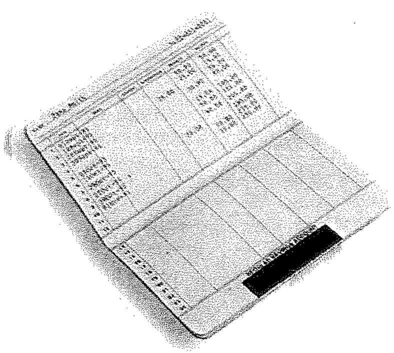
Use a graphing calculator to verify that the equation is correct.

An equation of the line is $y = \frac{1}{3}x + 8$.

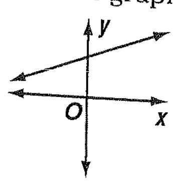
CHECK FOR UNDERSTANDING

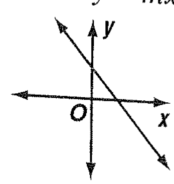
Communicating Mathematics

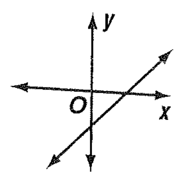
Study the lesson. Then complete the following.

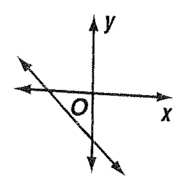


1. **Explain** how to write an equation in slope-intercept form and tell what each variable represents.
2. **Write** an equation of a line with slope of 5 and y-intercept of -4.
3. **Explain** how to write the equation of a line if you know the ordered pairs for two points on the line.
4. **Choose** the graph that shows $y = mx + b$, $m < 0$, $b > 0$.

a. 

b. 

c. 

d. 

5. **You Decide** Maribela thinks that the graphs of $y = 4x + 2$ and $4x - y = -2$ are different lines. Karen thinks that they are the same. Who is correct? Explain your reasoning.

MATH JOURNAL

6. **Assess Yourself** Suppose you have $250 in a savings account and decide to save an additional $10 each month. Write an equation to find the total amount, y, in your savings account after x months. Graph the equation. Then explain what the slope and y-intercept represent.

Guided Practice

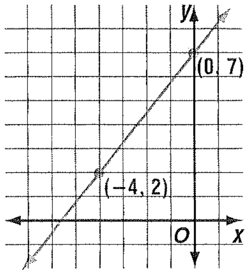
The slope and y-intercept of a line are given. Write the slope-intercept form of the equation for each line described.

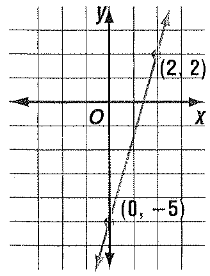
7. $m = 7, b = -3$
8. $m = 1.5, b = 0$
9. $m = -\frac{1}{3}, b = 4$

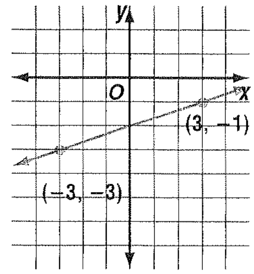
State the slope and y-intercept of the graph of each equation.

10. $y = 2x - 5$
11. $2y = 4x + 6$
12. $3x + 2y = 10$

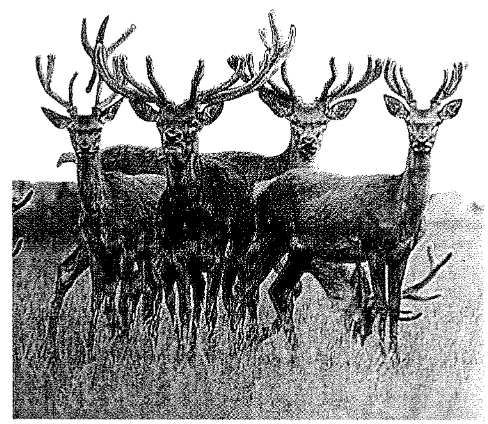
Write an equation in slope-intercept form for each graph.

13. 

14. 

15. 

Write an equation in slope-intercept form that satisfies each condition.



16. slope = 0.5, passes through (6, 4)
17. passes through (6, 1) and (8, −4)
18. passes through (0, 5) and is parallel to the graph of $y = 4x + 12$
19. passes through (0, −2) and is perpendicular to the graph of $y = x - 2$
20. **Ecology** A nature-preserve worker estimates there are 6000 deer in Sharon Woods Park. She also estimates that the population will increase by 75 deer each year thereafter. Write an equation that represents how many deer will be in the park in x years.

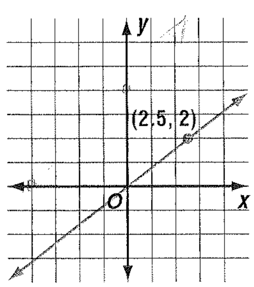
EXERCISES

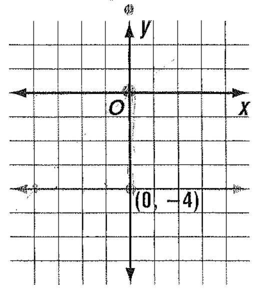
Practice

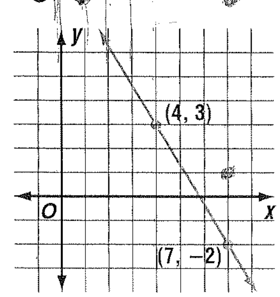
State the slope and y-intercept of the graph of each equation.

21. $y = -\frac{2}{3}x - 4$
22. $y = \frac{3}{4}x$
23. $-y = 0.3x + 6$
24. $4y = 2x - 10$
25. $-5y = 3x - 30$
26. $y = cx + d$

Write an equation in slope-intercept form for each graph.

27. 

28. 

29. 

Write an equation in slope-intercept form that satisfies each condition.

30. slope = 0.25, passes through (0, 4)
31. slope = −0.5, passes through (2, −3)
32. slope = 4, passes through the origin
33. passes through (−2, 5) and (3, 1)
34. passes through (7, 1) and (7, 8)
35. passes through (−2, −3) and (0, 0)
36. x-intercept = −4, y-intercept = 4

37. x-intercept $= \frac{1}{3}$, y-intercept $= -\frac{1}{4}$

38. x-intercept $= 0$, y-intercept $= 4$

39. passes through $(4, 6)$, parallel to the graph of $y = \frac{2}{3}x + 5$

40. passes through $(-2, 0)$, perpendicular to the graph of $y = -3x + 7$

41. passes through $(-3, -1)$, parallel to the line that passes through $(3, 3)$ and $(0, 6)$

42. passes through $(6, -5)$, perpendicular to the line whose equation is $3x - \frac{1}{5}y = 3$

Find the value of k in each equation if the given ordered pair is a solution of the equation.

43. $5x + ky = 8$, $(3, -1)$

44. $4x - ky = 7$, $(4, 3)$

45. $3x + 8y = k$, $(0, 0.5)$

46. $kx + 3y = 11$, $(7, 2)$

Graphing Calculator

Compare and contrast the graphs of each pair of equations. Use a graphing calculator to check your answers.

47. $2y = 6x + 14$
$3x - y = 6$

48. $2y - 4 = x$
$y = -2x + 2$

49. $3x + 5y = 15$
$y = -\frac{3}{5}x + 3$

Critical Thinking

50. **Geometry** Given ΔABC with $A(-6, -8)$, $B(6, 4)$, and $C(-6, 10)$, write the equation of the line containing the altitude from A. Remember that the altitude from A is perpendicular to $\overline{BC}$.

Applications and Problem Solving

Real World

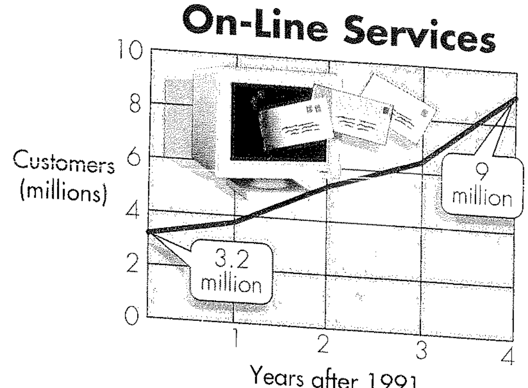
51. **Telecommunications** Refer to the application at the beginning of the lesson. Estimate the number of hours of long-distance calls that were made in 1999.

52. **Geometry** The equation $d = 180(c - 2)$ can be used to find the number of degrees, d, in any convex polygon with c sides.
   a. Write this equation in slope-intercept form.
   b. Identify the slope and y-intercept.
   c. Find the number of degrees in a pentagon.

53. **Science** At 20°F, 1 gallon of water weighs approximately 8.33 pounds. Write a linear equation to represent this situation.

54. **Business** Benny's Floral Shop charges \$3 per mile for delivery of a \$20 floral arrangement. Carmelita's Floral Shop charges \$2 per mile for delivery of a \$30 arrangement. When is it less expensive to buy from Carmelita?

55. **Technology** The graph shows the increase in the number of subscribers to on-line computer services from 1991 until 1995.
   a. Assuming that the increase is linear, write an equation to represent this situation.
   b. Predict the number of subscribers in the year 2000.
   c. What do the slope and y-intercept represent?



Source: Information and Interactive Services Report, Dataquest

Mixed Review

56. **Zoology** According to the *World Almanac*, a zebra can run at speeds up to 40 mph. Suppose a zebra could run at 40 mph for a long period of time. If at 1:30 P.M, a zebra had already traveled 32 miles, use the definition of slope to find how many miles the zebra could have traveled by 3:00 P.M. if it was running at top speed. (Lesson 2–3)

57. Determine if $g(x) = x(2 - x)$ is a linear function. (Lesson 2–2)

58. Find the value of $h(a + 1)$ if $h(x) = 3x - 1$ and $a = 4$. (Lesson 2–1)

Solve each inequality.

59. $|x - 2| \leq -99$ (Lesson 1–7)

60. $2(r - 4) + 5 \geq 9$ (Lesson 1–6)

61. Solve $|x - 3| = 2x$. (Lesson 1–5)

62. Write an algebraic expression to represent *one-fifth the sum of four and a number.* (Lesson 1–4)

63. **ACT Practice** What is the slope of the line that contains the points (15, 7) and (6, 4)?

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{3}{8}$ D $\frac{2}{3}$ E 3

For **Extra Practice**, see page 879.

64. Name the property illustrated by $11 + a = a + 11$. (Lesson 1–2)

65. Evaluate $(5a + 3d)^2 - e^2$ if $a = 3$, $d = 0.5$, and $e = 0.3$. (Lesson 1–1)

Mathematics and Society

Honeybee Counting

The excerpts below appeared in an article in *New Scientist* on March 4, 1995.

HONEYBEES CAN COUNT, ACCORDING TO two researchers in Germany. Lars Chittka and Karl Geiger of the Free University of Berlin say that bees measure the distance from their hive to a food source by counting landmarks as they fly past them . . . Chittka and Geiger trained bees to collect sugar solutions from a feeder more than 250 metres from their hive, which was in the centre of a large featureless meadow. Then they placed a series of obvious landmarks—tents about 3.5 metres high—along the bees' line of flight from the hive to the feeding station. To start with, there were four tents, spaced 75 metres apart, so the feeder was between the third and fourth landmarks. Even if sugar was also available between the second and third tents, most of the bees flew to the original feeder. The researchers then changed the number or position of the landmarks . . . Many of the bees simply stopped at the first feeder they encountered after the third landmark, even though this meant that they were nowhere near the feeder on which they had been trained . . . Chittka and Geiger say that bees clearly react to the number of landmarks they have passed, rather than to the distance they have flown, which means they must have the beginnings of an ability to count. ■

1. Under natural conditions, what types of landmarks might bees use to find their way to food sources?
2. If bees do in fact count landmarks, compare their behavior with the methods you use to find your way when traveling.
3. How is the bees' behavior similar to your use of graphs or measuring lines when illustrating or solving problems?

2-5 Integration: Statistics Modeling Real-World Data Using Scatter Plots

What YOU'LL LEARN

- To draw scatter plots, and
- to find and use prediction equations.

Why IT'S IMPORTANT

You can use scatter plots to display data, examine trends, and make predictions.

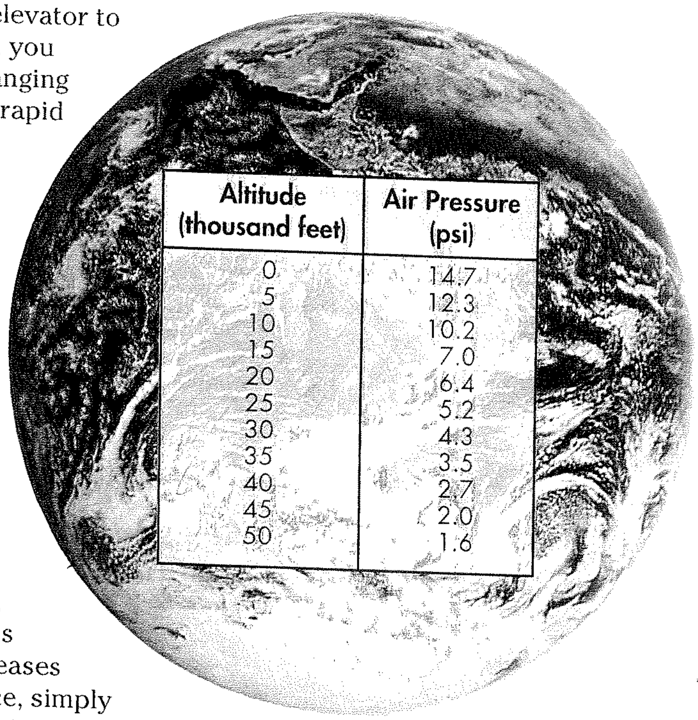
$\frac{4-7}{6-15} = \frac{-3}{-9}$

$-\frac{1}{3}$

CONNECTION

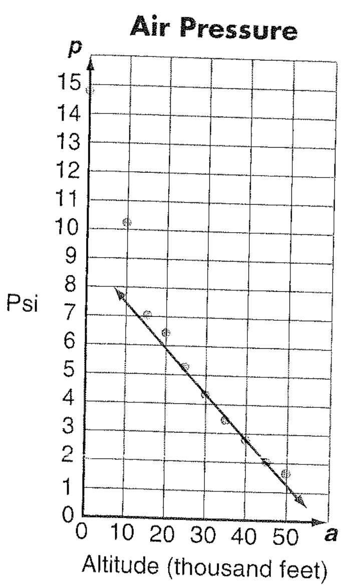
Science

When you ride the express elevator to the top floor of a skyscraper, you experience the effects of changing air pressure. Because of the rapid increase in altitude, the air pressure outside your eardrum is lower than the air pressure inside your eardrum. This difference in air pressure causes your eardrums to push out until some air finally forces its way out of your ears and goes pop!



Altitude (thousand feet)	Air Pressure (psi)
0	14.7
5	12.3
10	10.2
15	7.0
20	6.4
25	5.2
30	4.3
35	3.5
40	2.7
45	2.0
50	1.6

The weight of the air pressing down around us produces air pressure. Generally, air pressure is greatest near Earth's surface, where it averages 14.7 pounds per square inch (psi). It decreases as you move out toward space, simply because there is less air pressing down. The chart at the right shows average air pressure measured at different altitudes.



To determine the relationship between altitude and air pressure, graph the data points in a **scatter plot.** When real-life data is collected, the points graphed usually do not form a straight line, but may *approximate* a linear relationship. When this is the case, a **best-fit line** can be drawn, as shown at the left.

A **prediction equation** can be determined by employing a process similar to that used to determine an equation of a line when you know two points. You can use the prediction equation to estimate, or predict, one of the variables given the other.

Use two points from the line, (45, 2.0) and (25, 5.2), to find the slope.

$$m = \frac{2.0 - 5.2}{45 - 25} \text{ or } -0.16$$

Let x represent the altitude, and let y represent the air pressure. Use the slope and one of the points in the slope-intercept form to find a prediction equation.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

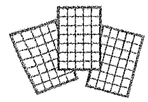
$y - 2 = -0.16(x - 45)$ *Let $(x_1, y_1) = (45, 2)$.*

$y = -0.16x + 9.2$

A prediction equation is $y = -0.16x + 9.2$.

MODELING MATHEMATICS

Head Versus Height

Materials:  tape measure  grid paper

In this activity, you will collect data to determine whether there is a relationship between the horizontal circumference of a person's head and his or her height.

Your Turn

a. Collect data from several of your classmates. Measure the circumference of each person's head and his or her height. Record the data as ordered pairs (circumference, height).

b. Graph the data in a scatter plot.

c. Choose two ordered pairs and write a prediction equation.

d. Explain the meaning of the slope in the prediction equation.

e. Predict the head size of a person who is 66 inches tall.

f. Predict the height of an individual whose head size is 18 inches.

Example 1

Real World APPLICATION

Keyboarding

Draw a scatter plot and find two prediction equations to show how keyboarding speed and experience are related. Predict the keyboarding speed in words per minute (wpm) of a student who has 11 weeks of experience.

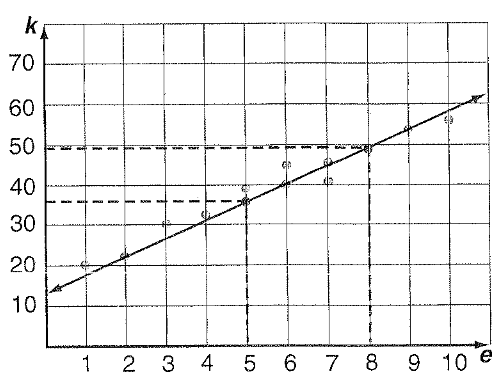
Experience (weeks)	4	7	8	1	6	3	5	2	9	6	7	10
Speed (wpm)	33	45	49	20	40	30	38	22	52	44	42	55

Explore The problem asks for two prediction equations.

Plan Make a scatter plot to determine the relationship between experience and speed. Since keyboarding speed is dependent on experience, the independent variable is the number of weeks of experience.

The best-fit line does not necessarily contain any points from the data.

Solve The pattern of points suggests a possible line that passes through (5, 36) and (8, 49).



$$m = \frac{49 - 36}{8 - 5}$$
$$= \frac{13}{3} \text{ or about } 4.3$$

Let e stand for experience in weeks. Let k stand for keyboarding speed in words per minute.

$$y = mx + b$$
$$k = 4.3e + b$$
$$36 = 4.3(5) + b$$
$$14.5 = b$$

One prediction equation is $k = 4.3e + 14.5$.

Another line can be suggested by using (2, 22) and (9, 52). Using these points results in a prediction equation of $t = 4.3e + 13.4$.

If a student had 11 weeks of experience, the first equation would predict that the student could type approximately 62 words per minute.

Examine Locate the ordered pair (11, 62) on the scatter plot. The point lies close to the graph of the prediction equation. Therefore, the solution is reasonable.

fabulous FIRSTS

Rita Moreno (1931–)

Rita Moreno, a Hispanic-American actress, singer, and dancer, was the first and only artist to win an Oscar®, a Tony, an Emmy, and a Grammy award.

The procedure for determining a prediction equation is dependent upon your judgment. You decide where to draw the best-fit line. You decide which two points on the line are used to find the slope and intercept. Your prediction equation may be different from someone else's. The prediction equation is used when a rough estimate is sufficient.

Example 2

Real World APPLICATION

Entertainment

Each year, the entertainment industry publishes a list of the top earners. The table at the right shows the top ten earners for 1998.

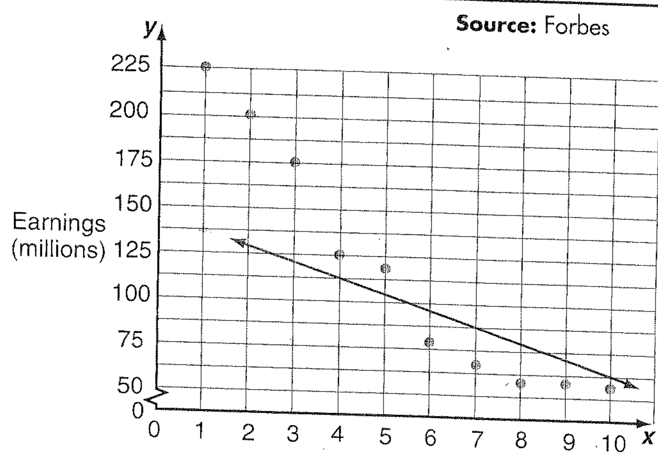
a. Draw a scatter plot for the data.

b. Predict the earnings of the fifteenth person on the list.

Rank	Person or Group	Earnings (millions)
1	Jerry Seinfeld	$225
2	Larry David	200
3	Steven Spielberg	175
4	Oprah Winfrey	125
5	James Cameron	115
6	Tim Allen	77
7	Michael Crichton	65
8	Harrison Ford	58
9	Rolling Stones	57
10	Master P	56.5

Source: Forbes

a.



b. The pattern of dots suggests a possible line that passes through (3,120) and (7, 87.5), as shown above.

Find the slope.

$$m = \frac{87.5 - 120}{7 - 3}$$

$$= \frac{-32.5}{4}$$

$$\approx -8$$

Then find a prediction equation.

$$y = mx + b$$

$$120 = -8(3) + b$$

$$144 = b$$

YEEEK!

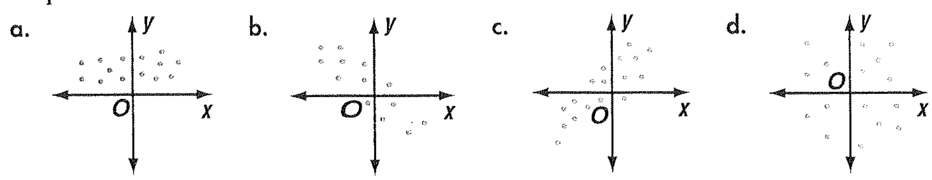
A prediction equation is $y = -8x + 144$. If an entertainer was fifteenth on the list, the equation would predict earnings of $-8(15) + 144$ or $24 million. *Do you think a line is a good predictor in this case?*

CHECK FOR UNDERSTANDING

Communicating Mathematics

Study the lesson. Then complete the following.

1. **Explain** why best-fit lines are helpful.
2. **Make** a scatter plot that shows the relationship between your test scores and the number of hours you do homework for each class you take.
3. **Choose** the scatter plot that has a prediction equation with a positive slope.



4. **You Decide** Refer to Example 1. Yoki thinks a prediction equation is accurate for values of e greater than 25. Juanita thinks a prediction equation probably won't be accurate at those values. Who is correct? Explain your reasoning.

MODELING MATHEMATICS

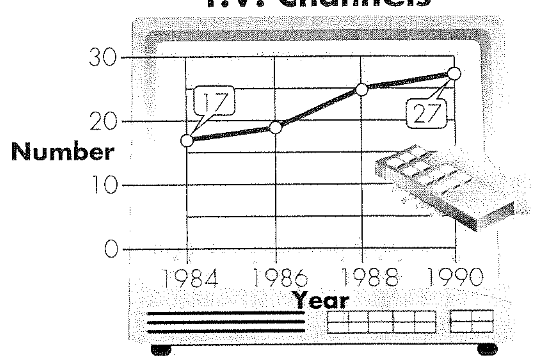
5. Collect data to determine whether there is any relationship between the circumference of a person's wrist and neck. If so, write a prediction equation for the relationship.

Guided Practice

In a study of the relationship between the height (h), in inches, and the ideal weight (w), in pounds, of adult men, a prediction equation is $w = 5h - 187$. Predict the ideal weight for each height.

6. 66 inches
7. 72 inches
8. 78 inches

T.V. Channels



Source: Nielsen Media Research

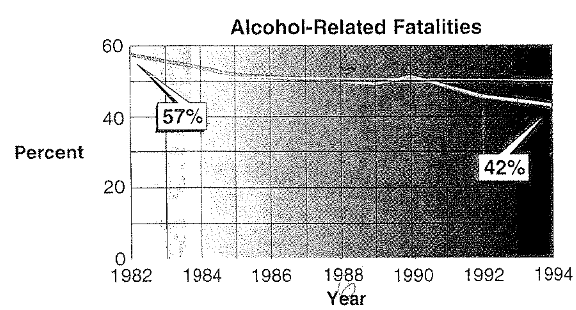
9. **Television** The graph at the left shows the average number of television stations that were received in U.S. households from 1984 until 1990.
   a. Copy the graph and draw a best-fit line.
   b. Find the slope and y-intercept of a prediction equation.
   c. Predict the average number of television stations that a household received in 1999.

EXERCISES

Applications and Problem Solving

Real World

10. **Safety** The graph at the right shows how the percent of traffic fatalities that were in alcohol-related crashes decreased from 1982 through 1994. Predict the percent of fatalities for the year 2000 if this trend continued.



Source: National Highway Traffic Safety Administration

11. **Health** The table below shows the age and systolic blood pressure for a group of people who recently donated blood.

Age	35	24	48	50	34	55	30	26	41	37
Blood Pressure	128	108	140	135	119	146	132	104	132	121

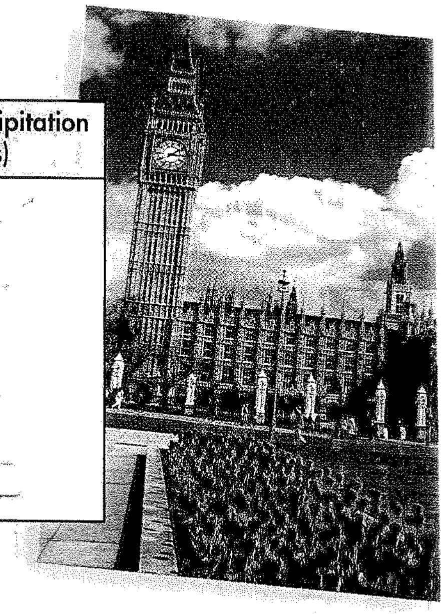
a. Draw a scatter plot to show how age x and systolic blood pressure y are related.

b. Write a prediction equation that relates a person's age to their approximate systolic blood pressure.

c. Find the approximate systolic blood pressure of a person 54 years old.

12. **Geography** The table below shows elevation and average precipitation for selected cities.

City	Elevation (feet)	Average Precipitation (inches)
Beirut, Lebanon	111	35
London, England	149	23
Paris, France	164	22
Montreal, Canada	187	41
Algiers, Algeria	194	30
Bucharest, Romania	269	23
Warsaw, Poland	294	22
Oslo, Norway	308	27
Rome, Italy	377	30
Toronto, Canada	379	32
Budapest, Hungary	394	24
Moscow, Russia	505	25



a. Draw a scatter plot to show how elevation e and precipitation p are related.

b. Write a prediction equation.

c. Check your equation by using the elevation of Dublin, Ireland, which is 155 feet with an average precipitation of 30 inches.

13. **Agriculture** Farmers will sometimes hold their crops from market until the price goes up to a level they think is satisfactory. The table below records the price per bushel and how many thousand bushels of wheat were sold at that price during a 10-day selling period in Iowa.

Price ($ / bushel)	3.84	3.66	3.87	3.96	3.60	4.05	3.63	3.60	3.72	3.87
Bushels Sold (thousands)	50	47	38	28	49	23	47	46	39	42

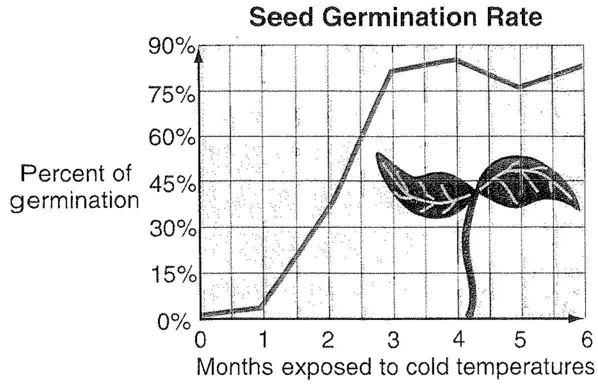
a. Draw a scatter plot and find a prediction equation for the data.

b. If the market price of wheat is $3.90/bushel next week, how many bushels of wheat can you predict will be sold?

c. Estimate what the price of wheat was when 25,500 bushels were sold.

Critical Thinking

14. **Botany** The graph shows the germination rate of batches of bristlecone pine seeds that have been exposed to cold temperatures. Explain why one best-fit line may not be the best solution to finding a prediction equation.



Mixed Review

15. Suppose there are three lines in a plane. Line a passes through Quadrants I, II, and IV. Line b passes through Quadrants I, III, and IV. Line c only passes through Quadrants I and II. Line a is perpendicular to Line b. All three lines pass through (3, 3). Given that the slope of Line b is 3, write the equations of the three lines in slope- intercept form. (Lesson 2–4)

16. Find $g(3)$ if $g(x) = -\frac{4x}{2} + 7$. (Lesson 2–1)

17. Solve $|x + 4| > 3$. (Lesson 1–7)

18. **ACT Practice** If one side of a triangle is three times as long as a second side and the second side is s units long, then the perimeter of the triangle can be:

A $3s$ **B** $4s$ **C** $5s$ **D** $6s$ **E** $7s$

For Extra Practice, see page 879.

19. Simplify $3(2x + 2) - 2(x - 1)$. (Lesson 1–2)

WORKING ON THE Investigation

Refer to the Investigation on pages 60–61.

To determine if their data are characteristic of a species or just random observations, naturalists often plot their observances on a graph such as a scatter plot. Then they draw conclusions from the patterns they observe.

1. Graph the data in your two charts in a single scatter plot. Use different colors for the data from each chart. Let one axis represent the height of the image and let the other axis represent the distance the viewer stood from the wall. Look for a pattern or relationship in the number pairs. How does the distance from the wall compare to the height of the image? Find a mathematical relationship between those two measures. Then explain your findings.
2. Write a function for each scope you used that relates distance from an image and its height. Explain the similarities and differences between the functions. Include a description of a reasonable domain and range in your discussion.
3. How does the size of the scope compare with the other data? Find a ratio between the width and the length of each scope. Explain how the equations relate to the sizes of the scopes.
4. Find two best-fit lines for the points on the scatter plot. How does the ratio for the size of the scope compare to the slope of the corresponding best-fit line? Explain the relationships that exist.

Add your results to your Investigation Folder.

2–5B Graphing Technology Lines of Regression

An Extension of Lesson 2–5

You can use a graphing calculator to draw scatter plots and a line that best fits the points in the scatter plot. This line is called a **regression line.** Once you have drawn the regression line, you can use the TRACE feature on the graphing calculator to make predictions about the data.

Example

Some scientists believe that global warming is a result of carbon dioxide emissions from fuel consumption. The table below shows world carbon dioxide emissions for 1950–1990. Draw a scatter plot and regression line to show how the year is related to the level of carbon dioxide emissions.

Year	Emissions (millions of metric tons)
1950	6002
1955	7511
1960	9475
1965	11,556
1970	14,989
1975	16,961
1980	19,287
1985	19,672
1990	22,588

Source: Carbon Dioxide Information Analysis Center

Let the independent variable be the number of years since 1940, and let the dependent variable be the emissions. First, set the window parameters. The values of the data suggest a viewing window of [0, 55] by [5000, 25000] with Xscl = 5 and Yscl = 1000.

Next, enter the data. Press STAT 1 to display lists for storing data. If old data has previously been stored, enter STAT 1 ▲ CLEAR ENTER ▶ ▲ CLEAR ENTER to clear the lists. The years will be entered into column L1.

Enter: 10 ENTER 15 ENTER 20 ENTER . . . 50 ENTER

Use ▶ to move the cursor to column L2 and then enter the carbon dioxide emissions data.

Enter: 6002 ENTER 7511 ENTER 9475 ENTER . . . 22588 ENTER

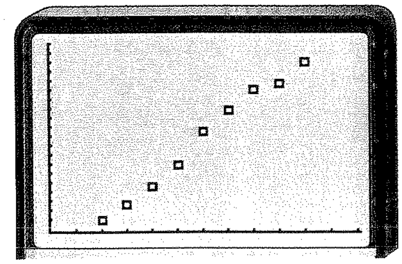
We are now ready to draw the scatter plot.

Enter: 2nd STAT PLOT 1 ENTER ▼ ENTER ▼ ENTER ▼ ▶ ENTER ▼ ENTER GRAPH

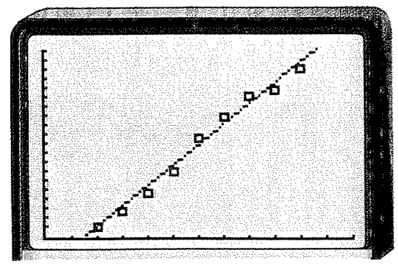
(continued on the next page)

Next, graph the equation for the regression line.

Enter: STAT ▶ 5 2nd L1 , 2nd L2 ENTER Y= VARS 5 ▶ ▶ 7 GRAPH



The TRACE feature allows you to move a cursor along the graph or scatter plot and read the coordinates of the points. Press TRACE and any of the arrow keys to observe what happens. Approximately what would you expect the carbon dioxide emissions to have been in 1971?



EXERCISES

Use a graphing calculator to draw a scatter plot and a regression line for the data in each table.

1.

x	−2	1	4	10	23	25
y	1	−0.5	2	−4.5	−10	−11.5

2.

x	0.2	1	3	4	5	8
y	0.11	0.31	0.9	1.25	1.75	2.3

3. **Employment** The table below shows the average yearly incomes (in dollars) of men and women in the United States.

Year	1960	1970	1980	1985	1990	1995
Women	3257	5323	11,197	15,624	19,822	22,497
Men	5368	8966	18,612	24,195	27,678	31,496

Source: U.S Census Bureau

a. Use a graphing calculator to draw a scatter plot and regression line to show how the year is related to women's salaries for 1960 to 1995. Let the independent variable be the years since 1950, and let the dependent variables be the incomes. Predict the average salary for women in the year 2005.

b. Repeat part a with the men's salaries.

c. Compare and contrast the data in the two scatter plots.

2-6 Special Functions

What YOU'LL LEARN

- To identify and graph special functions.

Why IT'S IMPORTANT

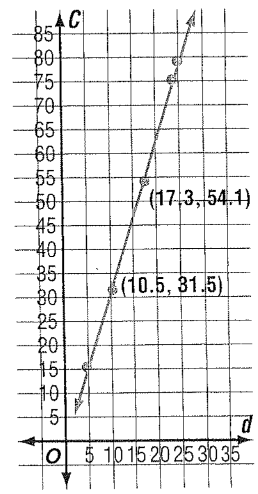
You can use functions to explore relations in technology and finance.

INTEGRATION
Geometry

Rebeca's little brother, Jorge, was doing a project for his mathematics class in which he measured the diameter and circumference of several circular objects. The data are shown below.

Diameter (centimeters)	4.8	10.5	17.3	23.8	25.0
Circumference (centimeters)	15.4	31.5	54.1	75.2	79.1

Rebeca helped Jorge graph the data, and it appeared to be a linear function. Using the points (10.5, 31.5) and (17.3, 54.1), the slope of the prediction equation is about 3.32. It appears that the y-intercept is the origin.

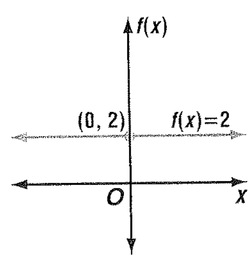


Of course, you know that the formula $C = \pi d$ describes the relationship between the diameter and circumference of a circle. Notice that the slope, 3.32, is close to π.

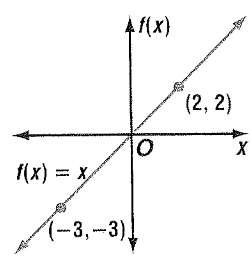
Whenever a linear function in the form $y = mx + b$ has $b = 0$ and $m \neq 0$, the function is called a **direct variation.** In this situation, the circumference varies directly as the diameter. In other words, as the diameter gets larger, the circumference also gets larger.

There are other special cases of linear functions. Two of these, the **constant function** and the **identity function** are shown below.

constant function
$m = 0$



identity function
$m = 1, b = 0$



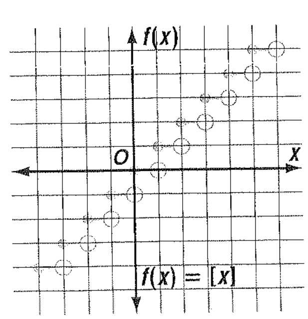
CAREER CHOICES

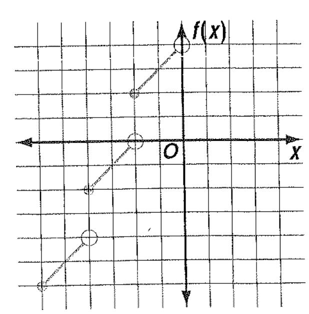
A **financial planner** helps clients plan how to make the most from their money. In their careers, they frequently translate data into various graphical forms.

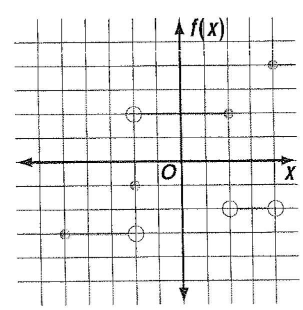
For more information, contact:

American Economics Association
1313 21st Ave.
Nashville, TN 37212

Step functions like the ones shown below are also related to linear functions. The open circle means that the point is *not* included in the graph.







One type of step function is the **greatest integer function.** The symbol $[x]$ means *the greatest integer not greater than x*. For example, $[7.3] = 7$ and $[-1.5] = -2$ because $-1 > -1.5$. The greatest integer function is given by $f(x) = [x]$. Its graph is the first step function graph shown on the previous page.

The graphs of step functions are often used to model real-world problems.

Example 1

Real World APPLICATION

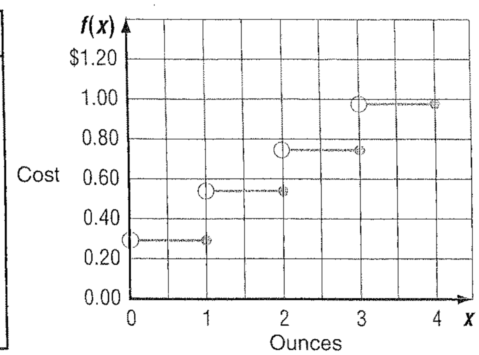
Postal Service

When you go to the post office to mail a first-class letter, you may need to ask the clerk how much it will cost to mail it—that is, how much postage is required. In 2000, first-class mail cost 33¢ for the first ounce and 22¢ for each additional ounce. So if your letter weighed one ounce or less, it cost you 33¢ to mail. A letter weighing 1.1 ounces cost 33 + 22 or 55¢ to mail. Graph the function that describes the relationship between the number of ounces and the cost of postage.

The equation that describes this function is $f(x) = 0.33 + 0.22[x - 0.1]$, where x is the number of ounces.

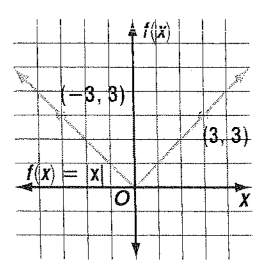
Make a table of values to help you draw the graph.

x	$0.33 + 0.22[x - 0.1]$	$f(x)$
0.1	0.33 + 0	0.33
0.7	0.33 + 0	0.33
1.0	0.33 + 0	0.33
1.5	0.33 + 0.22(1)	0.55
1.9	0.33 + 0.22(1)	0.55
2.0	0.33 + 0.22(1)	0.55
2.3	0.33 + 0.22(2)	0.77
2.8	0.33 + 0.22(2)	0.77
3.0	0.33 + 0.22(2)	0.77
3.2	0.33 + 0.22(3)	0.99



Another function that is closely related to linear functions is the **absolute value function.** Consider $f(x) = |x|$ or $y = |x|$. Look for a pattern when studying the values in the chart.

x	-3	-2	-1	0	1	2	3
$f(x)$	3	2	1	0	1	2	3



You can see that when x is positive or zero, the absolute value function looks like the graph of $y = x$. When x is negative, the absolute value function looks like the graph of $y = -x$.

LOOK BACK

You can refer to Lesson 1-5 for information about absolute value equations.

Example 2 **Graph $f(x) = |x| + 2$ and $g(x) = 3|x| + 2$ on the same coordinate plane. Determine the similarities and differences in the two graphs.**

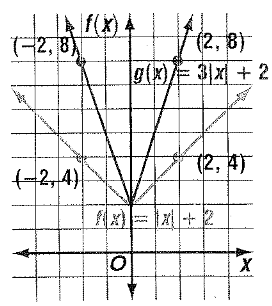
Find several ordered pairs for each function.

x	$\lvert x\rvert + 2$
−2	4
−1	3
0	2
1	3
2	4

→

x	$3\lvert x\rvert + 2$
−2	8
−1	5
0	2
1	5
2	8

Graph the points and connect them. Both graphs have the same y-intercept. The graph of $g(x) = 3|x| + 2$ is narrower than the graph of $f(x) = |x| + 2$.



Recall that families of graphs are groups of graphs that display one or more similar characteristics. Graphs of absolute value functions may also display similar characteristics.

EXPLORATION **GRAPHING CALCULATORS**

When a linear equation is written in the form $y = mx + b$, m is the slope, and b is the y-intercept. In this Exploration, you will use a graphing calculator to investigate absolute value functions of the form $y = a|x|$.

The parent graph of most families of absolute value functions is the graph of $y = |x|$.

Your Turn

a. Graph $y = |x|$, $y = 2|x|$, $y = 3|x|$, and $y = 5|x|$ on the same screen.

b. Describe this family of graphs. What pattern do you see?

c. In $y = mx + b$, as m increases, the slope of the line also increases. Describe how the graph of $y = a|x|$ changes as a increases.

d. Write an absolute value function whose graph is between the graphs of $y = 2|x|$ and $y = 3|x|$.

e. Graph $y = |x|$ and $y = -|x|$ on the same screen. Then graph $y = 2|x|$ and $y = -2|x|$ on the same screen.

f. Describe this family of graphs. What pattern do you see?

g. In a linear equation $y = mx + b$ with $m < 0$, the line falls to the right. Describe how the graph of $y = a|x|$ changes when $a < 0$.

h. Write an absolute value function whose graph opens down.

CHECK FOR UNDERSTANDING

Communicating Mathematics

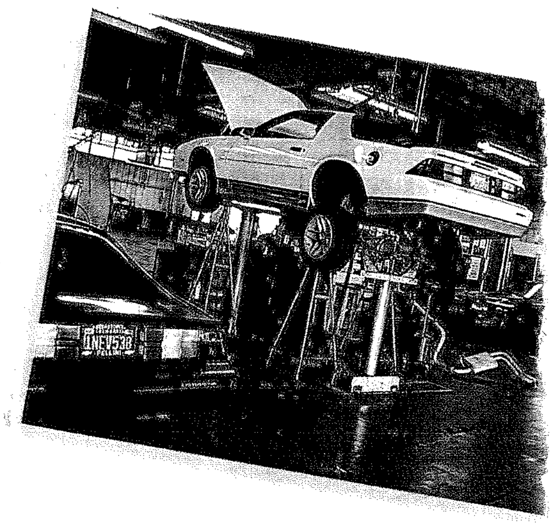
Study the lesson. Then complete the following.

1. **Explain** why the slope was not 3.14 in the application at the beginning of the lesson.
2. **Choose** which function is a direct variation.
   a. $f(x) = 4$ b. $y = -3$ c. $y = |x| + 2$ d. $f(x) = 5x$
3. **Explain** why the value of $[4.3]$ is 4, but the value of $[-4.3]$ is -5.
4. **Describe** the pattern for graphing an absolute value function.
5. **Compare and contrast** the graphs of $f(x) = |x|$ and $f(x) = |x| - 2$.

Guided Practice

Identify each function as C for constant, D for direct variation, A for absolute value, or G for greatest integer function. Then graph each function.

6. $f(x) = |3x - 2|$
7. $g(x) = -[x]$
8. $f(x) = 2x$
9. $h(x) = 0.5$
10. If $g(x) = |x - 5|$, find $g(4)$.
11. If $f(x) = 2[x - 1]$, find $f(6.9)$.



12. **Business** The Fix-It Auto Repair Shop has a sign in the Service Department that states that the labor costs are \$35 per hour or any fraction thereof. What type of function does this relationship represent?

EXERCISES

Practice

If $h(x) = [2x + 3]$, find each value.

13. $h(2)$
14. $h(-2)$
15. $h(-2.3)$
16. $h\left(\frac{1}{4}\right)$

Identify each function as C for constant, D for direct variation, A for absolute value, or G for greatest integer function. Then graph each function.

17. $h(x) = x$
18. $f(x) = |2x|$
19. $g(x) = -3$
20. $f(x) = [2x + 1]$
21. $f(x) = \left|x - \frac{1}{4}\right|$
22. $f(x) = -\frac{2}{3}x$
23. $f(x) = x + 3$
24. $f(x) = |x + 3|$
25. $f(x) = [x + 3]$
26. $g(x) = |x| + 3$
27. $g(x) = [x] + 3$
28. $g(x) = 3|x|$

Graph each pair of equations on the same coordinate plane. Discuss the similarities and differences in the two graphs.

29. $y = |x + 2|, y = |x - 2|$
30. $y = |x| + 4, y = |x| - 4$
31. $y = |x + 2|, y = |x + 2| - 1$
32. $y = 2[x], y = [2x]$
33. $y = [x + 5], y = [x] + 5$
34. $y = |2x|, y = 2|x|$
35. $y = -2|4x|, y = 4|-2x|$
36. $y = -3[x], y = [-3x]$

Graph each equation.

37. $y = [\,|x|\,]$ **38.** $y = |\,[x]\,|$ **39.** $y = x - [x]$ **40.** $y = x + |x|$

Graphing Calculator

Use a graphing calculator to solve each equation graphically.

41. $|2x - 4| = 6$ **42.** $|x + 2| = 4$

Critical Thinking

43. Compare and contrast the graphs of $y = |x|$ and $|y| = x$. Identify the domain and range of each.

Applications and Problem Solving

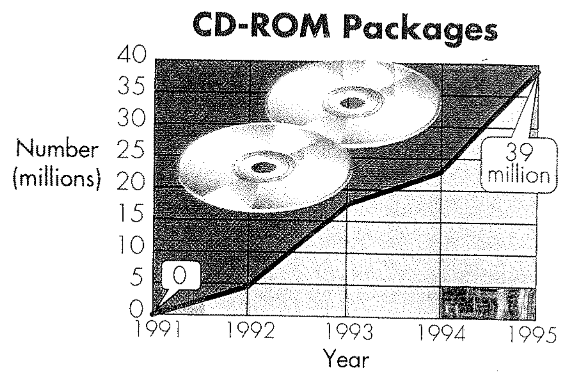
Real World

44. Technology The graph shows the number of CD-ROM multimedia packages shipped between 1991 and 1995.

a. Write an equation to represent a best-fit line for this graph.

b. What type of function does this relationship represent?

c. Predict the number of software packages that were shipped in 1999.



Source: Information and Interactive Services Report, Dataquest

45. Transportation When the Westerville North boy's basketball team went to the Ohio state tournament, the school chartered buses so that the student body could attend the games. Each bus held a maximum of 60 students.

a. Make a graph that shows the relationship between the number of students that went to the game by bus x and the number of buses that were needed y.

b. What type of function does this relationship represent?

46. Finance Lupe earns $5.25 per hour working at a video store after school. About 25% of her earnings are taken out for taxes and other deductions.

a. Write an equation that shows the relationship between the number of hours worked per week and Lupe's take-home pay.

b. About how many hours will Lupe need to work to take home $100?

c. What type of function does this relationship represent?

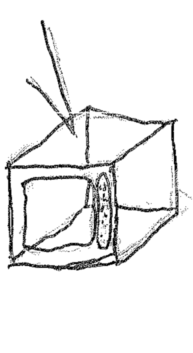
Mixed Review

47. Smoking The table shows the percent of people ages 18 to 24 who smoked for selected years. (Lesson 2–5)

a. Draw a scatter plot and find a prediction equation for the data.

b. Estimate what percent of the population in this age group will be smoking in the year 2005 if these trends continue.

Year	People Ages 18–24 Who Smoke
1965	45.5%
1970	38.0
1974	37.8
1978	34.4
1979	34.4
1980	33.3
1983	34.2
1985	29.3
1987	27.1
1988	25.9
1990	24.5
1993	25.8
1995	24.8



48. Write an equation in slope-intercept form for the line with slope $\frac{2}{3}$ that passes through $(6, -5)$. (Lesson 2–4)

49. **SAT Practice** If the product of $(2 + 3)$, $(3 + 4)$, and $(4 + 5)$ is equal to three times the sum of 40 and x, then $x =$

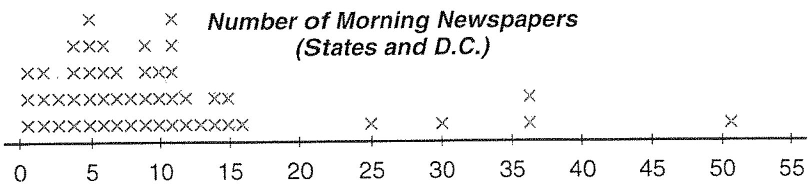
A 43 **B** 65 **C** 105 **D** 195 **E** 315

50. Graph $b = 3a - 2$. (Lesson 2–2)

51. Find $h(-1)$ if $h(x) = \frac{x^2 + 2x - 5}{x^2 - 2}$. (Lesson 2–1)

52. Solve $28 - 6y < 23$. (Lesson 1–6)

53. **Statistics** Use the line plot below to answer each question. (Lesson 1–3)



a. What is the greatest number of newspapers in one state?
b. What is the least number of newspapers in one state?
c. How many states have 10 to 20 newspapers?
d. How many newspapers do most states have?

For Extra Practice, see page 880.

54. Simplify $3 + \{8 \div [9 - 2(4)]\}$. (Lesson 1–1)

WORKING ON THE Investigation

Refer to the Investigation on pages 60–61.

Many species react when an invader to their territory comes too close. They often become defensive and may attack. So, it is often handy for naturalists to have equipment, such as a scope, to observe animals from a distance without creating a threat to the animal or to the naturalist's safety.

1. Select one of the scopes that you used in your experiment. Suppose you were 60 feet from a giraffe and the animal's image exactly filled the viewer. How tall would the giraffe be?
2. Using a second scope that was a different size than your first scope, how would the image of the giraffe change when looking through the second tube? Where would you need to stand to see the entire image of the giraffe fill the view of the second scope? Can you see the same image if you are standing in the same location and looking through two view tubes of different sizes?
3. Explain the methods you discovered for determining heights using the view tubes. Analyze those methods and discuss your findings in writing.

Add the results of your work to your Investigation Folder.

2–7A Graphing Technology Linear Inequalities

A Preview of Lesson 2–7

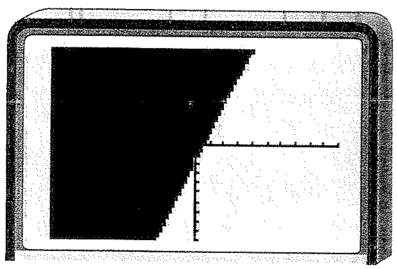
You can graph inequalities with a graphing calculator by using the Shade(command located in the DRAW menu. You must enter *two* functions to activate the shading. The first function defines the lower boundary of the region to be shaded. The second function defines the upper boundary of the region. The calculator will graph both functions and shade between the two. If the inequality is "$y \leq$", you can use the Ymin window value as the lower boundary since the points that satisfy the inequality are below the graph of the related equation. If the inequality is "$y \geq$", you can use the Ymax window value as the upper boundary since the points that satisfy the inequality are above the graph of the related function.

Before using the Shade(option, be sure to clear equations stored in the Y = list.

Example **Graph $y \geq 3x - 2$ in the standard viewing window.**

The inequality asks for points where y is greater than or equal to $3x - 2$, so we will use Ymax, or 10, as the upper boundary and $3x - 2$ as the lower boundary. This will shade all points on the graphics screen between $y = 3x - 2$ and $y = 10$.

Enter: $\boxed{\text{ZOOM}}$ 6 $\boxed{\text{2nd}}$ $\boxed{\text{DRAW}}$ 7
3 $\boxed{X,T,\theta,n}$ $\boxed{-}$ 2 $\boxed{,}$ 10
$\boxed{)}$ $\boxed{\text{ENTER}}$



Since both the x- and y-intercepts of the line $y = 3x - 2$ are within the current viewing window, the graph of the inequality is complete.

EXERCISES

Use a graphing calculator to graph each inequality. Then sketch your graph on a sheet of paper.

1. $y \geq 3$	**2.** $y \geq x + 2$	**3.** $y \leq -2x - 4$
4. $y > \frac{1}{3}x + 7$	**5.** $y \geq \frac{1}{2}x - 3$	**6.** $x - 7 \leq y$
7. $y + 1 \leq 0.5x$	**8.** $y - 3 > -2x$	**9.** $2 \geq x - 2y$

2–7 Linear Inequalities

What YOU'LL LEARN

- To draw graphs of inequalities in two variables.

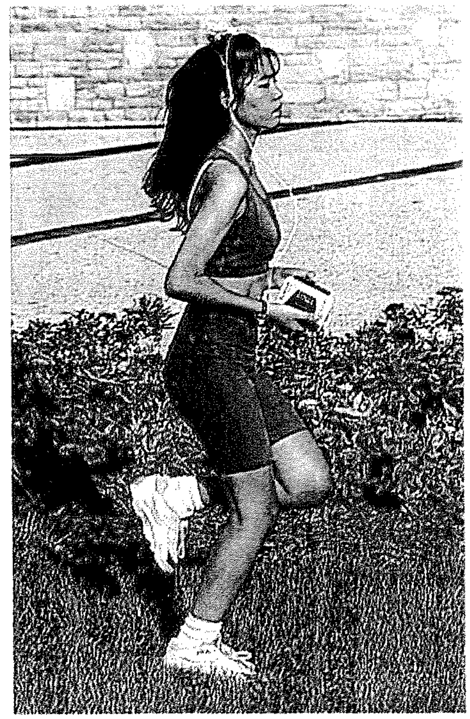
Why IT'S IMPORTANT

You can use inequalities to solve problems involving manufacturing and shopping.

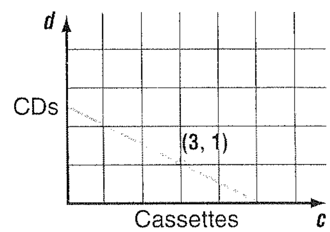
Real World APPLICATION

Shopping

Suppose you have \$35 to spend and want to purchase some cassettes that cost \$7 and some CDs that cost \$14. If c represents the number of cassettes, and d represents the number of CDs, the equation $7c + 14d = 35$ describes the different ways for you to spend *exactly* \$35.

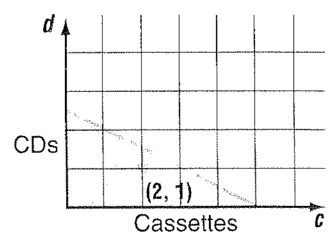


The graph at the right shows all of the ordered pairs that are solutions of the equation $7c + 14d = 35$. One solution is (3, 1).

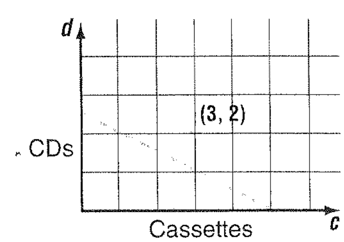


However, when you're shopping, it's not necessary to spend exactly \$35. You want to spend *at most* \$35. You know that if you spend *more than* \$35, you won't have enough money left to go out with your friends. These situations can be represented by the linear inequalities $7c + 14d \leq 35$ and $7c + 14d > 35$, respectively. The graph of $7c + 14d = 35$ separates the coordinate plane into two regions. The line is called the *boundary* of the regions. To graph an inequality, first graph the boundary and then determine which region to shade.

The graph of $7c + 14d \leq 35$ contains points for which the cost of the cassettes and CDs is less than or equal to 35. For example, the ordered pair (2, 1) is in the shaded region and therefore, is one of the solutions. Note that the boundary is a solid line. If the inequality uses the symbols $\leq$ or $\geq$, the boundary is solid to show that it is included.



The graph of $7c + 14d > 35$ contains points for which the cost of the cassettes and CDs is greater than \$35. For example, the ordered pair (3, 2) is in the shaded region and therefore, is one of the solutions. Note that the boundary is a dashed line. If the inequality uses the symbols $<$ or $>$, the boundary is dashed to show that it is *not* included.



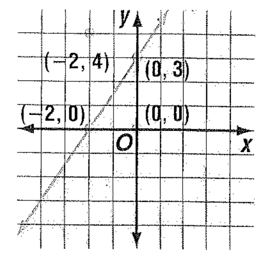
You can graph an inequality by following these steps.

1. Graph the boundary. Determine whether it should be solid or dashed.
2. Test a point in each region.
3. Shade the region whose ordered pair results in a true inequality.

Example 1 **Graph $2y - 3x \leq 6$.**

The boundary will be the graph of $2y - 3x = 6$. Let's use intercepts to graph the boundary more easily.

x-intercept	*y-intercept*
$2(0) - 3x = 6$	$2y - 3(0) = 6$
$-3x = 6$	$2y = 6$
$x = -2$	$y = 3$



Since the inequality is "less than or equal to," draw a solid line connecting the two intercepts. This is the boundary.

Try to test the origin because it is easy to substitute 0 for x and y.

Now test a point in each region.

Try (0, 0).

$2(0) - 3(0) \leq 6$

$0 \leq 6$ true

Try (−2, 4).

$2(4) - 3(-2) \leq 6$

$8 - (-6) \leq 6$

$14 \leq 6$ false

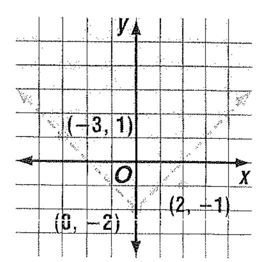
The region that contains (0, 0) should be shaded.

Sometimes inequalities contain absolute value. When this occurs, you must consider that the expression within the absolute value bars may be positive *or* negative.

Example 2 **Graph $y > |x| - 2$.**

This absolute value function has two conditions to consider.

when $x < 0$		*when $x \geq 0$*
$y > -x - 2$	and	$y > x - 2$



Graph each inequality for the specified values of x. The lines will be dashed.

When one ordered pair results in a true inequality, it is not necessary to test a point in the other region.

Test (0, 0).

$y > |x| - 2$

$0 > |0| - 2$ *Note that the boundary is not included.*

$0 > -2$ true

The shaded region should include (0, 0).

Inequalities can sometimes be used to analyze a situation and determine the trends in business and profitability.

Example 3

Real World APPLICATION

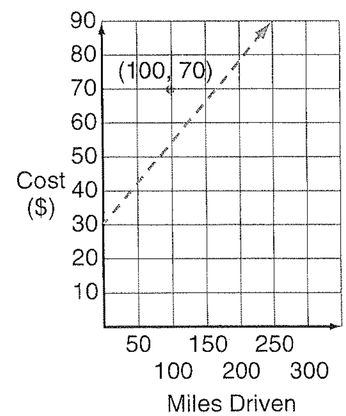
Business

Amanda Harris wants to rent a car for a business trip of about 100 miles. Reasonable Car Rental advertises that their daily rental rate is \$30 plus \$0.25 a mile. She knows that Executive Car Rental charges \$70 for a car rental with 100 free miles. Which company has the better rate?

Explore One way to solve the problem is to make a graph for one company that shows the relationship between the number of miles driven d and the total cost of the rental r.

Plan The initial cost of a car from Reasonable Car Rental is \$30. Since this is the point at which no miles are driven, it is the y-intercept of the graph. The slope would be the rate of change in the total cost. In this case, the rate is \$0.25 per mile. Thus, the slope is 0.25, and an equation of the line is $r = 0.25d + 30$.

Solve Graph the equation $r = 0.25d + 30$. Then graph the point (100, 70), which represents Executive Car Rental's charge of \$70 for a rental of 100 miles.



Since the point (100, 70) lies above the boundary, it is in the graph of $r > 0.25d + 30$. For all points in this region, the cost of renting a car is greater than the cost of renting from Reasonable Car Rental. Thus, Reasonable Car Rental has a better rate than Executive Car Rental for a car rental of 100 miles.

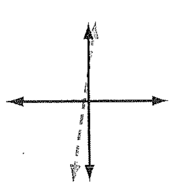
Examine For a rental of 100 miles, Reasonable Car Rental charges $0.25(100) + 30$ or \$55. This is a better rate than \$70. Thus, the solution seems reasonable.

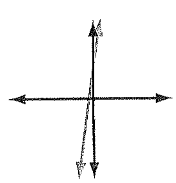
CHECK FOR UNDERSTANDING

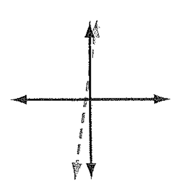
Communicating Mathematics

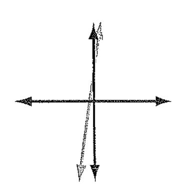
Study the lesson. Then complete the following.

1. **Explain** how you decide if the boundary of an inequality should be solid or dashed.
2. **Explain** why (0, 2) is a solution of $y \geq -8x + 2$.
3. **Choose** the graph of $y < 3x + 2$.

a. 

b. 

c. 

d. 

4. **Compare** the graphs of $y = |x| + 1$ and $y < |x| + 1$.

Guided Practice

State which points, (0, 0), (3, −4), or (−1, 3), satisfy each inequality.

5. $x + 2y < 5$
6. $4x + 3y \leq 0$
7. $5x - y \geq 6$

Graph each inequality.

8. $y > 2$
9. $x - y \geq 0$
10. $y > 2x$
11. Graph $y > |2x|$.
12. Graph all the points on the coordinate plane to the right of $x = 4$. Write an inequality to describe these points.

EXERCISES

Practice

Graph each inequality. $y < 3$

13. $x + y > -5$
14. $y + 1 < 4$
15. $y > 6x - 2$
16. $x - 5 \leq y$
17. $y \geq -4x + 3$
18. $y - 2 < 3x$
19. $y > \frac{1}{3}x + 5$
20. $y \geq \frac{1}{2}x - 5$
21. $3 \geq x - 3y$
22. $y \leq |x|$
23. $y + |x| < 3$
24. $y > |4x|$
25. Graph all the points on the coordinate plane to the left of $x = -2$. Write an inequality to describe these points.
26. Graph all the points in the first quadrant bounded by the two axes and the line $x + 2y = 5$.
27. Graph all second quadrant points bounded by the lines $x = -3$, $x = -6$, and $y = 4$.
28. Graph all points in the fourth quadrant bounded by the two axes and the lines $3x - y = 4$ and $x - y = 5$.

Graph each inequality.

29. $|x| \leq |y|$
30. $|x| - |y| = 1$
31. $|x| + |y| \geq 1$
32. $|x + y| > 1$

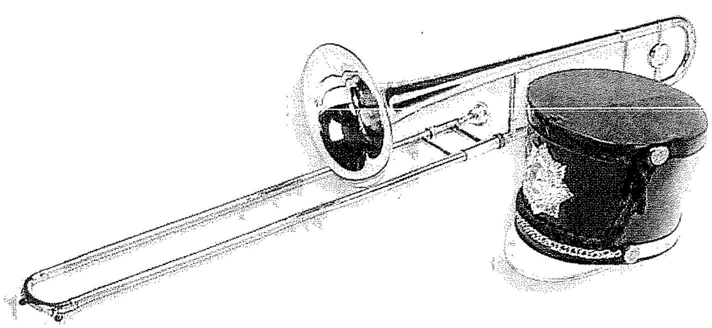
Critical Thinking

33. Describe the graph of $|y| < x$.

Applications and Problem Solving

Real World

34. **Business** The No-Drip Sponge Company must produce a certain number of sponges each day to keep the assembly-line staff busy. If production falls, then layoffs may be necessary. The inequality that describes this relationship is $s > 50e + 25$, where e is the number of employees and s is the number of sponges. Graph this inequality.



35. **Drama** Tickets for the Kingwood High School Drama Club's production of *The Music Man* cost \$5 for adults and \$4 for students. In order to cover expenses, at least \$2500 worth of tickets must be sold.
    a. Write an inequality that describes this situation.
    b. Graph the inequality.
    c. If 175 adult and 435 student tickets are sold, will the Drama Club cover its expenses?

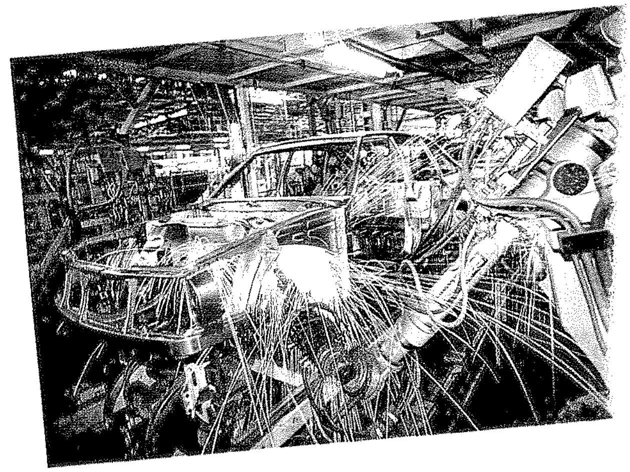
36. **Manufacturing** The Golden State Auto Company has a daily production quota of \$100,000 worth of cars. They produce two types of cars. Their compact model C is valued at \$10,000 and their luxury car L is valued at \$20,000. The equation $10{,}000C + 20{,}000L = 100{,}000$ describes the production quota.

   a. Make a graph of the quota equation.

   b. On December 17, the factory produced 5 compacts and 2 luxury cars. Was the company above, below, or on target with their quota? Write the equation or inequality that contains this point.

   c. On February 24, the factory produced 6 compacts and 2 luxury cars. Write the equation or inequality that contains this point.

   d. On March 5, the factory produced 9 compacts and 2 luxury cars. Write the equation or inequality that contains this point.



37. **Education** Your school counselor advises you that to be admitted to the college of your choice, you need to score at least 1200 for the combined verbal and mathematics parts of the SAT.

   a. Write an inequality that describes this situation.

   b. Graph the inequality.

   c. What restrictions, if any, are necessary to impose on the domain and range?

Mixed Review

38. Graph $y = [x] - 4$. (Lesson 2–6)

39. **Statistics** The table below shows the years of experience for eight encyclopedia sales representatives and the amount of sales during a given period of time. (Lesson 2–5)

Sales	\$9000	\$6000	\$4000	\$3000	\$3000	\$5000	\$8000	\$2000
Years	6	5	3	1	4	3	6	2

   a. Draw a scatter plot to show how the years of experience and the amount of sales are related.

   b. Write a prediction equation from these data.

   c. Predict the amount of sales for a representative with 8 years of experience.

40. Write an equation in slope-intercept form for the line with slope $\frac{2}{3}$ that passes through the point at $(6, -5)$. (Lesson 2–4)

41. **SAT Practice** **Grid-In** If n is a prime integer such that $2n > 19 > = \frac{7}{8}n$, what is one possible value of n?

42. Find the x-intercept and y-intercept of the graph of $4x + 5y = 10$. (Lesson 2–2)

43. **Geometry** The points at $(-2, 2)$, $(7, 1)$, and $(-2, 1)$ form three of the vertices of a rectangle. Find the coordinates of the fourth vertex. (Lesson 2–1)

For Extra Practice, see page 880.

INTERNET CONNECTION
Chapter Review For additional lesson-by-lesson review, visit: **www.algebra2.glencoe.com**

VOCABULARY

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Algebra

absolute value function (p. 104)
Cartesian coordinate plane (p. 64)
coordinate system (p. 64)
constant function (pp. 74, 103)
continuous function (p. 67)
dependent variable (p. 73)
direct variation (p. 103)
domain (p. 65)
family of graphs (p. 83)
function (p. 65)
greatest integer function (p. 104)
identity function (p. 103)
independent variable (p. 73)
linear equation (p. 73)
linear function (p. 74)
mapping (p. 65)
ordered pairs (p. 64)
origin (p. 64)
parent graph (p. 83)
point-slope form (p. 89)
quadrants (p. 64)
range (p. 65)
relation (p. 65)
slope (p. 80)
slope-intercept form (p. 88)
standard form (p. 73)
step functions (p. 103)
vertical line test (p. 65)
x-axis (p. 64)
x-intercept (p. 75)
y-axis (p. 64)
y-intercept (p. 75)

Discrete Mathematics

discrete function (p. 65)

Geometry

parallel lines (p. 83)
perpendicular lines (p. 83)

Problem Solving

look for a pattern (p. 80)

Statistics

best-fit line (p. 95)
prediction equation (p. 95)
regression line (p. 101)
scatter plot (p. 95)

UNDERSTANDING AND USING THE VOCABULARY

Choose the term from the list above that best completes each statement or phrase.

1. The equation of the line suggested by the points on a scatter plot is a ______________.
2. An ______________ is a linear function described by $f(x) = x$.
3. A ______________ is a function of the form $f(x) = b$, and the slope of the function is zero.
4. A linear function described by $f(x) = mx$, where $m \neq 0$, is known as a ______________.
5. An ______________ is closely related to a linear function, except that the graph of this function always forms a V-shape and is described by $f(x) = |x|$.
6. The ______________ of a linear function is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.
7. Two or more lines in the same plane having the same slope are ______________.
8. The ______________ is the set of all x-coordinates of the ordered pairs of a relation.
9. Two lines with slopes that are negative reciprocals of each other form right angles and therefore are ______________.
10. The set of all y-coordinates of the ordered pairs of a relation are known as the ______________.
11. The ______________ of a nonvertical line may be described in two ways, the first as the vertical change divided by the horizontal change and the second as $(y_2 - y_1)$ divided by $(x_2 - x_1)$.

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES	REVIEW EXERCISES

Upon completing this chapter, you should be able to:

- graph a relation, state its domain and range, and determine if it is a function (Lesson 2–1)

  $\{(-5, 6), (-3, -4), (-1, -6), (2, 6)\}$

  The domain is $\{-5, -3, -1, 2\}$.

  The range is $\{6, -4, -6\}$.

  It is a function because each element of the domain is paired with exactly one element of the range.

Use these exercises to review and prepare for the chapter test.

State the domain and range of each relation. Then graph the relation and identify whether it is a function or not.

12. $\{(4, -7), (-4, -7), (-4, 7), (4, 7)\}$
13. $\{(9, 1), (7, 2), (5, 3), (3, 4), (1, 5)\}$
14. $3x - 4y = -7$
15. $-2y + 4 = 1 - 5x$

- find values of functions for given elements of the domain (Lesson 2–1)

  If $f(x) = x^3 - 5$, find $f(-3)$.

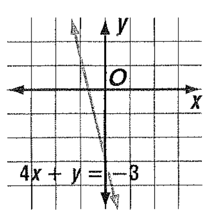
  $$f(-3) = (-3)^3 - 5 = -27 - 5 \text{ or } -32$$

Find each value if $f(x) = 4x^2 + 5x - 9$.

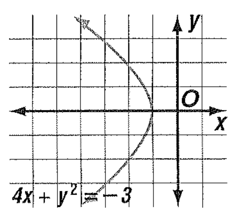
16. $f(6)$
17. $f(-2)$
18. $f(3y)$
19. $f(-2v)$

- identify equations that are linear and graph them (Lesson 2–2)

  $4x + y = -3$ is a linear equation.

  $4x + y^2 = -3$ is *not* a linear equation.

State whether each equation is linear. Write *yes* or *no*. If it is a linear equation, graph it.

20. $3x^2 - y = 6$
21. $2x + y = 11$
22. $y = -7$
23. $x^2 + y^2 = 25$

- write linear equations in standard form (Lesson 2–2)

  Write $2x - 6 = y + 8$ in standard form where A, B, and C are integers whose greatest common factor is 1.

  $2x - 6 = y + 8$

  $2x - y - 6 = 8$ *Subtract y from each side.*

  $2x - y = 14$ *Add 6 to each side.*

Write each equation in standard form where *A*, *B*, and *C* are integers whose greatest common factor is 1.

24. $y = 7x + 15$
25. $0.5x = -0.2y - 0.4$
26. $\frac{2}{3}x - \frac{3}{4}y = 6$
27. $-\frac{1}{5}y = x + 4$
28. $6x = -12y + 48$
29. $y - x = -9$

OBJECTIVES AND EXAMPLES

- determine the slope of a line (Lesson 2–3)

Determine the slope of the line that passes through $(-5, 3)$ and $(7, 9)$.

$$m = \frac{9-3}{7-(-5)}$$
$$= \frac{6}{12} \text{ or } \frac{1}{2}$$

REVIEW EXERCISES

Determine the slope of the line that passes through each pair of points.

30. $(-6, -3)$ and $(6, 7)$
31. $(5.5, -5.5)$ and $(11, -7)$
32. $(-3, 24)$ and $(10, -41)$

- write an equation of a line in slope-intercept form given the slope and one point (Lesson 2–4)

The slope-intercept form of the line that has a slope of $\frac{2}{3}$ and a y-intercept of 3 is $y = \frac{2}{3}x + 3$.

The standard form of this equation is $-2x + 3y = 9$.

Write an equation in slope-intercept form that satisfies each condition.

33. slope $= \frac{3}{4}$, passes through $(-6, 9)$
34. slope $= 2$, x-intercept $\frac{3}{2}$
35. passes through $(3, -8)$ and $(-3, 2)$
36. passes through $(0.35, 0.7)$ and $(0.7, 0.35)$

- write an equation of a line that is parallel or perpendicular to the graph of a given equation (Lesson 2–4)

An equation of a line parallel to $y = 2x - 2$ is $y = 2x + 1$.

An equation of a line perpendicular to $y = 2x - 2$ is $y = -\frac{1}{2}x + 1$.

Write an equation in slope-intercept form for each given situation.

37. passes through $(1, 2)$ and is parallel to the graph of $y = -3x + 7$
38. passes through $(-1, 2)$ and is parallel to the graph of $x - 3y = 14$
39. passes through $(3, 2)$ and is perpendicular to the graph of $4x - 3y = 12$
40. passes through $(1, 3)$ and is perpendicular to the graph of $y = -\frac{2}{3}x + \frac{11}{3}$

- draw a scatter plot and find a prediction equation (Lesson 2–5)

Construct a scatter plot for the given data. Then sketch the line that appears to best fit the points and find an equation of the line that best fits the data.

x	−1	0	0.5	1	2	3.5	4
y	3	3.5	1.5	2	0	−3.5	−2

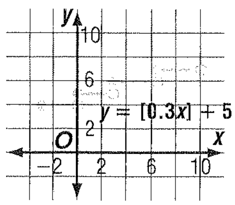
The slope-intercept form of the equation of the best-fit line is $y = -\frac{5}{4}x + \frac{17}{8}$.

41. On average, a person that is 70 inches tall (5′10″) weighs about 167 pounds. A person who is 62 inches tall (5′2″) weighs about 125 pounds. Let h represent the height, and let w represent the weight.
    a. Draw a scatter plot to show how height and weight are related.
    b. Find a prediction equation for the data.
    c. Predict the weight of a person who is 77 inches tall.
    d. Predict the height of a person who weighs 155 pounds.

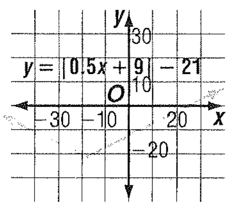
OBJECTIVES AND EXAMPLES

- identify and graph special functions

$y = [0.3x] + 5$ is a greatest integer function.



$y = |0.5x + 9| - 21$ is an absolute value function.



REVIEW EXERCISES

Identify each function as C for constant, D for direct variation, A for absolute value, or G for greatest integer function. Then graph each function.

42. $f(x) = |x| + 4$
43. $f(x) = [x] - 2$
44. $g(x) = \frac{2}{3}$
45. $h(x) = 3x$
46. $h(x) = [2x + 1]$
47. $g(x) = |x - 1| + 7$

2 − 1

1 + 7

- draw graphs of inequalities in two variables (Lesson 2–7)

Replace the inequality symbol with an equals sign to find the equation for the boundary. Graph the boundary, using a solid line for ≤ or ≥, or a dashed line for < or >. Test a point to determine which region to shade.

Graph each inequality.

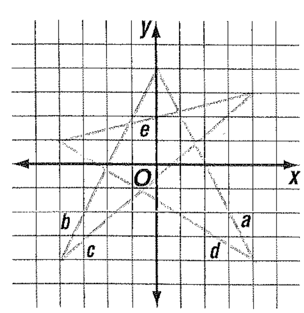
48. $y \leq 3x - 5$ $0 \leq -5$
49. $x > y - 1$
50. $y \geq |x| + 2$ $0 \geq 0+2$
51. $y + 0.5x < 4$

APPLICATIONS AND PROBLEM SOLVING

52. **Employment** Ley works at a clothing store for men. He earns \$7.00 an hour plus 50¢ for every item over 25 items that he sells. He works 40 hours a week. How much money will he make if he sells x items? (Lesson 2–6)

53. **Consumer Awareness** Monique's Fine Fashions allows its customers to charge their purchases on a delayed payment plan. When the customer receives a bill from Monique's, there is a \$5 charge for using the delayed payment plan, plus 2% interest on the purchase. Calculate the bill for a purchase of \$110. (Lesson 2–2)

54. **Geometry** What are the equations of the five lines that contain the sides of the star shown below? (Lesson 2–4)

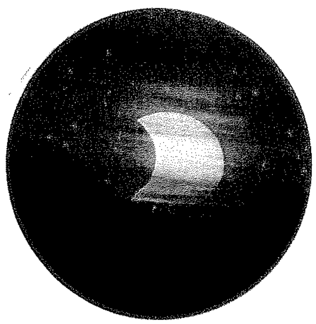


A practice test for Chapter 2 is provided on page 913.

ALTERNATIVE ASSESSMENT

COOPERATIVE LEARNING PROJECT

Predicting the Future The better we understand nature, the more accurately we are able to predict its behavior.



Ancient peoples had no understanding of the movements of the sun, the moon, or the stars. Predicting such events with accuracy was impossible. As a result, eclipses were believed to be a sign that the gods were angry. When astronomers discovered the cause of an eclipse and calculated the orbits of the sun and the other planets in our solar system, they were able to predict the timing of an eclipse with great accuracy.

In an effort to make predictions about natural occurrences that are not fully understood, scientists search for predictors or events which, for no apparent reason, seem to correlate with the occurrences. For example, geologist Ruth Simon discovered that cockroaches become more active a few hours preceding a major earthquake. Political analysts have noted that the candidate with the longer name wins nearly 75% of the presidential elections.

In this project, you will attempt to find a predictor that can be used to predict the behavior of some natural phenomenon. You will analyze several possible predictors to find the one with the best promise. Then you will use that one to make a prediction about the future.

Follow these steps to carry out your project.

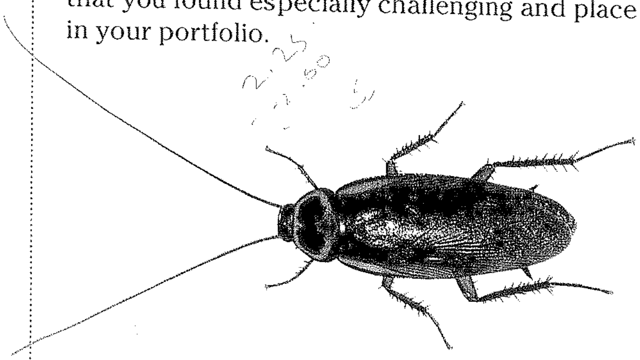
- Choose a phenomenon that interests your group, whose behavior cannot be predicted with complete accuracy. Be sure there is plenty of data available on the phenomenon you choose and that the phenomenon is not completely arbitrary in its behavior.
- Collect data on the phenomenon. Study it, looking for patterns, and discuss your findings with the rest of your group.
- Brainstorm with your fellow group members to find at least three possible predictors of the phenomenon you chose.
- Collect data on your predictors.
- Analyze your predictors. You may wish to draw scatter plots and find a prediction equation. You can use a graphing calculator to help you draw lines of regression. After you complete your analysis, choose the predictor that shows the best promise.
- Write a report summarizing your work as a group. Explain how you chose your predictors and how they were analyzed. Describe the results of your prediction. List any discrepancies found between predicted and actual results.

THINKING CRITICALLY

You have learned several ways to find the slope of a line: by using the vertical change divided by the horizontal change, by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, and by simply viewing the graph of a line. Is one way better than another? Write a one-page paper on the method you prefer, citing examples, and explaining why you prefer it.

PORTFOLIO

Select one of the assignments from this chapter that you found especially challenging and place it in your portfolio.

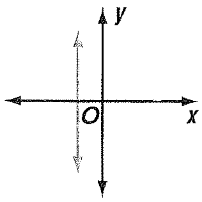


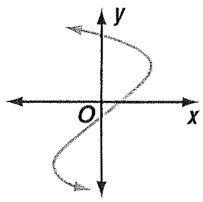
CHAPTERS 1–2

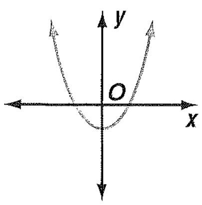
Section One: MULTIPLE CHOICE

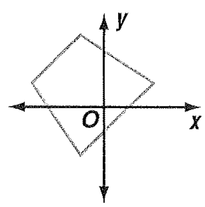
There are eight multiple-choice questions in this section. After working each problem, write the letter of the correct answer on your paper.

1. Determine which graph represents a function.

A.


B.


C.


D.


2. Choose the equation that is equivalent to $3(a + 2b) - c = 0$.

A. $a + 2b = c$
B. $b = \frac{c - 3a}{6}$
C. $a = \frac{2b - c}{3}$
D. $3a + 6b - 3c = 0$

3. What is the mean age of those in attendance at the Cruz family reunion if their ages are represented in the stem-and-leaf plot below?

Stem	Leaf
0	3 7
1	2 2 5 8 8
2	3 5 5 6 7
3	6
4	0 3 3 3
5	0 3
6	2 6 8

$4 \mid 3 = 43$ years

A. 32.5 years
B. 26.5 years
C. 43 years
D. 102 years

4. Evaluate the expression $\left(\frac{a}{x} + b\right)^2 - gh$ if $a = 9$, $b = 2$, $g = 4$, $h = 6$, and $x = 3$.

A. 19
B. -11
C. -3
D. 1

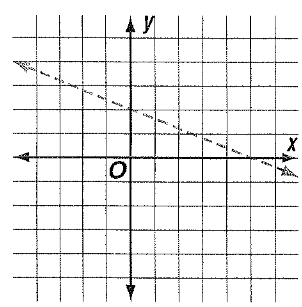
5. Determine which description of the graph of the equation $2x - 4y = 8$ is true.

A. It is parallel to the graph of the equation $2x - y = 8$.
B. Its slope is 2.
C. The point (6, 1) lies on it.
D. It is perpendicular to the graph of the equation $4x - 8y = 16$.

6. Choose the statement that is false for all real numbers a, b, and c except 1.

A. If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
B. $a \cdot \frac{1}{a} = 1 \cdot a$
C. If $a = b$ and $b = c$, then $a = c$.
D. $(a + b) + c = a + (b + c)$

7. Choose the inequality that describes the graph below.



A. $2x + 5y < 10$
B. $y > 2 - \frac{2}{5}x$
C. $y \geq -2x - 5$
D. $5x + 2y \leq 10$

8. Choose the type of function that represents the following relationship. For every yard Melissa mows, she earns $7.00.

A. absolute value
B. step
C. greatest integer
D. direct variation

THE PRINCETON REVIEW

Test-Taking Tip If you use the acronym PEMDAS to recall the order of operations, remember that multiplication and division are to be performed at the same time. Evaluate whichever comes first, reading left to right. The same is true of addition and subtraction.

Section Two: SHORT ANSWER

This section contains seven questions for which you will provide short answers. Write your answer on your paper.

9. Solve $8|2b - 3| = 64$.

10. The width of a rectangular rug is four feet more than one third its length. The perimeter is 64 feet. What are the length and width of the rug?

11. Intercity Car Rental offers a mid-size car that costs 20¢ per mile plus an initial fee of \$20. To rent the same car from Big Wheels Car Rental, each mile costs 23¢ with an initial fee of \$10. Barry plans to drive about 875 miles. Which company provides the cheaper offer, and how much will Barry save with this offer?

12. Solve the compound inequality
$\frac{x+8}{4} - 1 > \frac{x}{3}$ and $7 < 2x - 11$.
Then graph the solution set.

13. Determine the slope of a line perpendicular to the line that passes through $(-3, -1)$ and $(4, -7)$.

14. The marching band at Scoring High School is raising money for new uniforms by selling candy. They need to earn at least \$1025 to receive a 10% discount on the price of uniforms. During the first week of the sale, the amounts turned in by the music classes were \$125, \$86, \$98, \$72, \$63, and \$135. What is the minimum amount the band must raise to receive the 10% discount?

15. Describe the steps you would take to graph the equation $3y - 2x = 15$.

Section Three: COMPARISON

This section contains five comparison problems that involve comparing two quantities, one in column A and one in column B. In certain questions, information related to one or both quantities is centered above them. All variables used represent real numbers.

Compare quantities A and B below.

- Write A if quantity A is greater.
- Write B if quantity B is greater.
- Write C if the two quantities are equal.
- Write D if there is not enough information to determine the relationship.

Column A	Column B
16. the slope of a line perpendicular to the line that passes through $(-3, 2)$ and $(5, -1)$	$\frac{8}{3}$
$g = 3, k = -2, m = 5, p = 4$	
17. $g^2 + (k - p) \div g$	$\left(\frac{p}{k} + m\right)^2 \div g + k$
18. $y - 3 \geq 4x$	$3x + 5 < 10$
19. the median of 41, 33, 12, 38, 27, 19	the mean of 14, 23, 35, 18, 32, 27, 39
$a > b, b = c$	
20. $(b + a) - d$	$-d + a + c$

interNET CONNECTION

Test Practice For additional test practice questions, visit: **www.algebra2.glencoe.com**

CHAPTER 3

Solving Systems of Linear Equations and Inequalities

What You'll Learn

In this chapter, you will:

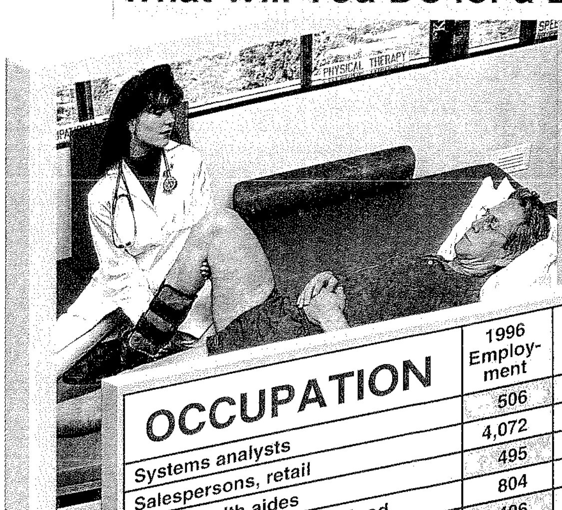
- solve systems of equations in two or three variables,
- solve systems of inequalities,
- use linear programming to find maximum and minimum values of functions, and
- solve problems by solving a simpler problem.

Why It's Important

Real World

Employment In the table, you can see that the number of jobs for each profession listed is expected to increase by the year 2006. You can see which professions have the same percent of change for the 10-year period. For example, cooks and secondary teachers have the same percent of change for the 10-year period. In Lesson 3–1, you will learn that equations where the change is the same are parallel lines when graphed.

What Will You Do for a Living?



OCCUPATION	1996 Employ-ment	2006 Employ-ment	Percent Change (1996-2006)
Systems analysts	506	1,025	103%
Salespersons, retail	4,072	4,481	10
Home health aides	495	873	76
Cooks, short order/fast food	804	978	22
Teachers, secondary	1,406	1,718	22
Truck drivers light and heavy	2,719	3,123	15
Child care workers	830	1,129	36
Computer engineers	216	451	109
Guards	955	1,175	23
Medical assistants	225	391	74
Maintenance repairers	1,362	1,608	18
Food preparation workers	1,253	1,487	19

Source: *Bureau of Labor Statistics, Occupational Outlook Program*
(Employment is given in thousands of jobs.)